Homework #7
Introduction to physical oceanography

1. For each of the following cases

   (a) A solid body rotation ($u_r = 0; u_\theta = \omega r$)

   (b) An "irrotational" vortex ($u_r = 0; u_\theta = \Lambda/(2\pi r)$); why is it called irrotational?

   (c) a velocity structure that is given by ($u_r = 0; u_\theta = a\exp(-r/r_0)$);

   please do the following

   (a) Write the velocity field in Cartesian coordinates (x, y) instead of polar coordinates (u_r, u_θ).

   (b) Plot the velocity vectors as function of (x, y) using Matlab's quiver function. You may assume $\Lambda = r_0 = a = \omega = 1$ for the plot.

   (c) Calculate the vorticity using the expression for curl in Cartesian coordinates.

   (d) Plot a contour of the vorticity field for the third case above using the Matlab "contourf" function; add a color bar as well.

   Are the results for the first two cases the same as obtained in class using cylindrical coordinates?

2. **Wind curl and Sverdrup relationship:** given the following wind stress field:

$$\vec{\tau} = (\tau^{(x)}, \tau^{(y)}) = \left(\tau_0 \cos\left[\frac{\pi}{20}(40 - \theta)\right], 0\right)$$

   where $\tau_0 = 0.7 dyn/cm^2$ and θ is the latitude in degrees, varying from 20N to 60N.

   (a) Plot the wind stress as arrows as function of both x and y using Matlab's quiver function.

   (b) Calculate the north-south velocity field using the Sverdrup relationship in units of cm/sec.

   (c) Assuming that the width of the ocean is 5000km and the depth of the wind driven circulation is $H = 1000m$, calculate and then plot using Matlab the total transport in units of Sverdrups as function of latitude.

   (d) Describe the ocean-interior flow you obtain as function of latitude. Explain how this flow is consistent with the existence of a "sub-polar gyre" and a "sub-tropical gyre" in the North Atlantic and North Pacific oceans.