Homework #4

Nonlinear dynamics and chaos

1. Do problem 6.6.8 from Strogatz, page 191.

2. Find fixed points, draw vector field around them, and calculate their index for the system:

$$\dot{x} = xy; \qquad \dot{y} = x + y$$

3. Do the following systems have a limit cycle solution?

a. (Construct a Lyapunov function...):

$$\dot{x} = y - x^3; \qquad \dot{y} = -x - y^3$$

b. If $\dot{x} = f(x,y); \dot{y} = g(x,y)$, is a gradient system, $\dot{\mathbf{x}} = -\nabla V$, where $\mathbf{x} \equiv (x,y)$, show that $f = -\partial V/\partial x; g = -\partial V/\partial y$. Similarly, show that if $\partial f/\partial y - \partial g/\partial x = 0$ this is a gradient system.

Using the above, does the following system have a limit cycle solution:

$$\dot{x} = y + 2xy; \qquad \dot{y} = x + x^2 - y^2$$

Optional/ extra credit:

(you can do each of the following problems in addition to, or instead of, one of questions 1-3 above)

4. Estimate the period of the limit cycle of the following system for $k \gg 1$:

$$\ddot{x} + k(x^2 - 4)\dot{x} + x = 1$$

5. A glider: Let $v =$ speed of glider and $u =$ angle flight path makes with the horizontal. In the absence of drag (friction), the dimensionless equations of motion are:

$$dv/dt = -\sin u; \quad v du/dt = -\cos u + v^2$$

a. Using numerical integration, sketch the trajectories on a slice of the u-v phase plane between $-\pi < u < \pi, v > 0$.

b. Obtain an exact expression for the trajectories

c. Using your result in part b, obtain an exact expression for the separatrix in this system.

d. What does the flight path of the glider look like for motions inside the separatrix versus motions outside the separatrix? Sketch the glider's flight path in both cases.

e. If there is also a drag force, then

$$dv/dt = -\sin u - Dv^2; \quad vdu/dt = -\cos u + v^2.$$

Describe what happens then.