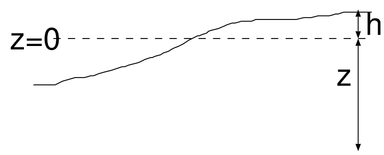
Homework #2
Introduction to physical oceanography

1. Calculating the transport of the gulf stream and circumpolar current: find and plot maps of the sea surface height in the area of the Gulf Stream and the Drake Passage. (One place to look is under the home page of Michael A. Chupa, `http://www.erc.msstate.edu/~chupa/`, see the picture `http://www.erc.msstate.edu/~chupa/istv/global16.jpg` which is a model result rather than data, but appropriate for our purpose here) What is the sea surface height difference across the gulf stream and circumpolar current? Ignoring density variations, Show that the pressure in the ocean is then

$$p(x,z) = g\rho_o(h(x) - z)$$



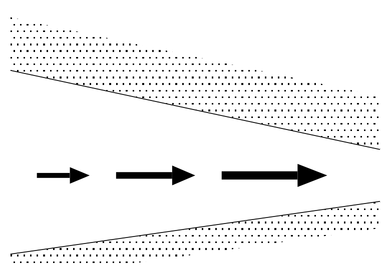
where $h(x)$ is the surface height as function of the east-west coordinate, ρ_0 is the constant water density, g is gravity, and z is the depth. Use one of the two geostrophic equations

$$fv = \frac{1}{\rho_0}\frac{\partial p}{\partial x} \quad (1)$$

$$fu = -\frac{1}{\rho_0}\frac{\partial p}{\partial y} \quad (2)$$

to calculate how much water is transported by the gulf stream/ circumpolar current per second. Hint: the transport is $\Delta X \int_{z=0}^{-H} v dz = \Delta x H v$ where Δx is the width of the Gulf stream/ circumpolar current, H is the maximum depth to which the current extends; assume the velocity is independent of depth within this range, and that the currents extend to $H = 2km$ depth for the Gulf stream and to the bottom in the circumpolar current.

2. (corrected...) Consider accelerating fluid particles in a narrowing channel.



Assume velocity is only in the x-direction, along the channel. Let the Lagrangian location be given by $x(t) = x_0 e^{ct}$ where x_0, c are constants and t is time. (i) Find the Lagrangian velocity and acceleration for such a particle. (ii) Find the Eulerian velocity field as function of x along the channel. (Hint: use the expression for $x(t)$ to find $t(x)$ and then write the velocity as function of x). (iii) Find the Eulerian acceleration using the expression for the material derivative. Note: Eulerian and Lagrangian results should agree.

3. Follow the argument in Knauss and explain why the term $\vec{\Omega} \times \vec{\Omega} \times \vec{\mathbf{x}}$ is negligible relative to the Coriolis acceleration $2\vec{\Omega} \times \vec{\mathbf{u}}$.

4. **An optional challenge problem:** Calculate the equilibrium shape of the surface of water in a bucket rotating about its axis with angular velocity Ω if it contains water of uniform density. Interpret the results in terms of the forces acting on the water. (Hint: The water is assumed to rotate with the bucket, so that it is at rest ($u = 0$) in the appropriate rotating frame. Use the equations derived in class to calculate the pressure, $P(x, y, z)$, and then use the boundary condition $P = 0$ at the surface to calculate the shape of the surface as $z = h(x, y)$.)