The Mediterranean Outflow as an Example of a Deep Buoyancy-Driven Flow

ELI TZIPERMAN[1]

Department of Earth, Atmospheric, and Planetary Sciences, Massachusetts Institute of Technology, Cambridge

Motivated by the Mediterranean outflow into the North Atlantic, a simple diffusive layer model of a deep buoyancy-driven circulation driven by a middepth inflow from the eastern boundary is developed. Cross-interfacial velocities are allowed and are related to the stratification through a density equation. The horizontal circulation, cross-interfacial (vertical) velocities, and density stratification are all coupled and are determined by the model as part of the solution. The importance of water masses injected into the ocean interior in driving the deep circulation is demonstrated. The inflow from the eastern boundary turns northward before reaching the western boundary current region and flows along the eastern boundary. The westward distance traveled by the inflow before turning northward can be derived by considering the dissipation of long westward traveling Rossby waves by the vertical density diffusion. Layers above and below the inflow are also set in motion, and their circulation is southward along the eastern boundary, opposite to the direction of the circulation in the inflow layer. The vertically integrated circulation of the model is similar to that in a Stommel-Arons model with the mass source on the eastern boundary: Most of the inflow flows to the western boundary current region, spreads there along the boundary, and reenters the interior as a broad northward current.

1. INTRODUCTION

The understanding of buoyancy-driven flows is important for determining the effects of buoyancy forcing on both the wind-driven and the deep circulation, but their modeling is complicated by many difficulties. Analytic solutions are difficult to obtain because of the high-order partial differential equations describing diffusive flows, and numerical methods suffer from the very long convergence times due to the long diffusive adjustment time scales.

In this work a simple model of a deep buoyancy-driven circulation is presented. The physical problem modeled is of a middepth inflow from the eastern boundary and the resulting large-scale interior circulation. The dynamics are geostrophic, hydrostatic, mass conserving, and diffusive. The mathematics is simplified by using layer formulation instead of continuous stratification. This allows us to include most of the important physical mechanisms relevant to diffusive flows. The model is generally motivated by the Mediterranean outflow into the North Atlantic.

Mixing, advection, and density stratification are all coupled and determined by the model. Previous efforts to model the interior buoyancy-driven circulation have usually dropped one or more of these ingredients by specifying them and proceeded to solve for the rest.

Olson [1985] used a density equation of the form $d\rho/dt = G$ where G is the density flux convergence. The horizontal structure of G was related to the heating or cooling at the surface by the atmosphere, while the vertical structure of G was specified and assumed not to depend on the stratification. *Rhines* [1986], *Pedlosky* [1986], and *Luyten and Stommel* [1986] all specified the interior cross-isopycnal velocity field and calculated the resulting horizontal circulation patterns in order to study the effects of buoyancy forcing on the wind driven circulation. *Arhan* [1987] used the same approach as Luyten and Stommel to model the effects of double diffusion on the dynamics of the Mediterranean outflow, by specifying the cross-interfacial velocity in the interior.

Specifying the interior cross-isopycnal velocities or relating them directly to atmospheric heating does simplify the problem but also causes difficulties. One does not expect the direct atmospheric heating to penetrate beyond the depth of the mixed layer and directly affect the deep flows, while observations indicate that interior mixing depends on the interior stratification through a density equation of the form $\mathbf{u} \cdot \nabla\rho = (\kappa\rho_z)_z$ [*Gargett*, 1984]. In the model presented below, the cross-isopycnal velocities (which force the horizontal circulation) are calculated from the density stratification using the density equation.

Other approaches to modeling buoyancy-driven flows include linearization of the equations about some mean specified state [e.g., *Gill*, 1985] and similarity solutions for the diffusive thermocline equations (*Veronis* [1969] or more recently *Young and Ierley* [1986]). In both cases the mathematical simplification involved enables one to obtain continuous solutions to the problem, but the physics suffers limitations. These models include a diffusion term in the density equation and therefore allow the model to determine the forcing of the horizontal circulation. But the average vertical stratification is still specified in the linearized case, and the similarity solutions are very restricted by the assumed similarity form.

In the following sections the layer model is formulated (section 2), equations of motion are derived (section 3), and then a series of problems of increasing complexity are solved. First, in section 4, the ideal case (no diffusion) is solved. Then the diffusive case of three moving layers is solved, with the vertical density equation $w\rho_z = \lambda_v\rho_{zz}$ only (section 5), and finally the effects of horizontal advection of density are included (section 6). The relation of the results to different observations is discussed in section 7, and some of the more important conclusions are summarized in section 8.

The purpose of the model is to demonstrate the general physical principles governing deep buoyancy-driven flows rather than to realistically model the Mediterranean outflow. The model does suggest, however, a possible mechanism for the spreading of the Mediterranean water and makes clear

[1]Now at Isotope Research Department, Weizmann Institute of Science, Rehovot, Israel.

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what parameters one needs to calculate from the data to better understand such a circulation. A one-moving-layer diffusive solution for the model developed here, as well as some more detailed results, can be found in the work by *Tziperman* [1987].

2. THE MODEL

The layer model used is shown in Figure 1. The inflow from the eastern boundary is confined to a limited latitude band, $y_1 < y < y_2$, in one layer (layer 0) only, which is assumed to be below the wind-driven circulation and above the bottom water circulation. No transport of water across the basin boundaries (e.g., cross-equatorial flow) is allowed, and to keep the total amount of water in layer 0 constant, the water entering layer 0 from the eastern boundary has to leave this layer through interior cross-interfacial fluxes. These fluxes are analogous to cross-isopycnal fluxes in a continuously stratified model and are present in the model because diffusion of heat and salt across the interfaces between layers is allowed and is represented by a diffusion term in the density equation.

The vertical circulation in the model can be described schematically as follows. Deep water masses are formed in some polar basin, sink to the bottom, and spread in the ocean interior. This water then upwells through cross-isopycnal velocities balanced by diffusion and appears in the model as an upwelling across the lower interfaces of the model. At the depth of layer 0 this upwelling is joined by the water coming from the eastern boundary, making the total upwelling at the top of layer 0 larger than that at the bottom. This larger upwelling leaves the upper layers of the model; some of it flows into the marginal sea to reappear later as the inflow from the eastern boundary; a larger portion flows poleward, where it loses heat and sinks to the bottom. Western boundary currents are needed to close the interior horizontal circulation and to connect the interior with the region of bottom water formation. The effects of mixing and cross-isopycnal fluxes in the western boundary current region are assumed negligible in comparison to those of mixing in the entire interior of the basin.

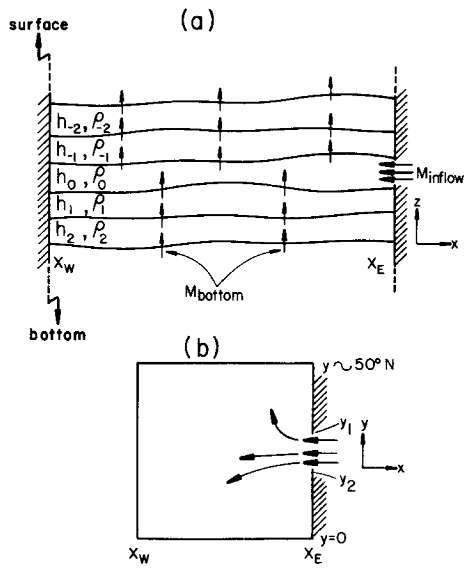

Fig. 1. A schematic picture of the inflow and layers (*a*) from above and (*b*) in a zonal section.

The dynamics we use are geostrophic, hydrostatic, and diffusive. The equations for the nth layer are

$$fu_n = -\frac{1}{\rho_0} p_{ny} \tag{1a}$$

$$fv_n = \frac{1}{\rho_0} p_{nx} \tag{1b}$$

$$p_{nz} = -g\rho_n \tag{1c}$$

$$\nabla \cdot \mathbf{u}_n = 0 \tag{1d}$$

where $x = R\phi \cos\theta$, $dy = R\, d\theta$, (ϕ, θ) are the longitude and latitude, $\mathbf{u}_n = (u_n, v_n, w_n)$, and R is the radius of the Earth. The function $f(y) = 2\Omega \sin\theta$ is the Coriolis acceleration, and we also use $\beta(y) = df/dy$. The β plane approximation is not made, and both f and β are functions of latitude. The density equation is

$$u\rho_x + v\rho_y + w\rho_z = \lambda_H \nabla_H{}^2 \rho + \lambda_V \frac{\partial^2 \rho}{\partial z^2} \tag{2}$$

and will be expressed later in terms of the layer thickness instead of continuous stratification. There are two observations we can make to simplify the problem.

First, following *Warren* [1977], consider the following simple scaling argument showing that the dominant balance in the density equation for the middepth circulation is simply

$$w\rho_z = \lambda_V \frac{\partial^2 \rho}{\partial z^2} \tag{3}$$

Let $\Delta_H \rho$ and $\Delta_V \rho$ be the variations of the density on the horizontal (L) and vertical (D) scales of motion, respectively. For the middepth circulation, where the density surfaces are nearly flat, the ratio $\varepsilon \equiv \Delta_H\rho/\Delta_V\rho$ is a small parameter (about 0.2 according to Warren). Nondimensionalizing the density equation (2) and using the scales $(u, v) \sim U$ and $w \sim UD/L$, we find that the nondimensional density equation is

$$\varepsilon(u\rho_x + v\rho_y) + w\rho_z = \left(\frac{\lambda_V}{WD}\right)\frac{\partial^2 \rho}{\partial z^2} + \varepsilon\left(\frac{\lambda_H}{UL}\right)\nabla_H{}^2 \tag{4}$$

With

$$W = 10^{-5} \text{ cm/s} \qquad D = 1 \text{ km} \qquad L = 5000 \text{ km}$$

$$\lambda_H = 5 \times 10^6 \text{ cm}^2\text{s}^{-1} \qquad \lambda_V = 1 \text{ cm}^2 \text{ s}^{-1}$$

the leading order balance in (4) is the vertical density equation (3). In section 6 we use a perturbation expansion in powers of ε and examine the effects of the horizontal diffusion and advection which are ignored now. In the same section, ε is also expressed in terms of the forcing (inflow) parameters, and the explicit conditions that make it small are made clear. It turns out that ε is small when the transport of the inflow is weak. Writing (3) in density coordinates and then in finite difference form, in terms of the layer thickness h_n, we have

$$w_n = \lambda_V \frac{\Delta_{n+1}\rho/h_{n+1} - \Delta_n\rho/h_n}{\rho_{n+1} - \rho_n} \tag{5}$$

where w_n is the vertical velocity across the interface between layers n and $n+1$, ρ_n is the density of the nth layer, and $\Delta_n\rho$ is the density range represented by the nth layer. See *Tziperman* [1986] for a more detailed derivation of the above parameterization of diffusion in layer models.

To further simplify the problem, it is assumed that the horizontal velocity fields above and below the inflow decay away from layer 0. For layers far enough from layer 0 the layer thicknesses then become uniform in (x, y) and by (5) so does the cross-interfacial upwelling w which is also depth independent. Although we were not able to prove that such a decay must occur, it can be justified heuristically by considering the forcing of each of the layers. It was already mentioned that the total upwelling at the top of layer 0 has to be larger than that at the bottom of this layer in order to balance the mass input by the inflow. This makes $w_{0,\text{top}} - w_{0,\text{bottom}}$ (or w_{0z}) positive, therefore forcing a northward flow in the interior through $\beta v = f w_z$. The circulation in layer 0 is a combination of the westward flow entering from the eastern boundary and the northward flow induced by $w_z > 0$. The motion in layer 0 induces variations in the thickness of that layer and of adjacent ones.

Consider next the situation in layers -1 and $+1$. Because there is no inflow into these layers, the horizontally averaged $w_{n,\text{top}} - w_{n,\text{bottom}}$ vanishes. Still, the variations in the thickness of these layers due to the motion in layer 0 induce local variations in $w_{n,\text{top}} - w_{n,\text{bottom}}$ through the density equation (5). Consequently, these layers are moving. Because the direct forcing due to the inflow and that due to the requirement that $\iint_{\text{whole basin}}(w_{n,\text{top}} - w_{n,\text{bottom}})\, dx\, dy > 0$ are missing now, the circulation in layers $+1$ and -1 and the variations in their thickness are weaker than for layer 0. Considering layers which are even further away from the inflow, we see that their circulation is even weaker, and their thickness becomes uniform in (x, y).

We assume below that the horizontal velocities vanish in layers far enough above and below the inflow and that only a finite number of layers around the inflow at layer 0 are at motion. (Note again that in the diffusive case only the horizontal velocities vanish far above and below the inflow, while the vertical velocity becomes uniform in (x, y), and constant in z.) This assumption will enable us to solve for the circulation in the layers assumed moving. In the diffusive solution to the three-moving-layer model (section 5) the circulation above and below the inflow is weaker than that at the inflow depth. This confirms the above argument about the decay of the solution away from the inflow. Ideally, the height scale of the circulation driven by the inflow should be determined by the model, but we are forced to assume only a finite number of moving layers due to the difficulty in solving for a large number of moving layers.

3. Equations of Motion for the Three-Moving-Layer Case

As explained in section 2, the horizontal velocity and pressure gradients are expected to weaken with increasing distance from layer 0. We now assume that the horizontal velocities vanish for all but three layers (-1, 0, and 1) and that the pressure gradients in layers -2 and $+2$ are already very small and can be taken to be zero. The actual pressure gradients in layers -2 and $+2$ are not exactly zero, and they affect the circulation in layers -1, 0, and 1. The solution derived below for the circulation in the three moving layers is valid as long as $|\nabla_H p_{\pm 2}| \ll |\nabla_H p_{0,\pm 1}|$. Physically, the vertical decay of the horizontal velocity fields should be determined by the model itself. The solution is truncated somewhat arbitrarily here because of the technical difficulty of solving for a large number of moving layers. The truncation is justified if the addition of more moving layers does not drastically change the circulation found when these layers were assumed at rest.

The equations of motion are given by (1), and the vertical diffusion equation (5) is used. In the ideal case discussed in section 4 the total three-dimensional velocity field vanishes for the layers assumed at rest. In the presence of diffusion (section 5), only the horizontal velocities vanish for the layers assumed at rest, while the vertical velocity is independent of depth but is not necessarily zero. The diffusive solution for the resting layers is analogous to an exponential density profile in the continuous diffusive case: $\rho = \exp(z\lambda_V/\bar{w})$, $\bar{w} = \text{const}$, and $u = v = 0$.

From the hydrostatic equation (1c)

$$\frac{1}{\rho_0} \nabla_H p_{n+1} = \frac{1}{\rho_0} \nabla_H p_n + \gamma_n \nabla_H z_n(x, y) \tag{6}$$

where $\gamma_n \equiv g(\rho_{n+1} - \rho_n)/\rho_0$ and $z_n(x, y)$ is the height of the interface between the n and $n+1$ layers. Because layers other than -1, 0, and 1 are at rest, $\nabla_H p_n = 0$ for $n \neq -1, 0, 1$. Using (6), we find

$$\nabla_H z_n(x, y) = 0 \qquad n \neq -2, -1, 0, 1 \tag{7}$$

so that only five layers (-2, -1, 0, 1, and 2) have varying thicknesses which need to be solved for. By (7) the thickness of all other layers is uniform in (x, y). From (6) we also find

$$\frac{1}{\rho_0} \nabla_H p_{-2} = 0 = -\gamma_{-2}\nabla_H(h_2 + h_1 + h_0 + h_{-1})$$
$$-\gamma_{-1}\nabla_H(h_2 + h_1 + h_0) - \gamma_0\nabla_H(h_2 + h_1) - \gamma_1\nabla_H h_2 \tag{8a}$$

$$\frac{1}{\rho_0} \nabla_H p_{-1} = -\gamma_{-2}\nabla_H h_{-2} \tag{8b}$$

$$\frac{1}{\rho_0} \nabla_H p_0 = -\gamma_1\nabla_H h_2 - \gamma_0\nabla_H(h_2 + h_1) \tag{8c}$$

$$\frac{1}{\rho_0} \nabla_H p_1 = -\gamma_1\nabla_H h_2 \tag{8d}$$

From (1) the vorticity equation for the moving layers is [*Pedlosky*, 1979]

$$\beta v_n(x, y) h_n(x, y) = f[w_{n-1}(x, y) - w_n(x, y)] \tag{9}$$
$$n = -1, 0, 1$$

Using (1) and (8), we can write (9) as

$$\frac{\partial h_2}{\partial x} = -\frac{f^2}{\beta\gamma_1}(w_0 - w_1)\frac{1}{h_1} \tag{10a}$$

$$\frac{\partial h_1}{\partial x} = -\frac{f^2}{\beta\gamma_0}(w_{-1} - \omega_0)\frac{1}{h_0} - \left(\frac{\gamma_1}{\gamma_0} + 1\right)\frac{\partial h_2}{\partial x} \tag{10b}$$

$$\frac{\partial h_{-2}}{\partial x} = -\frac{f^2}{\beta\gamma_{-2}}(w_{-2} - w_{-1})\frac{1}{h_{-1}} \tag{10c}$$

The vertical velocities can be written in terms of the h_n using the density equation (5). Denoting the thickness of the layers on the eastern boundary south of the inflow by H_n, we can derive two more equations from (7) and (8), expressing the assumptions that $\nabla_H p_{-2} = 0$ and that the interfaces enclosing layers $-2, \cdots, 2$ are flat

$$p_{-2} = -\gamma_{-2}(h_2 + h_1 + h_0 + h_{-1}) - \gamma_{-1}(h_2 + h_1 + h_0) - \gamma_0(h_2 + h_1) - \gamma_1 h_2 = \text{const}$$
$$= -\gamma_{-2}(H_2 + H_1 + H_0 + H_{-1}) - \gamma_{-1}(H_2 + H_1 + H_0) - \gamma_0(H_2 + H_1) - \gamma_1 H_2 \qquad (10d)$$

$$\sum_{n=-2}^{2} h_n(x, y) = \text{const} = H_{-2} + H_{-1} + H_0 + H_1 + H_2 \qquad (10e)$$

so that we have five equations (equation (10*a*)–(10*e*)) for the five unknowns $h_{-2}, \cdots, h_2$. Using (10*d*) and (10*e*) to express $h_0(x, y)$ and $h_{-1}(x, y)$ in terms of h_{-2}, h_1, h_2, and the H_n, and substituting them into (10*a*)–(10*c*), the set (10*a*)–(10*e*) can be reduced to three coupled first-order ordinary differential equations (ODEs) for h_{-2}, h_1, and h_2 of the form

$$\frac{\partial h_n}{\partial x} = \mathscr{F}_n[h_{-2}(x, y), h_1(x, y), h_2(x, y), H_m, f(y)] \qquad (11)$$

$$n = -2, 1, 2$$

The layer thickness on the eastern boundary is determined from the specified inflow. Let the inflow velocity as function of latitude y be (see Figure 1)

$$u_0(x_e, y) = U_0 f_0/f \qquad y_1 < y < y_2$$
$$u_0(x_e, y) = 0 \qquad \text{elsewhere} \qquad (12)$$

where $f_0 = f[(y_1 + y_2)/2]$ and $U_0 = \text{const}$. Using $fu_0 = (-1/\rho_0)p_{0y}$ and (8) with (12), we find

$$h_2(x_e, y) = H_2 \qquad (13a)$$

$$h_1(x_e, y) = H_1 + \frac{U_0 f_0}{\gamma_0}(y - y_1) \qquad (13b)$$

$$h_0(x_e, y) = H_0 - f_0 U_0\left(\frac{1}{\gamma_{-1}} + \frac{1}{\gamma_0}\right)(y - y_1) \qquad (13c)$$

$$h_{-1}(x_e, y) = (H_1 + H_0 + H_{-1}) - [h_0(x_e, y) + h_1(x_e, y)] \qquad (13d)$$

$$h_{-2}(x_e, y) = H_{-2} \qquad (13e)$$

where, again,

$$H_n \equiv h_n(x_e, y_1)$$

Before solving for the layer thickness and circulation in the diffusive case by integrating (11) from x_e, it is useful to examine the solution to the problem when the diffusive effects are ignored, assuming that density is conserved. This will give some measure of the importance of diffusion in this problem.

4. THE NONDIFFUSIVE CASE

Assume that the diffusive effects in the ocean interior are very weak, so that the diffusion coefficient λ_V can be set to zero, and use the ideal fluid geostrophic equations. The solution is then not unique, and there are many possible interior circulations consistent with a given specified inflow into layer 0. The circulation found will depend, for example, on the specified transport from the western boundary current into the layers above and below the inflow.

Consider now the simplest problem, where there is no inflow from the western boundary current into layers above and below layer 0. Only layer 0 is moving, and we can solve for its thickness and velocity for a given inflow from the eastern boundary. From (1), (8), and (9) the vorticity equation for the inflow layer is

$$\beta v_0 h_0 = -\beta \frac{\gamma_0}{f} h_{1x} h_0 = f(w_{0,\text{top}} - w_{0,\text{bottom}}) \qquad (14)$$

The vertical velocities are zero for the resting layers -1 and $+1$ and therefore also at the interfaces of layer 0 [*Pedlosky*, 1979]. The forcing on the right-hand side of (14) vanishes therefore, and the circulation in layer 0 is simply a zonal flow with the thickness of the layers equal to that on the eastern boundary as given by the boundary condition (13)

$$u_0(x, y) = u_0(x_e, y) \qquad v_0(x, y) = 0 \qquad w_0(x, y) = 0$$
$$h_n(x, y) = h_n(x_e, y) \qquad n = -2, -1, 0, +1, +2 \qquad (15)$$

Note that H_n, the thicknesses of all layers on the eastern boundary, south of the inflow, have to be specified. The H_n represent the basic density stratification and cannot be found by the model in the absence of diffusion in the physics.

We now proceed to the diffusive case, with several questions in mind: Can the addition of diffusion to the physics resolve the nonuniqueness problem? Can the H_n be found as part of the solution? How does the circulation change in the presence of diffusion, and does the diffusive solution reduce to the above ideal fluid solution as $\lambda_V \to 0$?

5. THE DIFFUSIVE CASE: THREE MOVING LAYERS

Consider now the diffusive solution for the three-moving-layer case. Simultaneously integrating the set (11) of ODEs from the eastern boundary, where the h_n are given by the boundary condition (13), we obtain the thickness of layers -2, 1, and 2 in terms of the eastern boundary stratification from the H_n, which must be specified at this stage. The solution for h_{-1} and h_0 is then found from (10*d*) and (10*e*). But with diffusion included in the physics, we expect to be able to determine the basic stratification of the model, as represented by the H_n. The H_n are now found by requiring the solution to satisfy integral constraints on the cross-interfacial fluxes, making sure that the total mass in each layer is constant in spite of the inflow into layer 0 and the cross-interfacial velocities.

Consider the integral constraints. The circulation in the moving layers depends on the values of the vertical velocities at the top and bottom interfaces of these layers through $\beta v_n h_n = f(w_{n-1} - w_n)$. With three moving layers there are four enclosing interfaces at which the vertical velocity must be calculated correctly by the model. Because the horizontal velocities are assumed to vanish for $|n| > 2$, we do not need to worry about the value of w at other interfaces. This leaves four constraints on the vertical velocities $w_{-2}(x, y), \cdots, w_1(x, y)$. To find what the constraints are, consider again Figure 1. The schematic figure shows that the total mass flux through the lower interface of layer 0 is equal to the amount of bottom water formed per unit time. The total flux through the upper interface of layer 0 is equal to that through the lower interface of that layer plus the inflow from the eastern boundary. A similar condition applies for the layers above and below the inflow. By assumption there is no fluid leaking from any of the layers, except through interior cross-interfaced velocities. All the fluid leaving the interior into the western boundary layer region, for example, is assumed to flow back into the same layer when returning to the interior. Once the transport of the inflow, M_{inflow}, and the amount of bottom water formation, M_{bottom}, are specified, the constraints become

$$\iint w_n(x, y)\, dx\, dy = M_{\text{bottom}} \qquad n = -1, 0$$
$$\iint w_n(x, y)\, dx\, dy = M_{\text{bottom}} + M_{\text{inflow}} \qquad n = 1, 2 \tag{16}$$

The double integral in (16) is taken over the entire area of the interior of the basin. The transport of the inflow from the eastern boundary is

$$M_{\text{inflow}} = \int_{y_1}^{y_2} u_0(x_e, y) h_0(x_e, y)\, dy$$
$$= \int \left[U_0 \frac{f_0}{f} \right] \left[H_0 - f_0 U_0 \left(\frac{1}{\gamma_{-1}} + \frac{1}{\gamma_0} \right) (y - y_1) \right] dy \tag{17}$$

Specifying the velocity $u_0(x_e, y)$ and the transport of the inflow, M_{inflow}, is equivalent by (17) to specifying H_0. The four equations (16) can then be used to solve for the four remaining stratification parameters H_{-2}, H_{-1}, H_1, and H_2 using the following procedure. For given values of the H_n the thickness of the layers $h_n(x, y)$ can be calculated everywhere by integrating (11) from the eastern boundary. The left-hand side of the constraints (16) can then be evaluated, using (5) to express w_n in terms of h_n, and used to define a function of the stratification parameters

$$\mathcal{G}(H_{-2}, H_{-1}, H_1, H_2) = \sum_{n=-1}^{2} \left(\iint w_n(x, y)\, dx\, dy - M_n \right)^2 \tag{18}$$

where w_n is the local upwelling velocity across the nth interface and M_n is M_{bottom} for $n = -1, 0$ and $M_{\text{bottom}} + M_{\text{inflow}}$ for $n = 1, 2$. The absolute minimum of this function, when $\mathcal{G} = 0$, corresponds to the H_n that satisfy the constraints (16). A quasi-Newton minimization routine was used to find the values of H_n for which $\mathcal{G} = 0$ and therefore complete the solution for the stratification and circulation in the model. Equation (18) is obviously highly nonlinear in the parameters H_{-2}, H_{-1}, H_1, and H_2, and we have found no mathematical proof for the uniqueness of the solution obtained for these parameters from (18). The physics of the problem and some experimentation with the routine used to solve for H_n suggest that the solution is unique.

Results and Discussion

Figure 2 shows the pressure in layers -1, 0, and $+1$. Note that although it was not so required a priori, the circulation in layers -1 and 1 is weaker than that of layer 0. This justifies (or at least is consistent with) the truncation of the model to a finite number of moving layers, in order to approximate a solution for the circulation in the layers assumed moving. (In the full problem, however, the vertical extent of the circulation must be determined by the dynamics). The solution is shown for the parameters $M_{\text{bottom}} = 2.5 \times 10^6$ m^3/s $= 2.5$ sverdrup (Sv), $U_0 = 0.2$ cm/s, $M_{\text{inflow}} = 0.3$ Sv, and $\lambda_V = 1$ cm^2/s. The difference $w_{0,\text{top}} - w_{0,\text{bottom}}$, integrated all over the basin, is positive and equal to the transport of the inflow. This ensures that the total mass in layer 0 is constant, although water is entering it from x_e and leaving through cross-interfacial upwelling. The positive $w_{0,\text{top}} - w_{0,\text{bottom}}$ induces a northward velocity through the vorticity equation (9), therefore turning the inflow northward, as seen in Figure 2. This northward flow is forced by the cross-interfacial velocities which are present due to the vertical mixing. It does not exist in the ideal fluid case presented in the previous section.

Examining Figure 2, note that the inflow proceeds westward a distance L_N, which is only about a half of the basin width before turning northward. One would like to know what determines this distance and whether it can become so small as to form a narrow eastern boundary current, with friction or nonlinearity dominating the dynamics and where the physics of the model is no longer valid.

L_N From Rossby Wave Argument

It is possible to derive an expression for L_N by considering a linearized time dependent problem. The argument is similar to that used to explain the existence of the western boundary current (WBC) in terms of Rossby waves [*Pedlosky*, 1979]. In the WBC case the balance is between eastward propagation of short Rossby waves and their dissipation (and trapping) by horizontal friction. Here westward propagating long Rossby waves are dissipated and trapped near the eastern boundary by the vertical density diffusion.

Consider a continuously stratified ocean, and linearize the equations of motion about a basic state of rest and a linear vertical density profile $\bar{\rho}(z)$. The linearized equations for the small perturbations about the basic state are

$$fu = -\frac{1}{\rho_0} p_y \tag{19a}$$

$$fv = \frac{1}{\rho_0} p_x \tag{19b}$$

$$p_z = -g\rho \tag{19c}$$

$$\rho_t + w\bar{\rho}_z = \lambda_V \rho_{zz} \tag{19d}$$

$$u_x + v_y + w_z = 0 \tag{19e}$$

From (19) a single equation for $\rho(x, y, z, t)$ can be derived

$$\rho_{zzt} + \frac{-\beta}{f^2/(\bar{\rho}_z/\rho_0)} \rho_x = \lambda_V \rho_{zzzz} \tag{20}$$

Substituting a wave solution

$$\rho = e^{ikx + imz - i\sigma t} \tag{21}$$

we find the dispersion relation

$$\sigma = \frac{-\beta k}{m^2(f^2/N^2)} - im^2\lambda_V \tag{22}$$

The real part of σ is the frequency of a westward propagating long Rossby wave. The imaginary part is the decay due to the vertical diffusion. A wave generated at the eastern boundary will travel westward at a speed $C_{g,x} = -\beta/(m^2 f^2/N^2)$, with its amplitude decaying with the e-folding diffusion time

$$t_d = (m^2\lambda_V)^{-1} \tag{23}$$

The distance such a wave travels westward is

$$L_N = |C_{g,x}| t_d = \frac{\beta}{m^4(f^2/N^2)\lambda_V} \tag{24}$$

which is the expression we were after, as can be further verified

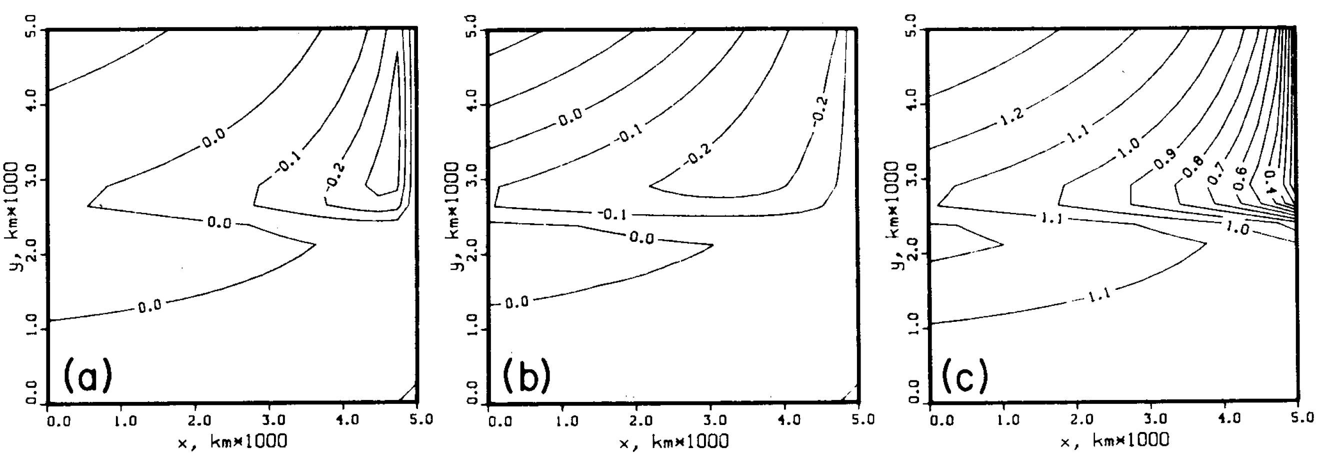

Fig. 2. Pressure in layers (*a*) 1, (*b*) −1, and (*c*) 0 for the three-moving-layer solution of section 5.

by experimenting with the size of the mixing coefficient and the latitude of the inflow [*Tziperman*, 1987].

After turning northward the flow becomes narrower. This effect is entirely due to the variation of the Coriolis parameters f and β with latitude and the resulting slower speed of the Rossby waves north of the inflow region, as given by the expression (24) for L_N. The dependence on the vertical wave number m simply expresses the fact that an inflow of smaller vertical scale will have a smaller L_N because it will be dissipated faster by the vertical diffusion.

The setup of a deep buoyancy-driven circulation was discussed by *Wajsowicz* [1983] and *Kawase* [1987], among others. There, Kelvin and Rossby waves complete against dissipation and are responsible for the penetration of the circulation from the boundaries or source region into the basin interior.

The vertical velocity difference across layer 0 is shown in Figure 2*d*. Its horizontal distribution is similar to that of the horizontal circulation in layer 0, and it is clear that specifying uniform vertical velocities would result in wrong circulation patterns.

As the fluid flows northward and the current becomes narrower, the Rossby and Ekman numbers may become large and violate the assumption of geostrophy. Writing these non-

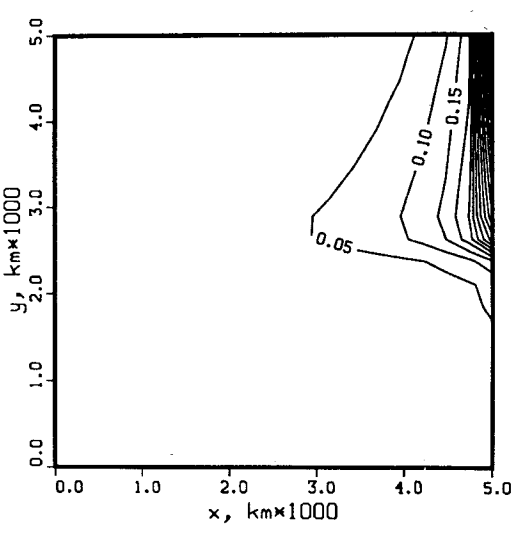

Fig. 2*d*. The vertical velocity difference across the inflow layer, $w_{0,\mathrm{top}} - w_{0,\mathrm{bottom}}$, in units of 10^{-5}.

dimensional numbers in terms of the problem's parameters, we find that they are, in fact, small for the parameters used to obtain the above solution. The northward flow, although narrow relative to the basin width, is therefore still in geostrophic balance, with the linear vorticity equation (9) governing the dynamics.

It is instructive now to compare the diffusive solution found above to the ideal fluid case examined in section 4. The circulation is obviously different, owing to the meridional flow forced by the vertical diffusion, as opposed to the purely zonal flow in the ideal fluid case. Another difference, perhaps more fundamental, is in the number of boundary conditions that may be applied and the amount of information that can be extracted from the model as part of the solution. The addition of vertical diffusion to the dynamics enables one to add boundary conditions on the upwelling across the layer interfaces and in return to calculate the average vertical stratification, as represented by the H_n. Note that we cannot specify the upwelling locally, but can only specify it in an integral sense, by specifying the total upwelling across a given interface. The local variations in the cross-interfacial velocity are determined by the model itself (Figure 1).

As $\lambda_V \to 0$, the diffusive solution changes in two ways. First, the interior is not diffusive enough to support the specified amount of cross-interfacial flux, and as a result the integral constraints (16) cannot be satisfied by realistic values of the H_n. The reason for this is that as λ_V becomes smaller, the depth scale of the density field, λ_V/w, becomes smaller, too, and at some stage the finite layers cannot resolve the vertical density structure and the solution breaks down. For the parameter range examined here, this happens for $\lambda_V \leq 0.5$ cm^2/s, and one must then specify the eastern boundary stratification and drop the additional boundary conditions (16). Then, with the H_n fixed and specified, as λ_V gets even smaller, the circulation reduces to a zonal flow as in the ideal fluid case. One assumes that mixing in the western boundary current takes care of the integral constraints on the total mass in each layer by allowing large enough cross-interfacial fluxes there. It is also possible that the inflow leaves layer 0 across the basin boundaries (e.g., across the equator).

The Circulation Above and Below the Inflow, the Vertically Integrated Circulation

To understand the circulation found for the layers above and below the inflow (Figure 2), consider the vertically inte-

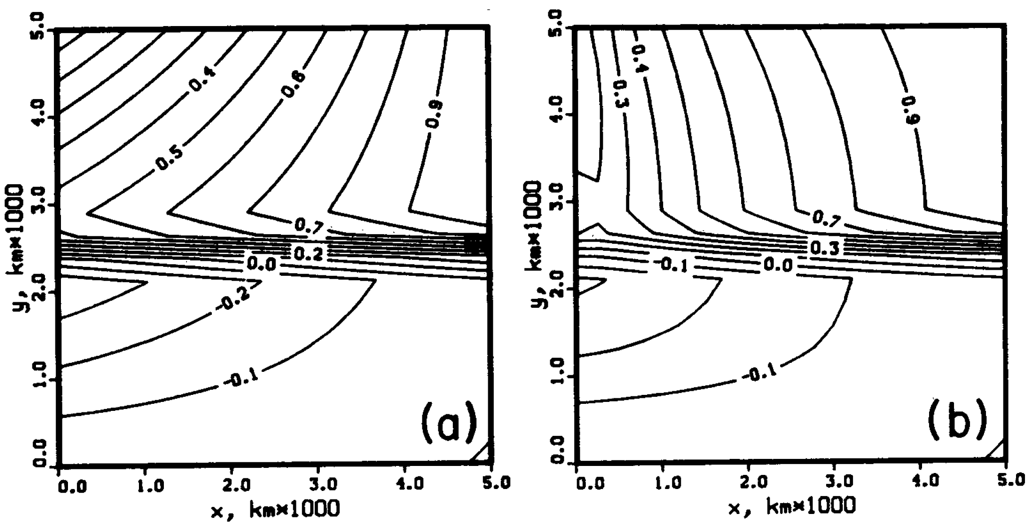

Fig. 3. Comparison of (*a*) the vertically integrated circulation forced by the inflow and (*b*) the combined transport of layers -1, 0, and 1 calculated by the three-layer model. See text.

grated circulation forced by the inflow. We have assumed that far above and below layer 0 the horizontal velocities vanish, the layer thickness becomes uniform in (x, y), and because the vertical velocity depends on the layer thickness through the density equation, it also becomes uniform in (x, y). The vertical velocity at layers far enough above the inflow is therefore

$$w_{top} = (M_{bottom} + M_{inflow})/\text{area} = \text{const}$$

and far below the inflow

$$w_{bottom} = M_{bottom}/\text{area} = \text{const}$$

Integrating the vorticity equation over all the layers in between, we obtain the total northward transport forced by the inflow,

$$\beta \sum_{\text{all moving layers}} v_n h_n = f(w_{top} - w_{bottom}) = f M_{inflow}/\text{area}$$

The transport stream function for this flow is shown in Figure 3*a*. Comparing Figure 3*a* and the circulation in layer 0 (Figure 2), we see that the vertically integrated circulation does not turn northward near the eastern boundary, as found for layer 0. Instead, most of the inflow flows westward to the western boundary current region and then returns to the interior and flows northward, filling the basin width as in a Stommel-Arons type model [*Stommel and Arons*, 1959].

The difference between the vertically integrated signal and the circulation in layer 0 means that somewhere above and below the inflow there must be a southward circulation near the eastern boundary, canceling the signal of the narrow northward flow in layer 0. Examining now the solution for layers $+1$ and -1 (Figure 2), we see that there is such a return flow in both layers near the eastern boundary. Figure 3*b* shows the transport streamlines for the three layers together, and one can see how the transport in layers $+1$ and -1 modifies that of layer 0, so that the total horizontal circulation of the three layers is very close to that of the Stommel-Arons type solution (Figure 3*a*). The small differences between Figures 3*a* and 3*b* are presumably taken care of by the weak circulation ignored by the present model in layers above layer -1 and below layer $+1$.

6. EFFECTS OF HORIZONTAL ADVECTION

So far, the horizontal advection and diffusion terms in the density equation were ignored on the assumption that isopycnal surfaces are nearly horizontal ($\varepsilon = \Delta_H\rho/\Delta_V\rho \ll 1$). In this section the conditions for these terms to be small are derived by expressing ε in terms of the forcing parameters. Then the effects of the horizontal advection are incorporated into the model using a perturbation expansion in powers of ε.

The scale for the horizontal variation in density, $\Delta_H\rho$, may be taken to be equal to the variation of the density across the inflow on the eastern boundary. The thermal wind relation to the inflow region, $fu_z = (g/\rho_0)\rho_y$, gives the scaling relation

$$\frac{fU_0}{H_0} = \frac{g\Delta_H\rho}{\rho_0(y_2 - y_1)}$$

or

$$\Delta_H\rho = \frac{[U_0(y_2 - y_1)H_0]\rho_0 f}{gH_0^2} = M_{inflow}\frac{\rho_0 f}{gH_0^2} \tag{25}$$

This gives

$$\varepsilon = \frac{\rho_0 f_0 M_{inflow}}{H_0^2 g\Delta_V\rho} \tag{26}$$

where $\Delta_V\rho = \Delta_0\rho$ is the density range represented by layer 0. As long as the transport of the inflow, M_{inflow}, is relatively weak, ε is small and the density equation is (3) to a good approximation. With the parameters used to obtain the solutions in Figure 2,

$$\rho_0 = 1\ \text{g/cm}^3 \quad f_0 = f\left(\frac{y_1 + y_2}{2}\right) = 7 \times 10^{-5}\ \text{s}^{-1}$$
$$H_0 = 350\ \text{m} \quad M_{inflow} = 0.3\ \text{Sv} \quad \Delta_V\rho = 0.0002\ \text{g/cm}^2 \tag{27}$$

we find $\varepsilon \approx 6 \times 10^{-2} \ll 1$.

Although small, the $O(\varepsilon)$ deviations from the vertical density equation (3) will affect the solutions for the thickness and circulation derived above. For $\varepsilon \ll 1$, it is possible to calculate the corrections due to horizontal advection by expanding all variables in powers of ε

$$u_n = u_n^{(0)} + \varepsilon u_n^{(1)} + O(\varepsilon^2) \quad h_n = h_n^{(0)} + \varepsilon d_n + O(\varepsilon^2)\cdots \tag{28}$$

Consider now the problem of three moving layers, as discussed in section 5, but with the full density equation (2) instead of the vertical one (equation (3)). After substituting the perturbation expansion in (1) and (2), the $O(1)$ equations are simply the equations used in section 5. It is possible to proceed, as was done in section 5, by calculating the $O(1)$ thickness fields in terms of the H_n and then to calculate the H_n by applying the constraints (16). In this section we are interested in the corrections due to horizontal advection, and for that purpose the $O(\varepsilon)$ equations and the appropriate boundary conditions need to be considered. The $O(e)$ equations are

$$fu_n^{(1)} = -\frac{1}{\rho_0} p_{ny}^{(1)} \tag{29a}$$

$$fv_n^{(1)} = \frac{1}{\rho_0} p_{nx}^{(1)} \tag{29b}$$

$$p_{nz}^{(1)} = -g\rho_n \tag{29c}$$

$$u_{nx}^{(1)} + v_{ny}^{(1)} + w_{nz}^{(1)} = 0 \tag{29d}$$

$$w^{(1)} = \lambda_V\left(\frac{\rho_{zz}}{\rho_z}\right)^{(1)} + \left[-u^{(0)}\rho_x^{(0)} + v^{(0)}\rho_y^{(0)} + \lambda_H \nabla_H^2 \rho^{(0)}\right]\frac{1}{\rho_z^{(0)}} \tag{29e}$$

The addition of the $O(\varepsilon)$ horizontal diffusion may cause the perturbation expansion to break down in places where $O(1)$ fields have discontinuous first derivatives. Also, boundary layers of $O(\varepsilon)$ width may be required near horizontal boundaries to assure the no-flux condition at the boundaries. To avoid these problems, we assume that the horizontal diffusion is very weak even in comparison with the horizontal advection, set $\lambda_H = 0$, and solve for the second-order effects due to the horizontal advection only.

To derive an equation for the $O(\varepsilon)$ thickness corrections d_n, start with the $O(\varepsilon)$ voriticity equation, derived from (9) and (28),

$$\beta[v_n^{(0)} d_n + v_n^{(1)} h_n^{(0)}] = f[w_{n-1}^{(1)}(x, y) - w_n^{(1)}(x, y)] \tag{30}$$

In density coordinates, $w^{(1)}$ from (29e) can be written as

$$w^{(1)} = \lambda_V\left[\frac{\partial}{\partial\rho}\frac{1}{z_\rho}\right]^{(1)} + u^{(0)} z_x^{(0)} + v^{(0)} z_y^{(0)} \tag{31}$$

which can then be written in discrete layer form

$$w_n^{(1)} = \lambda_V(\rho_{n+1} - \rho_n)^{-1}\left[\frac{\Delta_{n+1}\rho}{h_{n+1}^{(0)} + \varepsilon d_{n+1}} - \frac{\Delta_n \rho}{h_n^{(0)} + \varepsilon d_n}\right]^{(1)} + u_n^{(0)} z_{n,x}^{(0)} + v_n^{(0)} z_{n,y}^{(0)} \tag{32}$$

Here $w_n^{(1)}$ is the $O(\varepsilon)$ correction to the vertical velocity across the interface between the n and $n+1$ layers and $z_n^{(0)}$ and is the $O(1)$ height of this interface. Expanding the first term on the right-hand side of (32) gives

$$w_n^{(1)} = \lambda_V(\rho_{n+1} - \rho_n)^{-1}\left[\frac{-\Delta_{n+1}\rho}{[h_{n+1}^{(0)}]^2} d_{n+1} - \frac{-\Delta_n\rho}{[h_n^{(0)}]^2} d_n\right] + u_n^{(0)} z_{n,x}^{(0)} + v_n^{(0)} z_{n,y}^{(0)} \tag{33}$$

To obtain an equation for the d_n from (30), it is necessary to express the $O(\varepsilon)$ velocity $v_n^{(1)}$ in terms of d_n. The hydrostatic equation applies to the $O(\varepsilon)$ fields, and by assumption the horizontal pressure gradients vanish for layers other than $+1$, 0, and -1. We can therefore express the $O(\varepsilon)$ pressure in terms of the $O(\varepsilon)$ thicknesses d_n, as in (8), with d_n replacing h_n and $p_n^{(1)}$ replacing p_n. Such a relation can be used together with the geostrophic equations (29) to express $v_n^{(1)}$ in terms of $\partial d_n/\partial x$. Substituting (33) and $v_n^{(1)}$ into (30), one gets a coupled set of first-order differential equations for the d_n in the form

$$\frac{\partial}{\partial x} d_n(x, y) = \mathscr{F}_n(h_m, d_m) \tag{34}$$

These equations can now be integrated from the eastern boundary to give the corrections to the layer thickness everywhere. At the eastern boundary, $d_n(x_e, y) = 0$, because the total thickness there, $h_n + \varepsilon d_n$, is given by the boundary condition (13) and is satisfied by the $O(1)$ thickness h_n.

Figure 4 shows the solution to (34) with the parameters as in the $O(1)$ solution of section 5. The effect of the $O(\varepsilon)$ horizontal advection in the density equation is to move the streamlines in the direction of the $O(1)$ flow. In regions of westward $O(1)$ flow the total $O(1) + O(\varepsilon)$ flow extends further westward, while north of this region, where the $O(1)$ flow is in the northeast direction, streamlines of the $O(1) + O(\varepsilon)$ circulation are pushed eastward. The results can be interpreted in terms of Rossby waves, extending the arguments of section 5 to include the effects of advection on the propagation of the waves. Here the waves are advected by the flow, in addition to propagating westward, so that they reach points more or less further westward, depending on the direction of the advecting flow, before being dissipated by the vertical diffusion.

7. RELATION TO OBSERVATIONS

The Mediterranean Outflow

The most obvious feature of the solutions presented in the previous sections is in the turning northward of the flow entering from the eastern boundary (Figure 2). *Arhan* [1987] presented and analyzed observations indicating that the Mediterranean outflow turns northward, in a way resembling the model results presented above, after entering the eastern North Atlantic. In particular, he showed dynamic height maps at the depth of the Mediterranean outflow from *Maillard* [1986] and salinity maps from *Käse and Zenk* [1987], all indicating that the Mediterranean outflow turns northward before getting to about 30°W. His analysis seemed to show a northward flow of the upper Mediterranean water, and a southwest flow of the lower Mediterranean water. He tried to explain both these features by cross-isopycnal velocities resulting from double-diffusive mixing activity in the Mediterranean tongue region. The resemblance of the above observations and present model results is appealing, and we would like now to study the sensitivity of these results to the model's assumptions in order to examine the relevance of the dynamics used in the model to the Mediterranean outflow.

An important assumption made here is that all the mass entering some density range (layer) from the eastern boundary must leave this density range through interior cross-isopycnal velocities. This assumption is the basis for the application of the integral constraint (16) of constant total mass of water of given density. There are two possible problems with this assumption. First, if the interior mixing is too weak, the interior

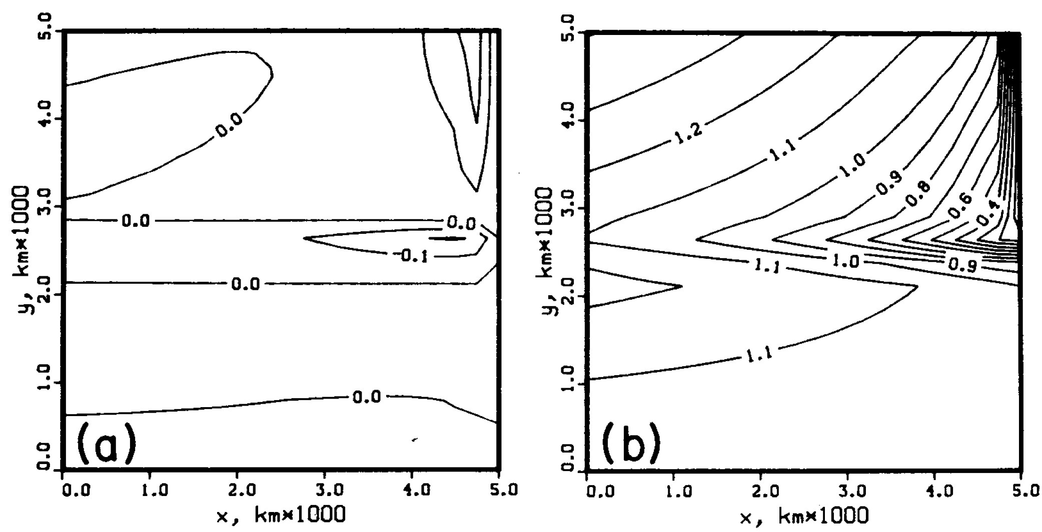

Fig. 4. Effects of horizontal advection terms in the density equation: (*a*) $O(\varepsilon)$ and (*b*) $O(1) + O(\varepsilon)$ pressure in layer 0 from the solution of section 6. The $O(1)$ solution is as in Figure 2.

cannot support a large enough cross-isopycnal mass flux, and the integral constraint based on this assumption cannot be used. This case was discussed in section 5, where the ideal limit of the diffusive solution was analyzed. Another possibility is that the diffusion (mixing) is large enough but some of the inflow into a given layer leaves this layer across the gyre boundaries (across the equator in the western boundary current region, for example), instead of through interior cross-interfacial velocities.

Figure 5 shows the circulation when none of the inflow mass is absorbed by the interior cross-isopycnal velocities. Solutions are shown for both the β plane case (f is constant unless differentiated, and β is constant) and the variable f and β case. The integral constraint applied in this case is

$$\iint w_n \, dx \, dy = M_{\text{bottom}} \qquad n = -1, 0, 1, 2 \qquad (35)$$

instead of (16). The averaged vortex stretching in the inflow layer in this case is zero,

$$\iint (\omega_{0,\text{top}} - w_{0,\text{bottom}}) \, dx \, dy = 0 \qquad (36)$$

and the vorticity equation $\beta v_0 h_0 = f(w_{0,\text{top}} - w_{0,\text{bottom}})$ implies a southward as well as a northward circulation in the interior. In the β plane case, with f and β constant, the inflow is equally split into southward and northward parts (Figure 5*b*). When the β plane approximation is not made and both f and β are functions of latitude, this symmetry is broken (Figure 5*a*) and the southward flow is much weaker.

This significant difference between the β plane case and the variable f and β case is a result of the integral constraints (16) and demonstrates the importance of calculating the correct basic stratification as part of the solution. In the β plane case a given difference $\Delta w = w_{0,\text{top}} - w_{0,\text{bottom}}$ forces the same meridional circulation everywhere. The condition (36) results, therefore, in a symmetric vertical velocity distribution and horizontal velocity field (Figure 5*b*). Note that because the vertical velocity field is coupled to the horizontal velocity field through the vorticity equation and the stratification, contours of Δw look like the streamlines of the horizontal flow.

When f and β are functions of y, there can be no flow across the equator, where $f = 0$, and streamlines of the southward flow must hit the western boundary before they reach $y = 0$. The constraint (36) is satisfied in this case by balancing a large region of small and negative Δw with a smaller region of large and positive Δw. This vertical velocity distribution forces, in turn, a weak southward flow and a stronger northward flow, as seen in Figure 5*a*. One may conclude that even when not all the inflow is absorbed by the interior cross-isopycnal velocities, it will still tend to flow northward as seen for the Mediterranean outflow.

Another assumption made in the model is of a constant diffusion coefficient in the density equation. Although a more realistic parameterization should probably have a mixing coefficient which varies with the buoyancy frequency [*Gargett*, 1984], this should not change the horizontal circulation found here very much. As long as $\iint (w_{0,\text{top}} - w_{0,\text{bottom}}) = M_{\text{inflow}} > 0$, the vorticity balance implies a northward flow, and this does not depend on the parameterization of the mixing. A variable diffusion coefficient means that the vertical velocities are balanced by the mixing in a different manner, the basic stratification may change, and the westward penetration distance of the inflow may be different. But the average structure of the vertical velocity field, the basic vorticity balance, and therefore the northward flow will not change significantly.

A final assumption we discuss in relation to observations is that the density surfaces are nearly horizontal ($\varepsilon = \Delta_H \rho / \Delta_V \rho \ll 1$). When leaving the straits of Gibraltar the Mediterranean water is heavier than the bottom water of the North Atlantic. There is, as a result, a large horizontal density variation there. This variation is not, however, the $\Delta_H \rho$ used for the scaling arguments of sections 2 and 6. The heavy Mediterranean water flows along the shelf, entrains lighter surrounding water, increases in volume, and decreases in density, until it reaches a depth where its density is equal to that of the surrounding stratification and it can spread horizontally. In the model presented above, the inflow from the eastern boundary represents the inflow of the diluted Mediterranean water, of density equal to that of the density surface it spreads on. There is no contrast in density between the water of the inflow and the interior stratification, and the scale for $\Delta_H \rho$ is

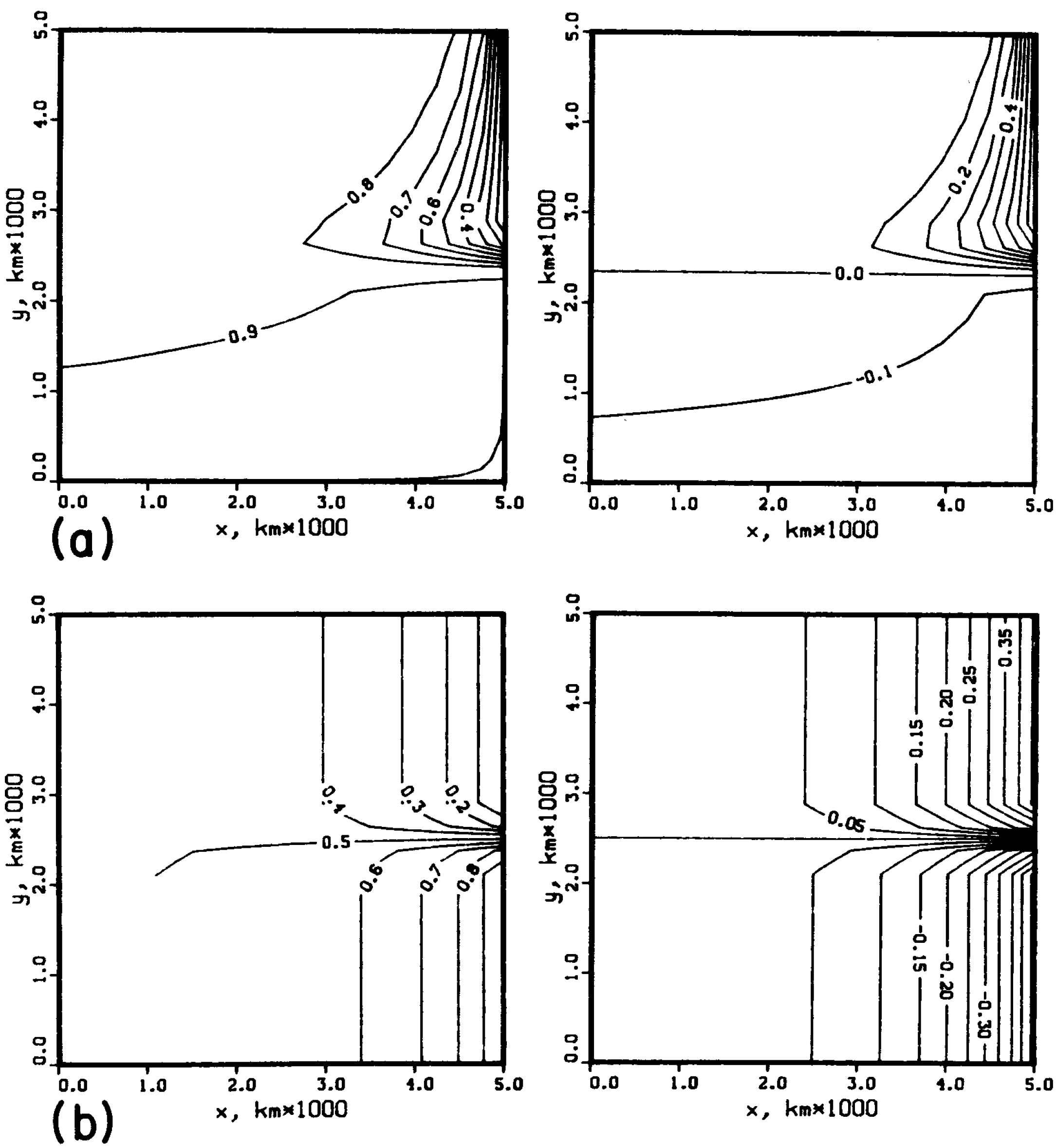

Fig. 5. The circulation in the diffusive case, when the mass flux of the inflow into a layer is not balanced by cross-isopycnal mass flux out of this layer in the interior. See text for more details. (*a*) The pressure in layer 0 (left) and $w_{0,\text{top}} - w_{0,\text{bottom}}$ (right), with f and β functions of latitude. (*b*) Same as in Figure 5*a*, for the plane β plane case.

calculated from the transport of the inflow, keeping ε not too large. It is difficult to estimate the actual transport of the diluted Mediterranean water from observations and therefore difficult to estimate ε for the Mediterranean outflow. As stated before, we have not tried to use "realistic" values for the problem and preferred concentrating on the understanding of the physics of the problem.

To summarize, the tendency of the inflow to turn northward is a fairly robust feature of the model, which also has some support in observations of the Mediterranean water circulation in the North Atlantic. For a more quantitative comparison of model and observations one needs to obtain estimates of the magnitude of the mixing coefficient, of the transport of the Mediterranean water after it entrains North Atlantic water and starts spreading horizontally, and of how much of the Mediterranean water is transformed to other density ranges by interior mixing.

Eastern Boundary Currents

The Rossby wave argument in section 5 indicates the possible existence of broad eastern boundary currents in a stratified ocean. These boundary currents exist due to the dissipation and trapping of long westward propagating Rossby waves by vertical diffusion of density and cannot exist without both the stratification and the diffusion. This is, perhaps, a possible explanation for the deep flow calculated by *Saunders* [1982] at a depth of 850–1200 m in the eastern North Atlantic.

A completely different mechanism for explaining these observations was suggested by *Schopp and Arhan* [1986]. They used an ideal ventilated model in which the northward mid-depth flow was driven by Ekman pumping far to the north.

8. CONCLUSIONS

We repeat some of the more general conclusions concerning the modeling of buoyancy-driven flows. When modeling deep flows, where the buoyancy forcing is due to mixing rather than to direct atmospheric heating, the model must determine the cross-interfacial velocities as part of the solution by relating them to the stratification through the density equation. Determination of the correct average vertical stratification (represented by H_n in the model presented above) is an important part of the solution. Wrong average vertical stratification will drive the wrong horizontal circulation and may not satisfy the condition of constant mass in a given density range (layer).

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E. Tziperman, Isotope Research Department, Weizmann Institute of Science, Rehovot, 76100, Israel.

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