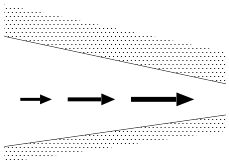
Homework #2
Introduction to physical oceanography
(due Wed Oct 12)

1. "Abyssal recipes":

   (a) Plot the vertical ocean temperature profile in the Pacific Ocean away from any continents, at a latitude of 45N, and at the equator; use the "Ingrid" ocean data home page at Columbia University
   (`http://iridl.ldeo.columbia.edu/SOURCES/.LEVITUS94/.ANNUAL/.temp/figviewer.html?plottype=line`)

   (b) Find and plot using Matlab an exponential profile that seems to fit each of the above profiles reasonably well at depth ranges from 500m to the ocean bottom. Assume the vertical velocity is $w = 10^{-4} cm/sec$, and find the appropriate vertical mixing coefficients. Can you fit both locations with the same value of κ, w? What do you conclude from this?

2. Consider an accelerating fluid parcel flowing along the center of a slowly narrowing channel.



   Assume the velocity is only in the x-direction, $\vec{u} = (u(x), 0, 0)$, along the channel and that it is constant in time. Use the following steps to calculate the acceleration of fluid parcels using the material derivative of the Eulerian velocity and the Lagrangian point of view.

   (a) Let the channel width be $W = W_0/x$, and assume the water depth is constant and equal to H. Given that the volume flux is constant along the channel, calculate the Eulerian velocity $u(x)$ in the x direction along the channel.

   (b) Find the Lagrangian location $x(t, x_0)$ for a fluid parcel that was at x_0 at $t = 0$;

   (c) Find the velocity and acceleration for the same Lagrangian parcel.

   (d) Find the acceleration using the expression for the material derivative of the Eulerian velocity. Note: Eulerian and Lagrangian results should agree.

3. **Challenge problem, optional:** Consider the Eulerian velocity field (u_1, u_2, u_3) in the Cartesian coordinate system (x_1, x_2, x_3)

$$\begin{aligned} u_1 &= U_0 \\ u_2 &= V_0 \cos(k[x_1 - ct]) \\ u_3 &= 0 \end{aligned}$$

   (a) Find the streamlines of the flow.

   (b) Find the trajectory of a fluid element which at $t = 0$ is at $(x_1, x_2, x_3) = (X_1, X_2, X_3)$.

   (c) What is the wavelength of the Eulerian streamline'?

   (d) What is the wavelength of the Lagrangian trajectory?

   (e) Discuss the case $U_0 - c \ll U_0$.