Homework #11
Nonlinear dynamics and chaos

1. **Embedding dimension:** Reproduce the heuristic argument from Ott's book explaining why embedding dimension using delay coordinates must be at least $2d+1$ where d is the actual dimension of the embedded attractor.

2. **Spectrum and harmonics in a nonlinear oscillations:** using as a base the Matlab program `spectrum_harmonics.m` from the course home page, plot the time series and the spectrum of a time series such as $x(t) = a\sin(\omega_1 t) + b\sin(\omega_2 t)$. How do the heights of the spectrum peaks correspond to the amplitudes a, b? why? Next, plot the time series and the spectrum for $x(t) = a\sin(\omega_1 t)$, and then for $x(t) = a\sin^3(\omega_1 t)$ and $x(t) = a\sin^5(\omega_1 t)$. Describe your results, and use standard trigonometric relations to derive an analytic expression for $x(t)$ that explains the appearance of additional peaks for the nonlinear time series. The powers of sine make the time series nonlinear as the oscillation is no longer a simple sine function. The additional spectrum peaks are called harmonics and are typical of nonlinear time series.

3. **Intermittency type I:** Show that the generic form of the intermittency map studied in class, $x_{n+1} = \varepsilon + x_n + x_n^2 = f(x_n)$ is self-similar, by deriving the full explicit expression for $F(x_n) = f(f(x_n))$, and showing that to lowest order in x_n it is similar to $f(x_n)$.

4. **Length of non-chaotic intervals for type III intermittency:** Find a map that appropriately describes a type III trapping region in the intermittency route to chaos. Explain why this map is the right one (you can find hints in Schuster Table 6 section 4.4 (1st edition) or Ott's book problem 3, page 303). Use $x_{n+2} - x_n \approx dx/dn$ (why is it appropriate to use this approximation in the trapping region? why $n+2$ rather than $n+1$?) and show that the length of non-chaotic intervals in this case behaves like ε^{-1}. Plot the iterates of the map in the trapping region.