A note on calculating age from tracer observations

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Abstract. Given measurements of transient tracers, radioactive or with a time-dependent surface boundary condition, we propose a functional approach to deriving a general expression for an approximate age tracer that is a function of these observed tracers. The functional form of the age tracer is determined such that the resulting age tracer equation is as close as possible to that of an ideal age tracer, and such that the resulting age tracer satisfies the ideal age tracer boundary conditions at the surface. The method is demonstrated using a simple model system, as well as using a WOCE CFC section.

1. Introduction

An ideal age tracer, τ_{ideal}, measures the time since a given water mass was in contact with the ocean surface. It satisfies an advection diffusion equation with a constant source term on the RHS, and with the boundary condition of zero age at the surface [Haidvogel and Bryan, 1992; England, 1995; Wunsch, 2002],

$$(\partial_t + u_i\partial_i - \partial_i\kappa_{ij}\partial_j)\tau_{ideal} = 1, \quad (1)$$

$$\tau_{ideal}(x,y,z=0,t) = 0. \quad (2)$$

We represent the mixing effects using a three by three tensor (matrix) κ_{ij}, as required for example by some recent parameterization of the mixing by meso scale ocean eddies [Gent and McWilliams, 1990; Griffies, 1998]. Also, ∂_i is a short form of $\partial/\partial x_i$ where x_i with $i = 1,2,3$ are the three spatial coordinates. We also use the summation convention such that when an index is repeated a sum is implied: $u_i\partial_i\tau_{ideal} = \sum_{i=1}^{3} u_i\partial_i\tau_{ideal}$. There are quite a few works dealing with the calculation of the age of water masses from various tracer measurements. In particular, an approximate age tracer may be composed from a combination of Tritium and Helium observations [Jenkins and Clarke, 1976; Jenkins, 1982, 1987, 1998; Robbins and Jenkins, 1998]. Different tracers such as CFC partial pressure age or CFC-11 and CFC-12 ratio age have also been used for age estimates [Doney and Bullister, 1992; Doney et al., 1997; Fine et al., 2002] as well as other related diagnostic purposes [e.g., Rhein et al., 2002]. It is well understood now, that in the presence of mixing, a simple age estimate cannot be used for a complete characterization of a given water mass. Instead, each water parcel is influenced by many surface sources and is thus more properly characterized by an age spectrum, or transit time probability distribution function [Holzer and Hall, 2000; Haine and Hall, 2002; Hall and Haine, 2002; Khatiwala et al., 2001; Waugh et al., 2003]. The simple ideal tracer (1) is the first moment of age spectrum mentioned above (the mean age or mean transit time) when equation (1) has become statistically stationary [Hall and Haine, 2002]. To further complicate things, the oceanic and atmospheric nomenclatures are different, and many different relevant time scales may be derived for a given simple advection-diffusion problem [Wunsch, 2002]. However, it seems that it is still useful to use the observed tracer concentrations to calculate a simple age estimate. A simple example of doing so is of using the Tritium (T) and Helium-3 (H) concentrations to calculate the approximated age τ using

$$\tau = \frac{1}{\lambda}\ln(1+\frac{H}{T}) \quad (3)$$

[Jenkins and Clarke, 1976; Jenkins, 1982]. However, τ from (3) does not satisfy the ideal tracer equation (1) because of terms nonlinear in T,H, which make τ deviate from the ideal age tracer τ_{ideal} satisfying (1) [Jenkins, 1987].

For non-radioactive transient tracers such as CFC, the time history of the increasing atmospheric concentration was used to derive an age-like tracer [e.g. Doney et al., 1997; Warner et al., 1996], by simply examining the interior concentration of CFC, and identifying the age of that interior water mass with the time at which the surface concentration was equal to the observed concentration. This clearly ignores mixing effects that dilute the tracer concentration.

In this note we propose a simple functional approach to determining the form of an age tracer from observed tracer concentrations. This approach is used to find the functional form of an age tracer as function of observed radioactive tracers, or as function of a CFC-like transient tracer and its time-dependent surface boundary conditions. The functional form is chosen by solving a simple differential equation such that the resulting age equation will be as similar to (1) as possible. We first discuss the case of radioactive tracers, starting with Tritium and Helium (section 2) and proceeding to any two or more arbitrary radioactive tracers with different half-life times. Next, we derive the functional form for an age tracer from a conservative transient tracer such as CFC (section 3). We conclude in section 4.

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2. Age tracer from radioactive tracer observations

Consider first the equations for Tritium and He-3

$$\begin{aligned}\partial_t T + u_i\partial_i T &= \partial_i(\kappa_{ij}\partial_j T) - \lambda T \\ \partial_t H + u_i\partial_i H &= \partial_i(\kappa_{ij}\partial_j H) + \lambda T. \end{aligned} \quad (4)$$

We would like to define an age tracer τ that is a function of H,T as follows

$$\tau = f(T,H). \quad (5)$$

Now, one should be aware that the relationship between tau, tritium, and helium is not necessarily well-behaved. The function f in (5) may not be single-valued because of unsteadiness in the flow. In addition, we need to stay away from areas where anthropogenic tritium concentrations vanish, as the function will be singular there. In general, we cannot expect ages larger than a few decades to be accurately determined using these tracers. The age tracer which is estimated from tracers with a finite time history, such as Tritium and Helium or CFC, cannot represent the full mean age because the longest transit times sampled are limited to the time for which these tracers have been present. The finite time these tracers are present makes the age estimate from them dependent on the initial conditions for τ, which is assumed zero here. Having noted these potential practical limitations, we proceed to find out what the functional form of $f(T,H)$ needs to be to approach an ideal age tracer equation (1). Substitute $\tau = f(T,H)$ into an advection diffusion equation, using $\partial_t\tau = f_T\partial_t T + f_H\partial_t H$ with $f_H \equiv \partial f/\partial H$, $f_T \equiv \partial f/\partial T$, $f_{TT} \equiv \partial^2 f/\partial T^2$, etc, to find

$$\begin{aligned} &\partial_t\tau + u_i\partial_i\tau - \partial_i(\kappa_{ij}\partial_j\tau) = f_T\left(\partial_t T + u_i\partial_i T - \partial_i(\kappa_{ij}\partial_j T)\right) \\ &+ \; f_H\left(\partial_t H + u_i\partial_i H - \partial_i(\kappa_{ij}\partial_j H)\right) \\ &+ \; \kappa_{ij}\Big((\partial_j H)(f_{HH}\partial_i H + f_{TH}\partial_i T) \\ &+ \; (\partial_j T)(f_{TT}\partial_i T + f_{TH}\partial_i H)\Big). \end{aligned}$$

Substituting now the equations (4) for T,H we find

$$\partial_t\tau + u_i\partial_i\tau - \partial_i(\kappa_{ij}\partial_j\tau) = -\lambda f_T T + \lambda f_H T + \mathcal{R} \quad (6)$$

where we define the residual $\mathcal{R}$, due to the nonlinear mixing terms [Jenkins, 1987], as

$$\begin{aligned}\mathcal{R} &= -\kappa_{ij}\Big(f_{HH}(\partial_i H)(\partial_j H) + 2f_{TH}(\partial_i T)(\partial_j H) \\ &+ \; f_{TT}(\partial_i T)(\partial_j T)\Big). \end{aligned} \quad (7)$$

In (6) we can require the RHS apart from the residual to be equal to one. This leads to an equation for the functional form of the age tracer function $\tau = f(T,H)$

$$-\lambda f_T T + \lambda f_H T = 1 \quad (8)$$

To solve for f, transform to $(X,Y) = (H-T, H+T)$ so that $f_T = f_Y - f_X$ and $f_H = f_Y + f_X$ and (8) becomes

$$\frac{\partial f}{\partial X} = \frac{1}{\lambda}\frac{1}{Y-X}$$

and the solution is $f = (-1/\lambda)\ln(Y-X) + F(Y)$ for any arbitrary function $F(Y)$, or in terms of H,T

$$\tau \equiv f(T,H) = \frac{-1}{\lambda}\left(\ln(T) + F(H+T)\right) \quad (9)$$

for any arbitrary $F(H+T)$. Jenkins' age tracer [Jenkins and Clarke, 1976] corresponds to

$$F(H+T) = -\ln(H+T) \quad (10)$$

which then gives $\tau = \frac{1}{\lambda}\ln(1+\frac{H}{T})$ but our derivation clearly shows that this expression is far from being unique. Note that the Jenkins formulation satisfies the boundary condition

$$\tau = 0 \text{ when } H = 0. \quad (11)$$

This has the advantage that the age is zero at the surface where Helium concentration vanishes. The more accurate boundary condition for the age tracer is (2) above. It should be noted that the various tracers used to deduce age satisfy a boundary condition in which their air-sea flux depends on the difference between the surface and air concentrations, while we would like the age tracer to just have a specified concentration at the surface. In the following we shall assume that the surface concentration of the observed tracers is specified and known. For our purpose in this work, we will not need to consider explicitly the air-sea flux mechanism that results in this observed and specified surface tracer concentration.

Next, let us very briefly consider now any two arbitrary transient tracers with different half-life times. Denote the tracers by A,B, such that their advection diffusion equations are

$$(\frac{\partial}{\partial t} + u_i\partial_i - \partial_i\kappa_{ij}\partial_j)\begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} -\lambda_A A \\ -\lambda_B B \end{pmatrix}. \quad (12)$$

We now write the age tracer as a function of the two tracer concentrations $\tau = f(A,B)$ and substitute it in an advection-diffusion equation. Using the A,B equations as we did above for the Tritium-Helium age, we find that the right hand side contains a nonlinear mixing term, plus the term $-f_A\lambda_A A - f_B\lambda_B B$ which we would like to be as close to one as possible,

$$-f_A\lambda_A A - f_B\lambda_B B = 1, \quad (13)$$

and a particular inhomogeneous solution in this case is

$$f^{(i)} = -\ln(A^{(1/\lambda_A)} + B^{(1/\lambda_B)}).$$

To find a homogeneous solution, use separation of variables $f^{(h)} = \mathcal{A}(A)\mathcal{B}(B)$ which, after being substituted into (13) with the rhs set to zero, results in the equation

$$-\lambda_A\mathcal{A}'\mathcal{B} - \lambda_B\mathcal{A}\mathcal{B}' = 0$$

which, after dividing by $\mathcal{A}\mathcal{B}$, implies

$$\lambda_A A\mathcal{A}' = \mu\mathcal{A} \quad (14)$$

$$\lambda_B B\mathcal{B}' = -\mu\mathcal{B} \quad (15)$$

So that the solution is $\mathcal{A}(A) = A^{\mu/\lambda_A}$ and $\mathcal{B}(B) = B^{-\mu/\lambda_B}$ so that $f^{(h)}$ is a sum of polynomials of the form

$$f^{(h)} = \left(\frac{A^{1/\lambda_A}}{B^{1/\lambda_B}}\right)^{\mu}$$

This means that the homogeneous solution is any function F of the above ratio in the brackets, so that we can write the general solution to (13) as The solution is

$$\tau \equiv f(A,B) = -\ln(A^{1/\lambda_A} + B^{1/\lambda_B}) + F\left(\frac{A^{1/\lambda_A}}{B^{1/\lambda_B}}\right). \quad (16)$$

where F is any function of its argument, determined using surface boundary conditions, etc. Suppose A is Argon, whose b.c. is $A = 100$ at the surface. Then The function F is determined to be

$$F = \ln[100^{1/\lambda_A}(1 + B^{1/\lambda_B}/A^{1/\lambda_A})] \quad (17)$$

so that the final expression for the age is

$$\tau = \ln\left[\frac{1 + B^{1/\lambda_B}/A^{1/\lambda_A}}{A^{1/\lambda_A} + B^{1/\lambda_B}} 100^{1/\lambda_A}\right] \quad (18)$$

We also note that one can derive an age tracer estimate from more than two tracers. Starting, for example, from three transient tracer equations A, B, C instead of with (12), we can derive an equation for the functional form of $\tau(A,B,C)$ along the same lines of the above derivation.

3. Age tracer from transient, non-radioactive tracers

Waugh et al. [2003] show that there is an equivalence between a radioactive decaying tracer, and a non decaying tracer which has an exponential surface boundary condition. In general, tracers with transient surface boundary conditions have been used to age estimates [e.g. Warner et al., 1996]. Consider next a transient tracer such as CFC, which is conservative in the ocean interior,

$$\frac{\partial C}{\partial t} + \mathbf{u}\cdot\nabla C - \nabla\kappa\nabla C = 0, \quad (19)$$

yet has a surface boundary condition that is a function of time. In this equation, $\mathbf{u}\cdot\nabla C = u_i\partial_i C$ and $\nabla\kappa\nabla C = \partial_i\kappa_{ij}\partial_j C$, as used above. Assuming for simplicity that the surface boundary condition is not a function of location (e.g. ignoring variations in CFC solubility due to variations in sea surface temperature), we can write

$$C(x,y,z=0,t) = S(t). \quad (20)$$

Following our above functional approach, we write the CFC age as an unknown function of the CFC concentration and of the surface boundary condition,

$$\tau_{CFC} = f(C(x,y,z,t), S(t)), \quad (21)$$

where our objective is again to find the functional form which results in a rhs of the tracer equation that is as close to that of an ideal tracer equation as possible. Substituting this functional form for τ_{CFC} into an advection diffusion equation, and using (19) we find

$$\begin{aligned} &\frac{\partial\tau_{CFC}}{\partial t} + \mathbf{u}\cdot\nabla\tau_{CFC} - \nabla\kappa\nabla\tau_{CFC} \\ &= f_S\frac{\partial S}{\partial t} - f_{CC}\kappa_{ij}(\partial_i C)(\partial_j C). \end{aligned} \quad (22)$$

Now, apart from the usual nonlinear mixing residual term [Jenkins, 1987], the right hand side contains the term $f_S\frac{\partial S}{\partial t}$ which we would like to simply be equal to one, for the equation to be closer to the ideal tracer equation. In other words, we need to solve the equation

$$\frac{\partial f}{\partial S}\frac{\partial S}{\partial t} = 1 \quad (23)$$

In addition, we would like to satisfy the boundary condition that the CFC age vanishes at the surface, which from (20) we write as

$$\tau_{CFC} = 0 \quad \text{when} \quad C = S(t). \quad (24)$$

This form of boundary conditions is clearly not exactly equivalent to having the desired $\tau_{CFC} = 0$ at $z = 0$, and using (24) is equivalent to the similar choice made above for the Tritium-Helium age, for example.

The solution of (23) depends on the specific form of $S(t)$, and we shall consider several examples here. Suppose, first, that

$$S(t) = at^b. \quad (25)$$

so that $t = (S/a)^{1/b}$. This implies

$$\frac{\partial S(t)}{\partial t} = abt^{b-1}.$$

and therefore

$$\frac{\partial S(t)}{\partial t} = ab\,(S/a)^{(b-1)/b}$$

such that (23) becomes

$$\frac{\partial f(C,S)}{\partial S} = (ab)^{-1}\,(S/a)^{(1-b)/b}$$

and the general solution for the functional form of the desired age tracer before applying the boundary conditions is therefore

$$\tau_{CFC}(C,S) = f(C,S) = (S/a)^{1/b} + F(C) \quad (26)$$

where $F(C)$ is an arbitrary function to be determined by the boundary conditions. Applying the boundary conditions (24) we finally find

$$\tau_{CFC}(C,S) = \left(\left(\frac{S}{a}\right)^{1/b} - \left(\frac{C}{a}\right)^{1/b}\right) \quad (27)$$

Warner et al. [1996] use a method called "apparent age" based on CFC concentration. This method assumes that water maintains its ratio of observed to saturated CFC concentration, or pCFC (=CFC/solubility), due to the atmospheric concentration at the time of water mass formation. The pCFC apparent age can be obtained directly by comparing the pCFC to the CFC atmospheric time history (saturation factor assumed to be one).

The pCFC at the surface is in near-equilibration with the atmosphere. Let t be the time at which the tracer observation is made, and T_{age} be the time since a water parcel was at the surface so that T_{age} is the apparent age of the parcel. Assume

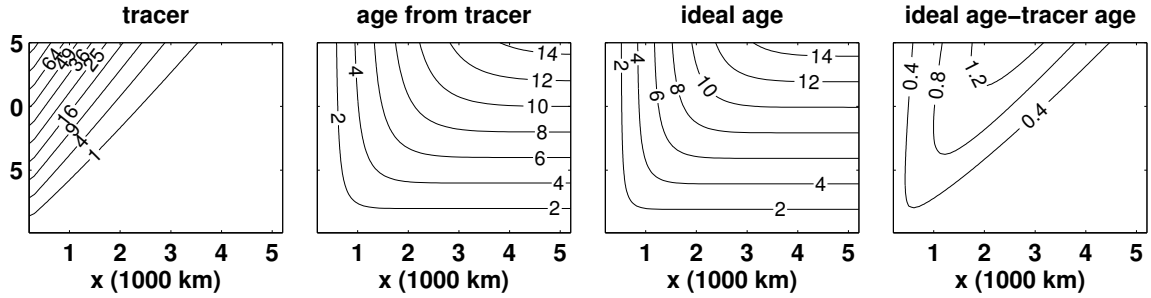

Figure 1. One dimensional example of calculating age from a transient tracer based on the functional formulation. From left to right: tracer concentration as function of space and time, approximate age calculated from the tracer concentration, solution to ideal age equation, and difference between ideal age and age calculated from tracer concentration.

Table 1. Approximate age tracer from tracer concentration for different temporal forms of the surface boundary conditions of non-radioactive transient tracers.

$S(t)$	$\tau(C,S)$
at^b	$\left(\left(\frac{S}{a}\right)^{1/b} - \left(\frac{C}{a}\right)^{1/b}\right)$
$a(e^{bt}-1)$	$(1/b)\ln\left(\frac{C+a}{S+a}\right)$

that the CFC concentration of this water parcel at the observed time, $C(t)$ is equal to the surface concentration at a time $t - T_{age}$ when the parcel was at the surface, $S(t - T_{age})$. If, for example, $S(t) = at^b$ for any time, we can write $S(t - T_{age}) = a(t - T_{age})^b = C(t)$. Therefore $t - T_{age} = (C/a)^{1/b}$. Now, we subtract the observed time written in terms of the surface boundary condition $t = (S/a)^{1/b}$ to get $T_{age} = (S/a)^{1/b} - (C/a)^{1/b}$, in agreement with our formulation. Thus, the relation between apparent age and the ideal tracer advection-diffusion equation for a particular surface boundary condition is clear.

Note that our formulation has several advantages. First, it allows to derive explicit expression for the age from tracer concentration. Second, it allows to use more than one transient (non radioactive) tracer in the functional approach, as shown for example for the radioactive tracers above.

Finally, our approach is easily repeated for other forms of $S(t)$, and a few specific examples of age tracers for different choices of the boundary condition $S(t)$ are shown in Table 1.

3.1. A one dimensional example

Let us demonstrate the above approach using a one dimensional pipe-like transient tracer C model. The equations for the tracer and for the ideal age τ_{ideal} are

$$\begin{aligned} \frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} &= \kappa\frac{\partial^2 C}{\partial x^2}, \\ \frac{\partial \tau_{ideal}}{\partial t} + u\frac{\partial \tau_{ideal}}{\partial x} &= \kappa\frac{\partial^2 \tau_{ideal}}{\partial x^2} + 1. \end{aligned} \tag{28}$$

where the spatial dimension x represents the distance of a water parcel from the surface into the deep ocean. Assume typical boundary conditions of a transient tracer such that, for example,

$$C(x=0,t) = at^b; \qquad \tau_{ideal}(x=0,t) = 0. \tag{29}$$

Setting the model parameters to be $\kappa = 1.0e7cm^2/sec$, $u = 0.5cm/sec$, $b = 2$, $a = 0.5/year^b$, and the size of the domain to $L = 5000km$, we plot in Fig. 1 the tracer concentration, the age estimated from this tracer based on the formula in Table 1, the ideal age tracer solution and the difference between the ideal and tracer ages. Clearly the fit between the ideal age and the age calculated from the tracer concentration is satisfactory, indicating the potential usefulness of our approach. Note that in our formulation for the case of a polynomial (in time) surface boundary condition there are two terms. The first is calculated directly from the surface boundary condition $S(t)$, and has therefore no contribution to the spatial structure of the age field. This term by itself would result in purely horizontal contour lines in Fig. 1. The second term is the contribution of the tracer, which represents the ventilation and renewal of the sub-surface water, and reduces the age based on the observed spreading of the tracer. The differences between the ideal age and that calculated from our formulas are due to the nonlinear mixing effects.

3.2. An example application: approximate age based on CFC concentration in the North Atlantic Ocean

We now illustrate the approach using CFC observations in the North Atlantic ocean [Doney and Bullister, 1992]. The time history of surface concentration of tracers is not known precisely, but estimates of atmospheric concentration exist based on model simulations and recent measurements of monthly mean tropospheric values; for a detailed description of how these annual mean mixing ratios were obtained, see Walker et al. [2000]. These values were converted to an equilibrated surface concentration ($S(t)$) assuming fixed gas exchanges conditions. CFC 12 has the steepest temporal history curve (left panel of Fig. 2) so we use it to compute an age on the World Ocean Circulation Experiment (WOCE) section A16, 20W, 15N - 60N (Iceland). A saturation factor needs to be used to convert atmospheric values to the surface concentration history $S(t)$ [Warner and Weiss, 1985]. For the present calculation, the saturation factor is assumed to be constant, based on the solubility at a salinity of 35ppt and temperature of 10.5°C. Clearly we are ignoring the effects of variable solubility due to surface temperature variations, and this might introduces biases into our estimated age field.

The dependence of surface CFC 12 concentration on time is roughly quadratic $S(t) = at^2$, where t is in years since 1950, and $a = 0.0008$. From Table 1 one finds an approximate age of $\sqrt{S(1988)/a} - \sqrt{C/a}$, where $S(t = 1988)$ is the surface concentration during the year 1988 of the hydrographic section.

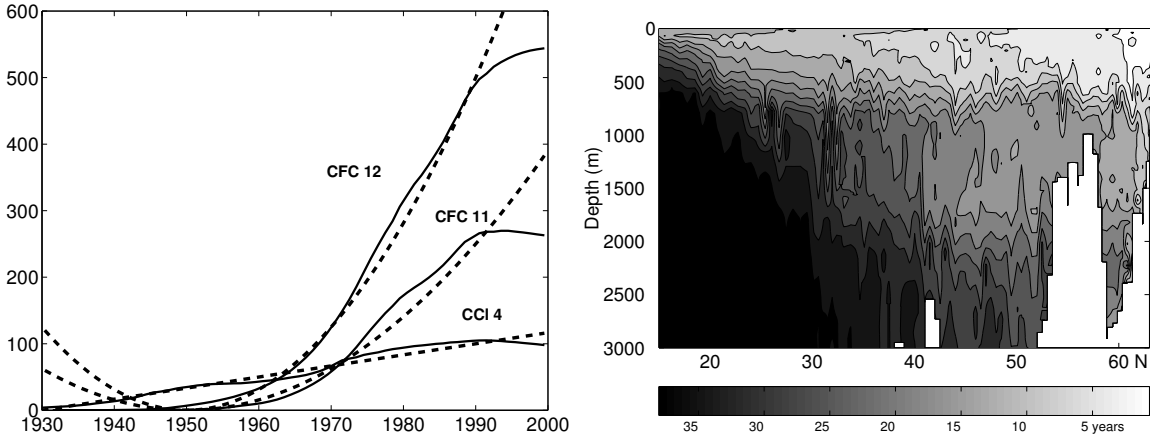

Figure 2. Left panel: atmospheric northern hemisphere mixing ratios from 1930 to 2000 $\pm$ error (which is small, about 1%, and so hardly seen). (Mole fraction, ppt; http://gaslab.ucsd.edu/pub/cfchist/). Quadratic fits to CFC and a linear fit to CCl 4 are also shown by the dash lines. Right panel: a section (depth vs latitude) of approximate age from CFC concentration (Table 1) using the quadratic fit to atmospheric time series for CFC 12, which has the steepest growth rate. White area near surface is zero age; numbers in years; contour interval is 3 years. Data are from the 1988 WOCE Atlantic hydrographic section A16 nominally along 20 W.

Results (right panel of Fig. 2) are consistent with other tracer-based estimates [Doney et al., 1997; Robbins and Jenkins, 1998]. We do not claim our age estimate is better or worse (it is most likely worse because we are ignoring solubility effects, which could lead to biases in the age estimates), but only wish to demonstrate that the polynomial approximation produces plausible results in at least one simple application. Further investigation using multiple tracers in combination with our approach could be useful in unraveling transport rates and mechanisms from the ventilated layers near the surface.

4. Conclusions

The age of water masses, reflecting their travel time from the surface, is a useful quantitative measure for understanding the oceanic circulation as well as in interpreting the ocean's role in climate. The determination of the age of ocean water masses may be formulated as an inverse problem in which the age equation as well as any tracer data are imposed as constraints [Wunsch, 2002]. However, it still seems useful to use the observed transient tracer concentrations in a more direct way to calculate and plot the age field in the ocean. We proposed here a functional approach to deriving approximate age tracers from any two (or more) radioactive transient tracers with two different half-life times, or from a non-radioactive transient tracer(s) with a time-varying surface boundary condition. We demonstrated our approach using a one dimensional advection diffusion equation, as well as using CFC WOCE data, and found the results to be encouraging. While both of these examples resulted in seemingly satisfactory results, indicating that the method is feasible, it should be noted that these are still weak tests of the method.

Our proposed approach is limited to finding the ideal age, which is equal at steady state to the first moment of the age spectrum (or transient time distribution) [Holzer and Hall, 2000; Haine and Hall, 2002; Hall and Haine, 2002; Khatiwala et al., 2001], which more accurately represents the age of a given water mass. The relation between the different moments of the age spectrum and different surface boundary conditions that may be used to calculate them has been explored by Waugh et al. [2003, 2002].

The usefulness of the approach proposed here will clearly need to be demonstrated by a more detailed application to various transient tracers than was possible here.

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