MODELING THE EARTH SYSTEM

Dennis Ojima, Editor

Papers arising from the 1990
OIES Global Change Institute

Snowmass, Colorado
16–27 July 1990

OIES

UCAR/Office for Interdisciplinary Earth Studies
Boulder, Colorado
1992

Methods of Testing Parameterizations: Vertical Ocean Mixing

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Introduction

Because the ocean is stratified, water parcels move more easily horizontally along surfaces of constant potential density (isopycnals) than across these surfaces. Any movement across isopycnal surfaces must involve a change in gravitational energy, as well as a density change of the moving water parcel. This change occurs through vertical mixing with water of different potential density above or below. Vertical mixing in the ocean has an important role in maintaining the oceanic stratification at a steady state: The air-sea fluxes continuously cool and heat the surface water, thereby forming cold bottom water and warm surface water. The vertical mixing processes balance the air-sea fluxes by mixing the cold and warm water, to form the mid-density water found throughout the water column in the ocean. But the vertical mixing also presents a difficult challenge to the oceanographer interested in modeling the large-scale ocean circulation.

The ocean's velocity field is characterized by an exceptional variety of scales. While the small-scale oceanic turbulence responsible for the vertical mixing in the ocean is of scales a few centimeters and smaller, the oceanic general circulation is characterized by horizontal scales of thousands of kilometers. In oceanic general circulation models (GCMs) that are typically run today, the vertical structure of the ocean is represented by a few tens of discrete grid points, whose separation varies from a few meters near the surface of the ocean to hundreds of meters in the deep water. Such models cannot explicitly model the small-scale mixing processes, and must, therefore, find ways to parameterize them in terms of the larger-scale fields. Finding a parameterization that is both reliable and plausible to use in ocean models is not a simple task. Vertical mixing in the ocean is the combined result of many complex processes, and, in fact, mixing is one of the less known and less understood aspects of the oceanic circulation. In present models of the oceanic circulation, the many complex processes responsible for vertical mixing are often parameterized in an oversimplified manner. Yet, finding an adequate parameterization of vertical ocean mixing is crucial to the successful application of ocean models to climate studies. We will see below that the results of general circulation models for quantities that are of particular interest to climate studies, such as the meridional heat flux carried by the ocean, are quite sensitive to the strength of the vertical mixing.

Below we try to examine the difficulties in choosing an appropriate vertical mixing parameterization, and the methods that are available for validating different parameterizations by comparing model results to oceanographic data. First, some of the physical processes responsible for vertically mixing the ocean are briefly mentioned, and some possible approaches to the parameterization of these processes in oceanographic general circulation models are described in the following section. We then discuss the role of the vertical mixing in the physics of the large-scale ocean circulation, and examine methods of validating mixing parameterizations using large-scale ocean models.

Physical Processes Responsible for Oceanic Mixing/ Mixing Parameterizations for Large-Scale Models

Many physical processes participate in creating the small-scale turbulence (e.g., wave dynamics, shear flow, salinity gradients, and wind forces) responsible for vertical mixing in the ocean (Turner, 1981). Each of these processes may be dominant under different circumstances, at different times, and at different geographic locations. In addition, each of these small-scale vertical mixing mechanisms depends on the larger-scale fields in a different way, and may therefore require a different parameterization. In this section we briefly describe some of the vertical mixing mechanisms and discuss some of the commonly used parameterizations of vertical mixing in oceanic GCMs. Our purpose here is not to present an extensive review of vertical mixing parameterizations, but merely to demonstrate the difficulty in making a vertical mixing parameterization that accurately represents the variety of vertical mixing mechanisms. The terms "vertical mixing" and "cross-isopycnal mixing" are often used interchangeably, and we will do so below, but it is useful to keep in mind that what is meant in both cases is the mixing of water across surfaces of constant potential density. Although isopycnal surfaces in the ocean are nearly horizontal, and therefore the cross-isopycnal direction is nearly the same as the vertical direction in most places, the distinction is still important, as we will see below.

Constant Coefficient Parameterization

Simple scaling arguments based on the shape of the vertical temperature profile in the ocean immediately lead to the conclusion that mixing must be much larger than can be explained by molecular diffusivity, indicating the presence of small-scale turbulent mixing. Assuming that the small-scale turbulence in the ocean interior is of uniform intensity, it can be modeled with constant turbulent mixing coefficients, analogous to the molecular diffusion coefficients. Allowing for different vertical and horizontal mixing due to the preference of horizontal mixing in the ocean, we obtain the following advection diffusion equation for the temperature (T),

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}+K_v\frac{\partial^2 T}{\partial z^2}+K_h\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{1}$$

where K_h and K_v are the horizontal and vertical mixing coefficients, and $K_h \gg K_v$; (u,v) and (x,y) are the horizontal velocity components and coordinates; w is the vertical velocity; z is the local vertical coordinate; and t denotes time. Such constant eddy coefficients, although clearly oversimplified, are still the most commonly used parameterization of oceanic mixing in general circulation models (K. Bryan, 1969; F. Bryan, 1987).

Tensor Diffusivities Parameterization

The strong horizontal mixing represented by the large horizontal mixing coefficient K_h in Equation (1) is due to oceanic mesoscale eddies, which are circulation features of horizontal scale of tens to hundreds of kilometers. These eddies do not mix water horizontally, but in fact along isopycnal surfaces that may be tilted relative to the horizon. The large horizontal mixing coefficient meant to represent these eddies in Equation (1) causes strong mixing in the horizontal direction and may therefore result in a much too strong cross-isopycnal mixing when the isopycnals are not exactly flat. In order to accurately separate long-isopycnal and cross-isopycnal mixing, and thus to better parameterize the cross-isopycnal mixing in which we are interested here, the mixing coefficients K_h and K_v in Equation (1) may be replaced by the more complex tensor diffusivities (Redi, 1982). These diffusivities guarantee that the mixing is stronger along the direction of the isopycnals, and prevent the artificially strong cross-isopycnal mixing evident in models using the constant coefficient parameterization in Equation (1).

Stratification-Dependent Mixing Coefficients

An important source for turbulent energy in the interior of the ocean is the breaking of internal waves. Internal waves, expressed as the motion of the density layers in the ocean, are characterized by scales of kilometers and hours, and the amplitude of vertical movement of a given isopycnal layer due to these waves may be on the order of 10 m. These waves may break, like surface ocean waves, and produce patches of turbulence. The resulting small-scale turbulence is not uniform, but varies in intensity from place to place, and therefore cannot be represented by the simple constant mixing coefficient model. There are theoretical as well as experimental indications (Gargett and Holloway, 1984; Gargett, 1984) that when the small-scale turbulence comes mainly from internal wave breaking, the dependence of the vertical mixing coefficient on the vertical density stratification $\rho(z)$ may be written in terms of the buoyancy frequency, N, as

$$K_v = a_0 N^{-q} \qquad \text{where } q \approx 1, \text{ and } N^2 = -\frac{g}{\rho_0}\frac{\partial \rho}{\partial z} \tag{2}$$

a_0 is an empirical constant, g is the gravitational acceleration, and ρ_0 is a constant reference density. This parameterization is simple enough to use in a numerical general circulation model, and has more of a physical justification than the constant coefficient parameterization. This is a good example of the way in which work on the physics of small-scale mixing processes can benefit general circulation modeling. This form (and other possible ones) of the vertical mixing coefficient still needs to be validated using oceanographic data, as will be discussed below.

Richardson Number Parameterization

A vertically sheared flow (that is, a velocity in the horizontal direction whose magnitude changes with depth) in a stratified fluid may be unstable to small perturbations when the Richardson number, *Ri*, which is the ratio of the buoyancy frequency and the vertical velocity shear, is less than the critical value of 1/4. So when the Richardson number is near its critical value we may expect strong mixing to occur. In order to parameterize this instability, the following simple form of vertical mixing coefficient was suggested by Pacanowski and Philander (1981) and used by various researchers:

$$K_v = \frac{K_{max}}{(1+\alpha Ri)^n} + K_{background} \tag{3}$$

where K_{max} and $K_{background}$ are two values for the vertical mixing coefficient, with $K_{max} \gg K_{background}$, and α and n are adjustable empirical parameters. With an appropriate choice for α and n, the vertical mixing coefficient calculated by Equation (3) is small in most cases, near the "background" value. When the Richardson number approaches its critical value of 1/4, so that instability and strong vertical mixing may be expected to occur, the vertical mixing coefficient becomes large, of the size of K_{max}.

Different Eddy Coefficients for Temperature and Salinity

Salt fingering is an instability of the stratification caused by the different molecular diffusivities of heat and salt, and it may occur where warm, salty water lies above cold, fresh water. The instability results in finger-shaped intrusions at the interface of the different water masses, and eventually leads to strong mixing that is seen as vertical steps in the temperature and salinity profiles. There are some indications that the turbulent fluxes of heat and salt resulting from the occurrence of double diffusive salt fingers are not equal. This may justify using different values of eddy mixing coefficients for temperature and salinity in general circulation models. (There are also some more sophisticated parameterizations of mixing due to salt fingering, based on the relative magnitude of the local salinity and temperature gradients.)

Overturning Mixing Parameterization

A particularly important vertical mixing mechanism that occurs only in limited regions of the world ocean is that involved in water-mass formation. A statically unstable water column (heavy water above light water) may lead to strong vertical motion and mixing (convection). This typically happens in polar regions, where strong cooling of the surface water results in its sinking to the bottom. To represent such a process in oceanographic models, regions of unstable density profile are often simply vertically mixed together to uniform temperature and salinity at every time step (Cox, 1984). This mixing parameterization is based on the assumption that the strong vertical velocities due to the convection rapidly mix the parts of the water column where convection occurred.

Surface and Boundary Mixing Parameterization

A large portion of the mixing in the ocean occurs at the surface, and near the bottom and side boundaries. The wind forcing at the surface of the ocean is an important energy source for mixing within the surface mixed layer. Similarly, the bottom boundary induces strong mixing in bottom boundary layers. To parameterize the wind mixing one could use one of the simpler models, such as the constant eddy coefficient parameterization, with a larger value for K_v near the surface, or some parameterizations of mechanical mixing depending on the wind stress strength are possible as well.

The side boundaries (i.e., continental slope, islands) are also a source of strong vertical mixing, through the breaking of internal waves incident on these boundaries, through upwelling or downwelling induced by the presence of the boundary, etc. It is not obvious what is the proportion of boundary mixing relative to interior mixing in the ocean, a question of obvious importance as far as mixing parameterizations for oceanographic models are concerned. Laboratory experiments have shown that a system where all the vertical mixing occurs near the boundaries, and where the vertically mixed water is rapidly advected from the boundaries into the interior by the velocity field or by the strong horizontal mixing, may seem very similar to a system where the vertical mixing occurs in the interior only. More work is required in order to improve our understanding of the boundary mixing mechanisms and in order to find ways to adequately parameterize them in general circulation models.

Turbulence Closure Models

Most of the above parameterizations are to some extent ad hoc solutions guided by the need to find a simple enough way of including the turbulent mixing in general circulation models. A more rigorous approach is that of turbulence closure models, which try to explicitly model the turbulent transports of heat and salt (and momentum) in terms of the larger-scale known fields (Mellor and Yamada, 1974). This approach was used mostly in upper ocean models, where it may be expected to improve the representation of the mixed layer dynamics.

There are various other instabilities and physical processes leading to vertical mixing, but for our purpose it is sufficient to note the large variety of processes leading to vertical mixing in the ocean. The variety of existing parameterizations indicates that the problem of finding an adequate parameterization for oceanic vertical mixing is far from being solved.

Role of Vertical Mixing in the General Circulation Dynamics

Before examining the efforts to validate mixing parameterizations using oceanographic data, it is instructive to first try and understand the role of mixing in the dynamics of the oceanic general circulation. Consider, for example, the equation for the temperature in the ocean. Using simple scaling arguments it is easy to show that the effect of mixing on the temperature field is *locally* negligible compared to that of the advection by the oceanic horizontal velocity field. In terms of the simple advection diffusion model of Equation (1) this implies

$$\left(u\frac{\partial T}{\partial x}, v\frac{\partial T}{\partial y}\right) \gg \left[K_v\frac{\partial^2 T}{\partial z^2}, K_h\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)\right] \tag{4}$$

But this does not mean that mixing, and in particular vertical mixing, may be ignored in ocean models. In fact, when it is ignored for the purpose of modeling the wind-driven ocean circulation, the basic stratification of the ocean cannot be determined from the model but must be specified externally. Such ideal fluid models that do not include mixing in the dynamics (Luyten, et al., 1983; Rhines and Young, 1982) must specify the density stratification on, say, the eastern boundary of the ocean, and can only calculate the horizontal variation of the stratification relative to this basic stratification. In order to be able to calculate the basic stratification as well, one must include in the model both air-sea fluxes and interior vertical mixing processes.

The interior vertical mixing determines the oceanic stratification by balancing the water mass formation by air-sea heat fluxes. These fluxes cool the surface water in polar regions, and the resulting colder and denser water then sinks to the bottom of the ocean to form the bottom water masses. The air-sea heat fluxes also warm the surface water in the tropical and midlatitudes. This production of cold and warm water by the air-sea interaction is continuously balanced through the mixing of warm surface water and cold bottom water. Warm and cold water produced by the air-sea fluxes are turned by this mixing into mid-density water found in the ocean interior, and the stratification is thereby kept at a steady state. A similar picture applies to the modification of the oceanic salinity by precipitation and evaporation, and its balance by the mixing of water masses of different salinities.

In summary, vertical mixing is, locally, a second-order effect; yet, it is a process of major importance due to its role in the global budget of heat and salt in the oceans: Vertical mixing balances the water-mass formation by air-sea fluxes, and therefore is responsible for maintaining the ocean stratification at steady state.

It is important to note that in the context of global warming and models addressing the greenhouse problem, only some of the vertical mixing processes are significant. The time scales involved in the thermocline problem and in the global heat and salt balances discussed above are of thousands of years, and the slow interior vertical mixing is therefore of major importance for these problems. The global warming problem is more concerned with the shorter time scales (10–100 years) required, for example, to mix atmospheric CO_2 into the upper ocean. For this purpose, faster mixing processes such as deep convection and mixing within the mixed layer and upper ocean are relevant and need to be carefully modeled and validated.

Validating Vertical Mixing Parameterizations Using Large-Scale Ocean Models

The variety of vertical mixing parameterizations mentioned above makes it obvious that no single satisfactory vertical mixing parameterization yet exists. New parameterizations and especially reliable methods for validating these parameterizations are clearly needed.

There are two main approaches to testing mixing parameterizations. The first involves laboratory or field experiments in order to compare the mixing predicted by a given parameterization with the mixing actually measured in the experiment. (See Gargett, 1984, for a review of such studies trying to deduce the dependence of the vertical mixing coefficient on the stratification.) But this approach does not directly test the effect of the different parameterizations on global ocean models, which are our main interest here. It is also not possible to examine in a single experiment the combined effect of the wide variety of mixing processes active in the ocean. We will, therefore, concentrate on efforts to validate vertical mixing parameterizations using general circulation data and the models themselves. We first discuss the sensitivity of general circulation models to vertical mixing parameterizations, and then describe the efforts to deduce mixing coefficients from general circulation data using inverse methods.

Sensitivity of Large-Scale General Circulation Models to Vertical Mixing

Before considering different parameterizations to be used in ocean models, it is sensible to try and find out the sensitivity of the model results to the choice of vertical mixing parameterization. Only if the results prove to be sensitive to the choice of parameterization, does it make sense to proceed and look for the optimal parameterization to use.

Sensitivity studies of ocean general circulation models involve running the model using a variety of choices for the model parameters. For this purpose, it is not possible to use the high-resolution (eddy resolving) ocean models, because the computational cost is forbidding even when using the most powerful present-day computers. As a result, the common practice for sensitivity studies is to use low-resolution models that are relatively inexpensive to run. In such a study, Bryan (1987) used a primitive-equation general circulation model, within an idealized ocean basin, to examine model sensitivity to vertical mixing. He ran the model using a constant coefficient parameterization for the vertical mixing, and varying the magnitude of the vertical mixing coefficient. Bryan examined the dependence of the meridional heat flux carried by the ocean on the value of the vertical mixing coefficient (Figure 1). The maximum poleward flux varies from about 0.2×10^{15} W for $K_v = 0.1$ cm^2/s to 1×10^{15} W for $K_v = 2.5$ cm^2/s. Note that both values for the vertical mixing coefficients lie in the range of values suggested by observations, which further emphasizes the difficulty of finding a reliable parameterization for the vertical mixing. The ocean carries the poleward transport of heat through the meridional circulation cell: Warm surface water moves poleward, where it is cooled by the atmosphere, sinks to the bottom, and returns equatorward as cold bottom water. His results show that the strength of the meridional circulation is clearly sensitive to the vertical mixing coefficient, which results in the sensitivity of the meridional heat flux to this parameter (Figure 2). The depth of the oceanic

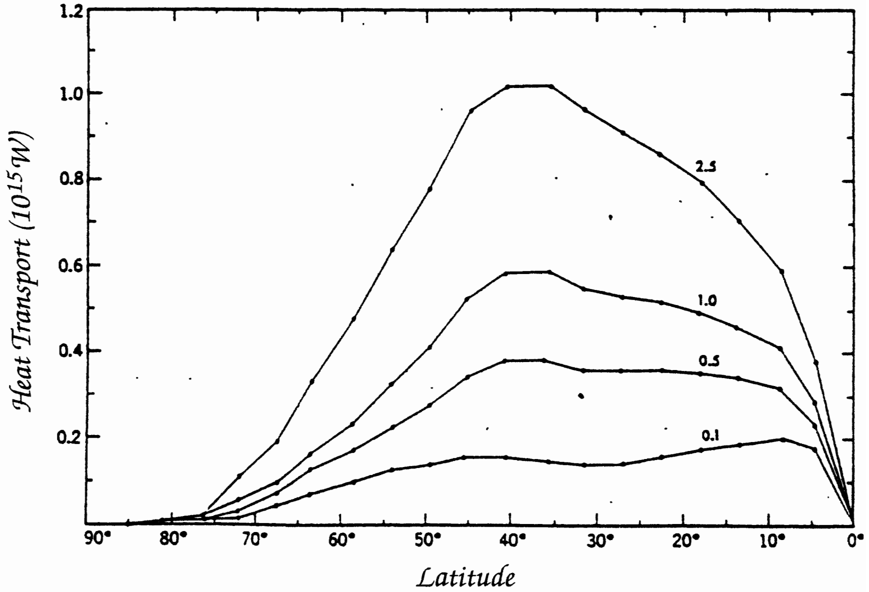


Figure 1. Dependence of the poleward heat transport in an oceanic primitive-equation model on vertical diffusivity (vertical mixing coefficient values vary from 0.1 to 2.5) (from Bryan, 1987; © American Meteorological Society).

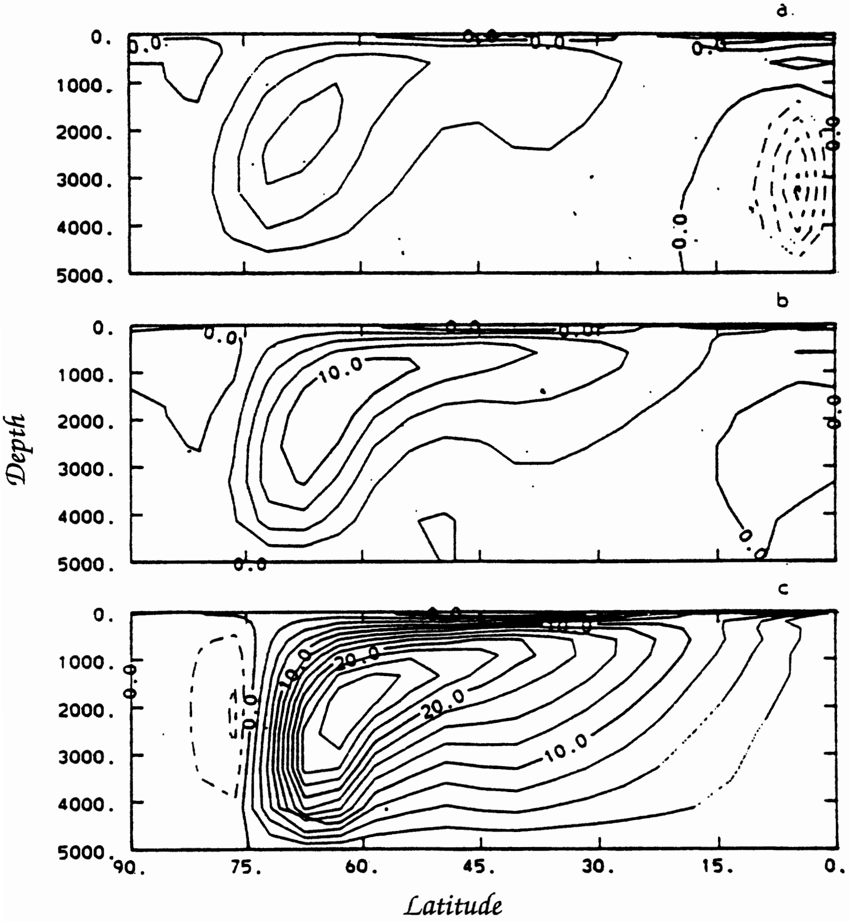


Figure 2. The meridional turning cell stream function calculated using a primitive-equation model, for three choices of the vertical mixing coefficient (from Bryan, 1987; ©American Meteorological Society).

main thermocline is another parameter of interest for climate studies, being relevant among other things to the total heat storage of the oceans. Figure 3 shows the zonally averaged thermocline structure calculated by Bryan for various choices of the vertical mixing coefficient. Again it is obvious that making the correct choice for the vertical mixing coefficient is critical for large-scale ocean modeling.

A similar sensitivity study was carried out by Colin de Verdiere (1988), using a simpler GCM that allows an even more extensive study of the sensitivity of the model results to various model parameters. He again found the meridional heat transport to be sensitive to the vertical mixing coefficient. Figure 4 summarizes his results, showing the sensitivity of various quantities of interest to the value of the vertical mixing coefficient.

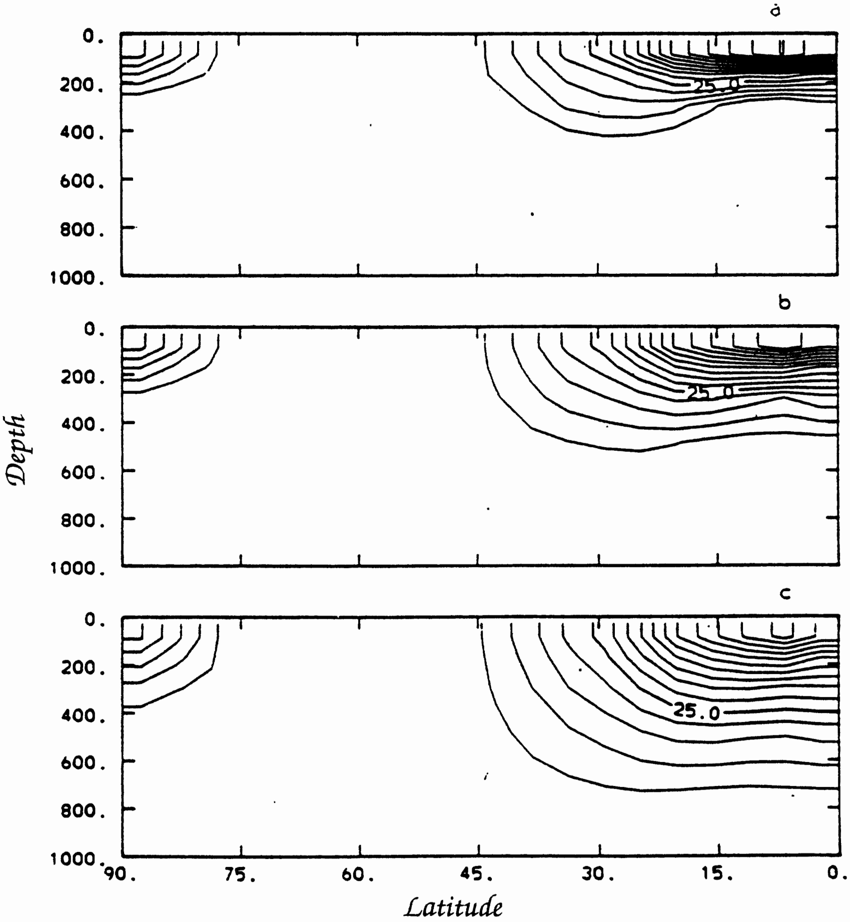
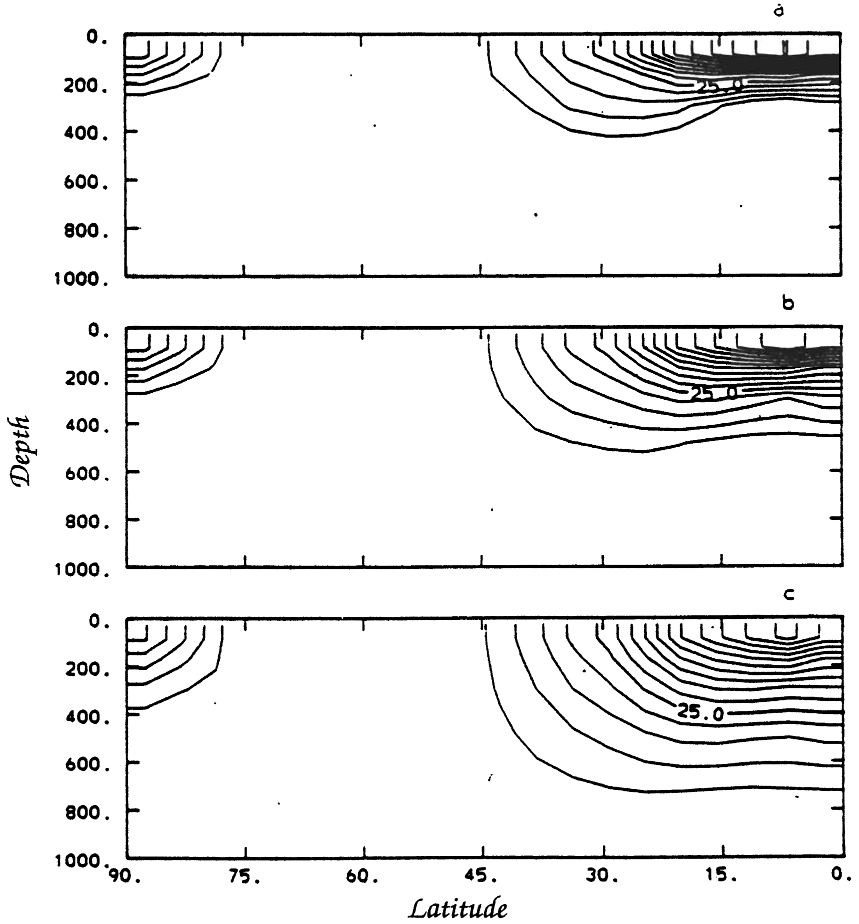


Figure 3. Zonally averaged thermocline in an oceanic primitive-equation model, for three choices of the vertical mixing coefficient (from Bryan, 1987; ©American Meteorological Society).

Validating Vertical Mixing Parameterizations Using Numerical GCMs

Having established that using the right value of the vertical mixing coefficient is crucial, and therefore more generally that using the right vertical mixing parameterization is important, we now consider the problem of choosing an appropriate parameterization for a numerical oceanic general circulation model from the many possible parameterizations presented above. The most straightforward method, which is often also the only one used, is to run the model with various possible parameterizations, and qualitatively compare the model results to the available observations for temperature, salinity, etc. By such a qualitative comparison of the main features of the data and model results, it is decided whether the parameteri-



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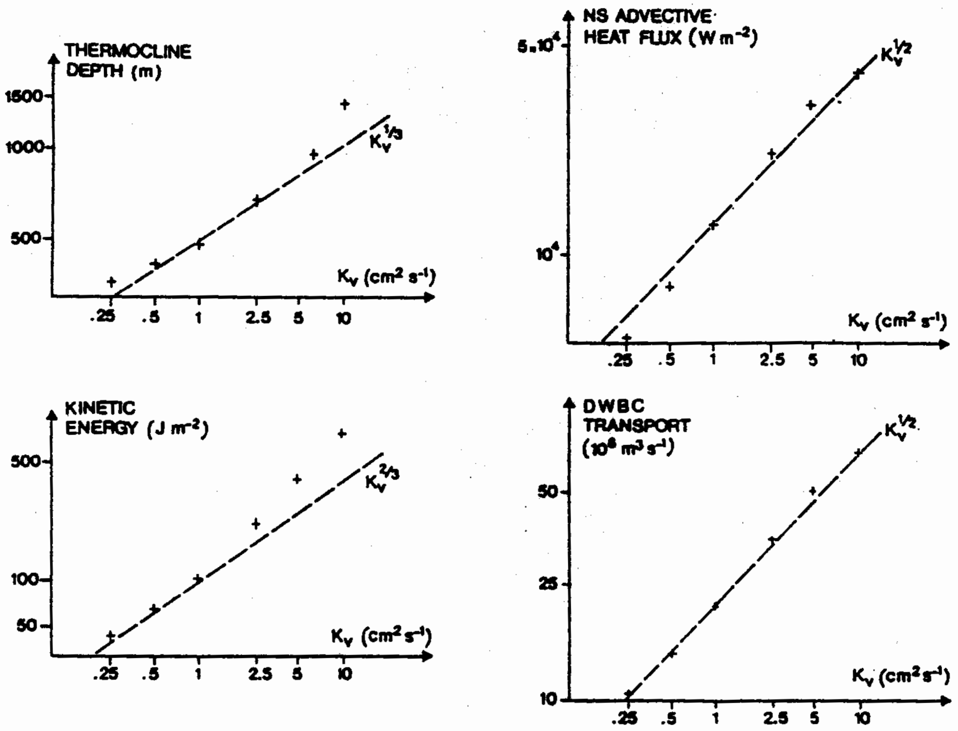


Figure 4. Mean thermocline depth, northward advective heat flux, kinetic energy, and deep western boundary current transport as function of the vertical mixing coefficient K_v, as calculated using a simple oceanic general circulation model (from Colin de Verdiere, 1988; used with permission).

zation is successful. Below are a few examples of validating vertical mixing parameterizations this way, and then a discussion of the limitations of this approach.

Richardson Number Parameterization

In an effort to develop an adequate vertical mixing parameterization for models of the tropical oceans, Pacanowski and Philander (1981) have demonstrated that using the Richardson number parameterization shown in Equation (3) results in a better fit to the data than using the constant coefficient vertical mixing parameterization. Figure 5 shows three temperature sections along the equatorial plane. The first is based on observations, the second is calculated by a general circulation model using constant vertical mixing parameterization, and the third is calculated using the Richardson number parameterization. It seems from these results that the Richardson number parameterization is indeed preferable, at least as far as equatorial ocean modeling is concerned.

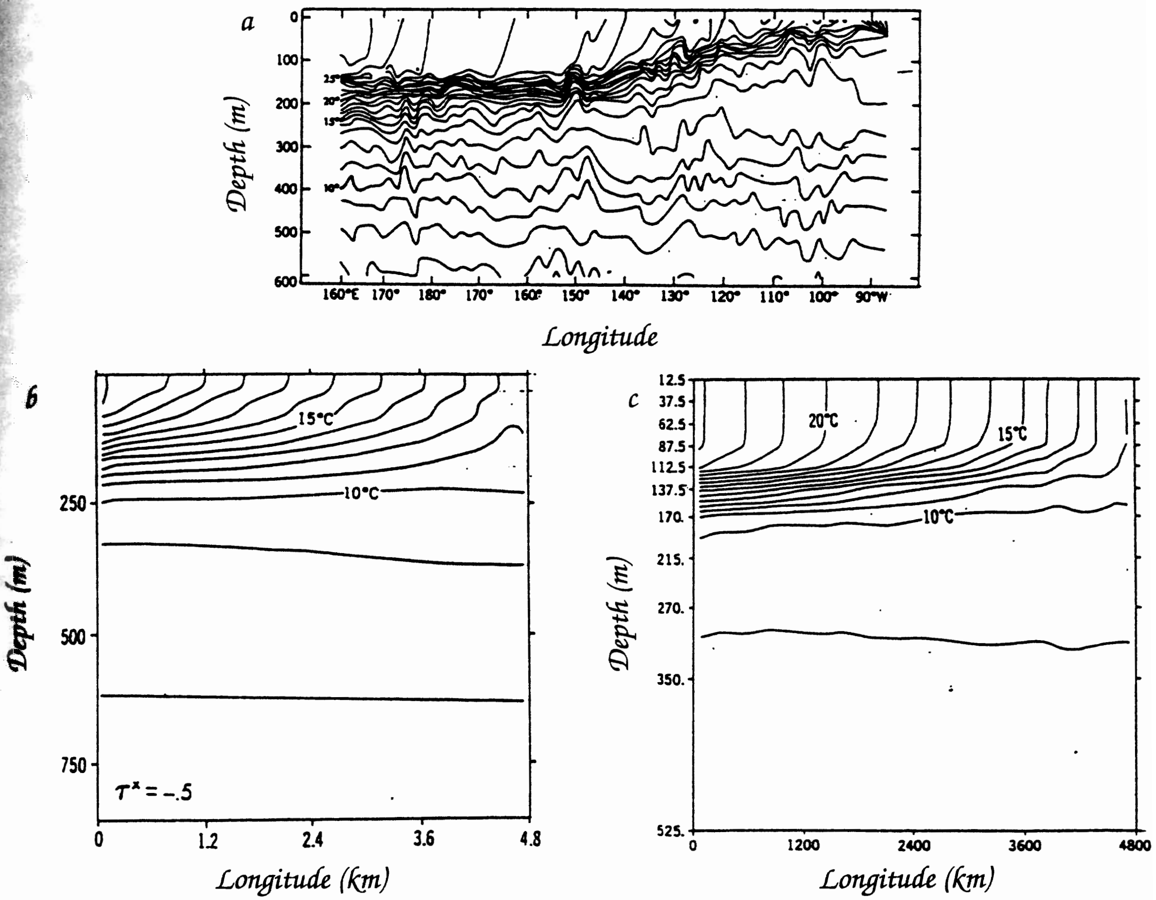


Figure 5. Three temperature sections along the equatorial plane: (a) based on observations, (b) from a model using constant vertical mixing, and (c) from a model using the Richardson number parameterization (from Pacanowski and Philander, 1981; ©American Meteorological Society).

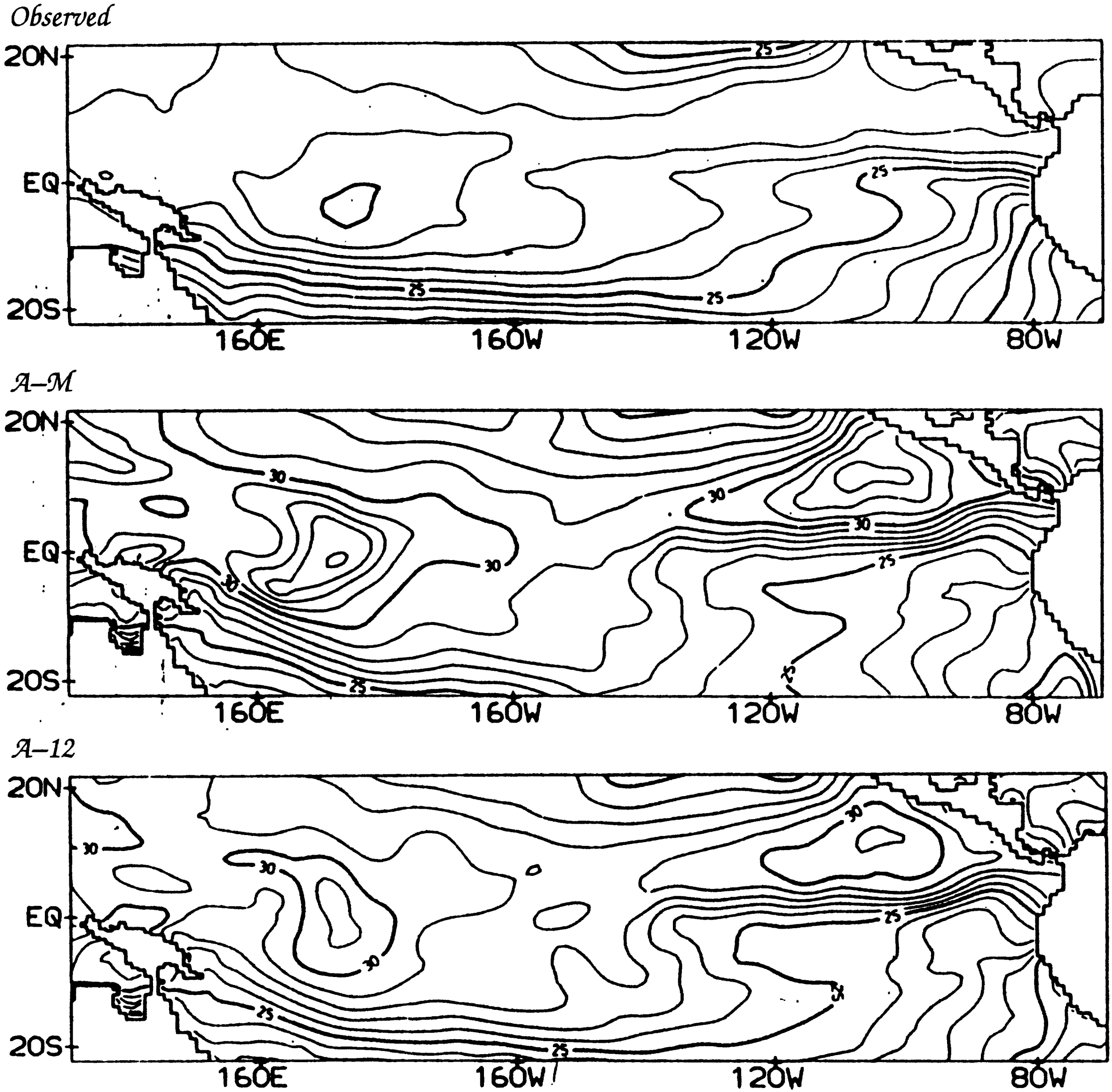
Turbulence Closure Models

The closure model of Mellor and Yamada (1974) was applied to several upper ocean models and offers an alternative to the simple constant coefficient vertical mixing parameterization. Rosati and Miyakoda (1988) compared the performance of the constant coefficient parameterization with that based on a closure model and nonlinear viscosity. Figure 6 compares the observed Pacific Ocean sea surface temperature to that calculated by a variety of models, differing in mixing parameterizations as well as in atmospheric forcing frequency. Rosati and Miyakoda concluded that the more sophisticated closure parameterization works better than the one using constant mixing coefficients, although there are still misfits to the data whose sources are unclear.

Tensor Diffusivities

In the next few years coarse-resolution models, rather than high-resolution eddy resolving models, may still be expected to be used for climate studies because of the limits set by the available computational resources. For such coarse-resolution models it is most

Sea Surface Temperature



Observed

A–M

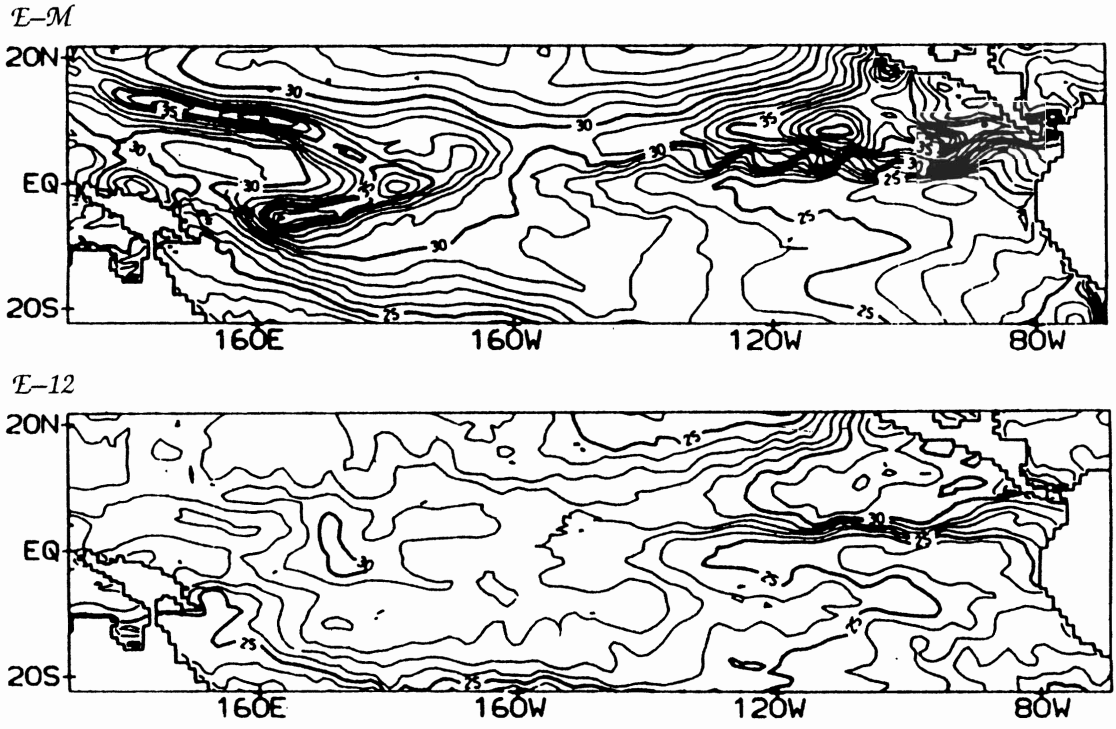
A–12

Figure 6. Pacific Ocean sea surface temperature from observations and calculated by a variety of models differing in the mixing parameterizations as well as atmospheric forcing frequency. A-M and A-12 are constant coefficient models. E-M and E-12 are models using a turbulence closure scheme. (M and 12 refer to monthly and 12-hour frequency of the atmospheric surface forcing used in the model) (from Rosati and Miyakoda, 1988; ©American Meteorological Society).

important to correctly parameterize the effect of mesoscale eddies. The mixing parameterization using tensor diffusivity, which can differentiate between long-isopycnal and cross-isopycnal mixing rather than between horizontal and vertical mixing, may be a reasonable substitute to using eddy resolving models in the near future. The tensor diffusivity prevents the artificially strong cross-isopycnal mixing created by the large horizontal mixing coefficients in regions where isopycnals are tilted relative to the horizon. Some long-term integrations with a coarse-resolution model using such tensor diffusivities were recently carried out by K. Bryan, resulting in an improved fit to some of the patterns of the oceanic stratification.

Discussion

The above common approach to validating parameterizations using GCMs suffers some obvious limitations. First, if there is a misfit between model results and data, it is not obvious which of the many model inputs is responsible. The problem could lie in using the wrong value for the vertical mixing coefficient, or in using the wrong model for the mixing process. But the problem could also be an error in the wind forcing at some location over the ocean, in the tracer boundary conditions that were not set correctly (e.g., in simulations



E–M

E–12

Figure 6, continued.

of tritium or ^{14}C), or in one of the many other different model inputs. Because of the large number of model inputs, it is not possible to vary each one separately in order to find its optimal value resulting in the best fit to observations. But changing only one of the inputs in order to improve the fit to observations may be misleading as well. By changing, for example, only the vertical mixing parameterization in order to better fit the data, we may be compensating for some completely different problem in another model input. Such a compensation will be an ad hoc solution only, and will most probably not work in another ocean basin, or under different physical conditions.

A second problem with the above approach to model validation is its inability to directly and quantitatively use the wealth of available oceanographic data to correct the model inputs or parameterizations. Model validation is mostly limited to a qualitative comparison of model results and data, and does not involve a quantitative comparison, use of error estimate for the data, etc. This inability to use the increasing number of oceanographic observations is in particular disturbing now that many new techniques of making oceanographic observations are becoming available, especially through remote sensing methods. A way must be found to use all of the available information in order to improve oceanic general circulation models.

An alternative to using general circulation models for validating model parameterizations that may overcome some of the above difficulties is the use of inverse methods. These methods use oceanographic data in order to calculate various unknown model parameters, and offer some advantages for model validation and parameter estimation, as discussed in the following subsection. Inverse methods are not meant to replace the general circulation models for the purpose of predicting the ocean's role in climate change, for example, but to quantitatively confront these models with the available data, in order to try and improve their less certain aspects.

Calculating Mixing Coefficients Using Inverse Methods

The original motivation for using the inverse methodology in oceanography was in trying to resolve the classical problem of level of no motion (Wunsch, 1978). The traditional dynamic method for calculating the oceanic circulation from temperature and salinity data used the simple geostrophic equations in order to calculate the horizontal velocity field in the ocean, relative to the velocity at some specified reference depth. In order to make that an absolute velocity estimate, one often assumed the velocity to vanish at some deep reference level—the level of no motion—and calculated the velocity relative to that level. The choice of this level was usually based on some subjective criteria, and the purpose of introducing the inverse methodology to this problem was to use our knowledge of the ocean dynamics in order to quantitatively calculate the absolute velocity field. Later the method was also used to estimate mixing coefficients from oceanographic temperature and salinity data.

Using inverse methods, the velocity field and mixing coefficients are calculated by requiring the ocean to satisfy the conservation equations of mass, heat, and salt. Let us demonstrate this using the simple steady state temperature equation,

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=K_v\frac{\partial^2 T}{\partial x^2}+K_h\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)\pm\varepsilon(x,y,z) \qquad (5)$$

where $\varepsilon(x,y,z)$ denotes the error in this equation due to measurement errors in the temperature field. Evaluating the temperature gradients appearing in Equation (5) from the data, we can obtain a linear system of equations for the velocity field and the mixing coefficients. (In fact the geostrophic equations are also used, to relate the three-dimensional velocity field to the known density and to an unknown reference velocity that is calculated together with the mixing coefficients.) In addition to solving for the unknown parameters, the inverse solution also provides the important resolution information and error estimates that together indicate which parameters can actually be solved for from the data, and what is the expected error for the estimated parameters.

Several variations on this approach have been used to try and calculate mixing coefficients from hydrographic data. Figure 7 shows the cross-isopycnal (vertical) mixing coefficient calculated by Olbers et al. (1985), using a beta spiral inverse method and sophisticated tensor diffusivities parameterization for the mixing processes. Olbers et al. found, however, that the mixing coefficients in general, and the vertical mixing coefficient in particular, are calculated with large error bars and cannot be distinguished from zero in most cases.

Hogg (1987) inverted the Levitus (1982) data for temperature, salinity, and oxygen in a region of the North Atlantic ocean. He fitted an advection diffusion equation to the data and calculated both the velocities and the mixing coefficients for various mixing models, including the vertical mixing low given in Equation (2). Hogg found that the velocities calculated by the inverse were not sensitive to the particular form of mixing parameterization used, but the mixing coefficients were very sensitive to the details of the mixing model used.

The difficulties found by these studies seem to be representative of a general problem concerning the calculation of mixing coefficients from data. It is often found that the coefficient could not be

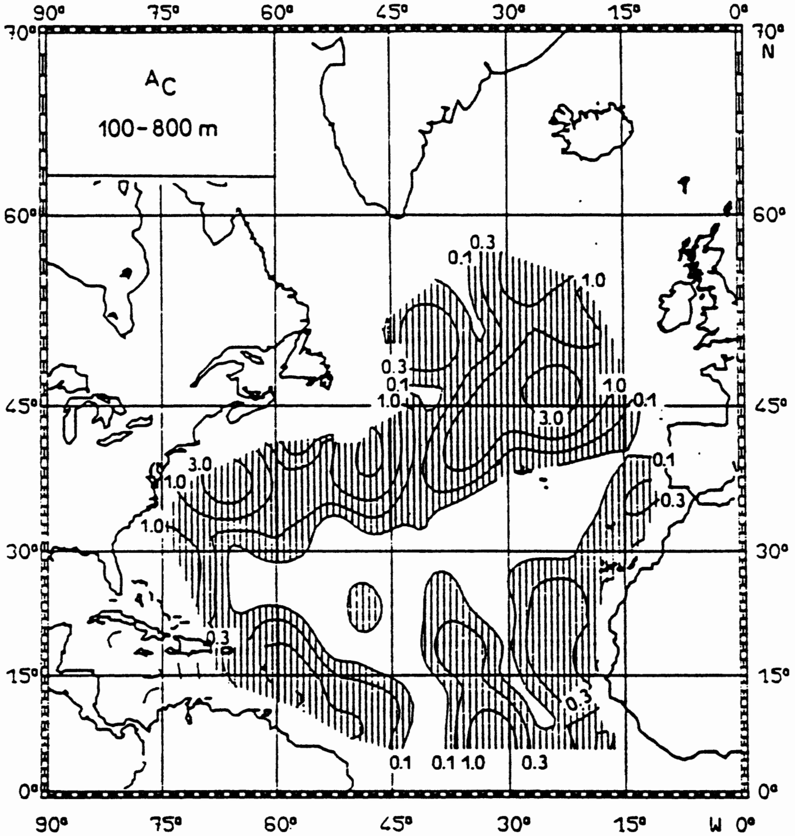


Figure 7. Cross-isopycnal mixing coefficient calculated for 100–800-m depths using a beta spiral inverse model of the North Atlantic (from Olbers et al., 1985).

resolved by the data, or that the large error estimates make them indistinguishable from zero, or that the values of the coefficients are too sensitive to the particular form of mixing model assumed.

The difficulty in calculating mixing coefficients can be explained using the fact that mixing is, locally, a second-order effect (Tziperman, 1988). As explained above, the mixing terms in the temperature equation are small compared to the advection terms. Figure 8 shows a vertical profile of the different terms in the temperature equation evaluated from data in the Mediterranean Sea where the velocities and mixing coefficients are calculated using an inverse model. The mixing terms are clearly smaller than the advection terms throughout the water column, and they can be as small as the noise in the equation [$\varepsilon(x,y,z)$ in Equation (5)]. The expected error in the horizontal advection terms [$u(\partial T/\partial x) + v(\partial T/\partial y)$] is on the order of ε, which is much smaller than these terms. As a result, the error in the calcu-

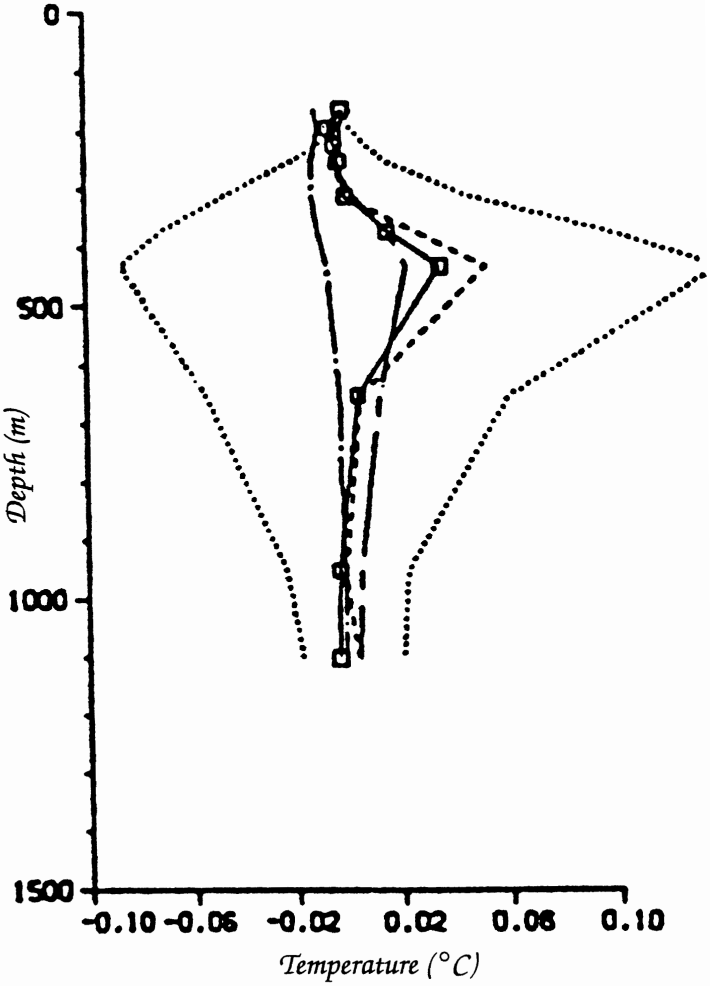


Figure 8. A vertical profile of the different terms in the temperature equation evaluated from data in the Mediterranean Sea where the velocities and mixing coefficients are calculated using an inverse model. Dotted lines are the horizontal advection terms, uT_x, vT_y; the dashed line is the sum of the horizontal advection terms, $uT_x + vT_y$; the chain-dotted line is the vertical advection term wT_z; and the chain-dashed line is the vertical diffusion term $[K_v(z)T_z]_z$ (from Tziperman, 1988; ©American Meteorological Society).

lated horizontal velocities (u, v) is relatively small. But because the mixing terms, and in particular the vertical mixing term $[K_v(\partial^2 T/\partial z^2)]$, are on the order of the noise ε, the error in the mixing coefficients could be on the order of the coefficients themselves.

In other words, the solution for the mixing coefficients is controlled by the noise level in the data. It is no surprise, therefore, that the coefficients calculated from hydrographic data are mostly statistically indistinguishable from zero, and that the solution for them is most sensitive to the exact parameterization used for the mixing processes. The problem clearly lies in our attempt to calculate the mixing coefficients from *local* balances, while they are physically significant only in determining the *global* balances in the ocean (and in the model). Furthermore, some of the inverse calculations, forced by the limitations on the number of equations that can be handled simultaneously, have not been able to calculate a mass conserving velocity field. It is difficult to justify the calculation of a second-order physical effect such as mixing, when the first-order requirement of mass conservation cannot be satisfied by the inverse solution.

In order to be able to obtain useful estimates for the mixing coefficients, it seems that an inverse model must have (at least) the following properties. First, its physics should include all second-order physical effects that may be of the same order as the vertical mixing we wish to estimate. This includes, for example, the nonlinear terms in the momentum equations that may be important in certain oceanic regions. In the absence of some second-order physical effects in the model, the inverse model may try to compensate for their lack by artificially modifying the mixing coefficients, resulting in a wrong solution for them. Next, the model should be of fairly high resolution in order to enable the estimation of various second-order terms in the model equations, which cannot be estimated in coarse-resolution box models. The inverse model should not be local in the sense that it needs to be mass conserving and satisfy all model boundary conditions for the velocity field. The above three requirements are equivalent to simply specifying that the physics and resolution of the inverse model should be similar to those used in oceanic general circulation models. Finally, although of high resolution, the inverse model should allow for global constraints such as requiring the total water-mass dissipation due to vertical mixing throughout the basin to exactly balance the water-mass formation by air-sea fluxes. Because vertical mixing has a dominant role in such global constraints, unlike its role in the local dynamics, using such constraints may enable us to obtain useful information about the vertical mixing.

But trying to formulate even a moderate-resolution inverse model that uses nonlocal constraints leads to a huge system of equations

(equations for the temperature, salinity, mass conservation, etc., at every grid point) that cannot be solved on any of the present-day computers. More sophisticated mathematical methods are needed for such large-scale optimization problems, and such a method has recently been introduced into the fields of meteorology and physical oceanography. This method, known as the optimal control or adjoint method, allows the efficient combination of a complex general circulation model and oceanographic data. The method has been suggested as an assimilation technique (Le Dimet and Talagrand, 1986; Long and Thacker, 1989), but can also serve as a powerful and efficient inverse procedure in oceanography (Tziperman and Thacker, 1989). Because a full numerical general circulation model is used in the inverse calculation, all second-order physical effects are included, and the resolution may be fairly high. Additional global balances may be added as constraints in the inverse calculation, as may be needed in order to resolve the mixing processes.

Given a numerical general circulation model, the adjoint method can be used to calculate the temperature, salinity, velocity, surface forcing by wind and air-sea heat fluxes, and mixing coefficients, all of which satisfy the model equations and boundary conditions, and at the same time fit the available data. A crucial component of the method is a numerical model composed of the adjoint equations of the GCM that is fitted to the data. The adjoint model is used, within an iterative procedure, to calculate the unknown parameters (surface forcing, mixing parameters, etc.) in an efficient and elegant way.

The use of the adjoint method in oceanography is still at a preliminary stage, but the method is most promising, and may be expected to serve as a useful tool for the purpose of model validation and parameter estimation. In particular, it should prove helpful in the effort to produce reliable vertical mixing parameterizations for large-scale oceanographic models. Given several vertical mixing parameterizations, they can all be included in a general circulation model, each with a weight factor determining its importance in the model equations. The weights may then be calculated by the adjoint method by requiring the inverse calculation to choose the parameterization that results in the best fit to the available data, thereby indicating which parameterization is the most appropriate.

Summary and Conclusions

Oceanic general circulation models have become quite sophisticated in recent years, and are now used with high eddy resolving resolution and realistic basin geometry and atmospheric forcing. Despite these improvements, these models are unlikely to be able to directly resolve the small-scale physical processes responsible for vertical mixing in the ocean. Yet, some of the most basic elements of the oceanic circulation, such as the vertical structure of the main thermocline and the meridional transport of heat by the ocean, are quite sensitive to the vertical mixing parameterization. Models that are used for climate studies can greatly benefit, therefore, from an adequate parameterization of the vertical mixing processes in the ocean. (Note that models addressing the problem of greenhouse warming during the next 100 years or so will mostly benefit from improved parameterizations of only some of the mixing processes—in particular, those that are responsible for upper ocean mixing and that act on the tens of years time scale.) Because vertical mixing is the combined effect of many different and complex processes in the ocean, it is still not well understood, nor is there a single best parameterization that can represent this mixing in oceanographic models. More research is needed to both develop and validate vertical mixing parameterizations for large-scale ocean models.

Oceanic general circulation models are the main tool for investigating the role of the oceans in the climate system, but they are difficult to use at present for testing different vertical mixing parameterizations. These models are very expensive to run and are affected by quite a few input parameters and subgrid parameterizations for which no sufficient information is available, and of which vertical mixing is only one example. It is most difficult, therefore, to run the general circulation models for all possible combinations of unknown model inputs in order to choose the optimal inputs that best fit the oceanographic data. In addition, general circulation models cannot make a direct use of most of the available oceanographic data for calculating unknown input parameters. The large quantities of oceanographic data now available from classical measurements and through remote sensing methods can be used mostly for qualitative comparisons to model results. One would like to be able to use these data in order to quantitatively calculate unknown model inputs.

Inverse models may, in principle, be used for estimating unknown input model parameters from oceanographic data, but still may encounter serious difficulties in inferring mixing processes from the available data. These difficulties arise from the fact that mixing processes, although important in setting the global balances of heat and salt in the ocean, are locally negligible compared with other physical processes in the ocean. The small local effect of the mixing, on the tracer fields in particular, is near the noise level of the data. An inverse model that is to calculate useful estimates for the mixing processes needs to be able to handle the global constraints that may be able to constrain the mixing processes in the ocean. In addition, such an inverse model should have both high resolution and the ability to handle second-order physical effects (such as nonlinear dynamics) that may have a comparable effect to that of vertical mixing. However, the limitations on the dynamics and resolution of inverse models set by the available computational resources have prevented so far the use of such inverse models for validating mixing parameterizations.

With new mathematical methods now becoming available, it may be possible to use the dynamics and resolution of general circulation models in inverse calculations, as well as to use global constraints for testing mixing parameterizations against oceanographic data. This must, of course, be accompanied by more work on the physics of the small-scale processes responsible for vertical mixing in the ocean, and the development of better parameterizations. The combination of better parameterizations and reliable validation procedures will, hopefully, result in useful vertical mixing parameterizations for oceanographic models, and therefore in better ocean models for climate studies.

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