Lecture 2

ENSO toy models

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2.3 A heuristic derivation of a delayed oscillator equation

Let us consider first a heuristic derivation of an equation for the sea surface temperature in the East Pacific, which will be followed by a more rigorous derivation in the following sections. Assume that the East Pacific SST affects the atmospheric heating and thus the central Pacific wind speed. The resulting wind stress, in turn, excites equatorial Kelvin and Rossby ocean waves. These waves affect the East Pacific thermocline depth and hence the East Pacific equatorial SST, and the whole feedback loop may be quantified as follows. Let τ_K and τ_R be the basin crossing times of equatorial Kelvin and Rossby waves, correspondingly. Now, a positive central Pacific equatorial thermocline depth anomaly $h_{eq}(x_c)$ at time $t-\frac{1}{2}\tau_K$ excites an eastward propagating downwelling Kelvin wave at the central Pacific that arrives after about $\frac{1}{2}\tau_K \approx 1$ month to the eastern Pacific and deepens the thermocline there. Similarly, a negative off-equatorial depth anomaly (that is, a shallowing signal of the thermocline) in the central Pacific $h_{off-eq}(x_c)$ at a time $t-[\frac{1}{2}\tau_R+\tau_K]$ ($\frac{1}{2}\tau_R+\tau_K \approx 6$ months) excites a westward propagating Rossby wave at the central Pacific that is reflected off the western boundary as an equatorial Kelvin waves and eventually arrives to the eastern Pacific at time t, shallows the thermocline there and causes cooling of the SST. We add a nonlinear damping term that can stabilize the system, and write an equation for the eastern Pacific temperature that includes the Kelvin wave, Rossby wave and local damping terms as follows

$$\frac{dT(t)}{dt} = \hat{a}h_{eq}(x_c, t-\frac{1}{2}\tau_K) + \hat{b}h_{off-eq}(x_c, t-[\frac{1}{2}\tau_R+\tau_K]) - cT(t)^3$$

where $\hat{a}, \hat{b}, c$ are positive constants. Note that we assume that once the thermocline deepening or shallowing signal reaches the East Pacific it immediately affects the SST there. This actually neglects the SST adjustment time and we will include this time scale in the more rigorous derivation below. Note that because the mean thermocline depth is shallower in the East Pacific than in the West Pacific, a deepening or rise of the thermocline in the East Pacific is able to affect the mixing between cool sub-thermocline waters and surface waters, and thus affect the SST; in the West Pacific, the thermocline is deeper, so that even if it rises somewhat, it is still too deep to affect the SST. Now, the thermocline depth in the equatorial central Pacific is a response to the equatorial central Pacific wind. The off-equatorial thermocline depth in the central Pacific will be shown below to be a response to the wind curl off the equator, which will be shown to be negatively correlated with the wind stress at the equator. We can therefore write the above equation as

$$\frac{dT(t)}{dt} = \bar{a}\tau_{eq}(x_c, t-\frac{1}{2}\tau_K) - \bar{b}\tau_{eq}(x_c, t-[\frac{1}{2}\tau_R+\tau_K]) - cT(t)^3$$

where $\bar{a}, \bar{b}$ are some new proportionality constants. Next, the wind stress in the Central Pacific is a pretty much simultaneous response to the East Pacific SST, so that we can actually write

$$\frac{dT(t)}{dt} = aT(t-\frac{1}{2}\tau_K) - bT(t-[\frac{1}{2}\tau_R+\tau_K]) - cT(t)^3 \tag{9}$$

where again the constants of proportionality a, b, c are all positive. The first term in this equation provides a positive feedback due to the Kelvin wave, with a short delay of about one month; the second term represents the Rossby wave with a longer-delayed negative feedback, and the last term is a nonlinear damping term. This last equation (9) is the desired delayed oscillator equation for El Nino.

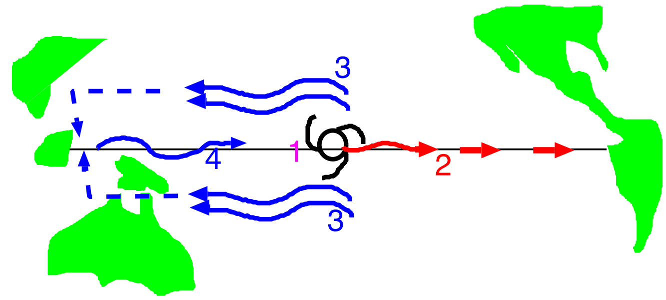

Figure 21: A schematic picture of the delayed oscillator mechanism

Based on the delayed oscillator model, the El Nino cycle may be described as follows (Fig. 21). A wind weakening (1 in Fig. 21)) creates an equatorial warm (downwelling) Kelvin wave (2) that travels to the East Pacific within 1-2 months, where the thermocline deepening induces an SST heating and starts an El Niño event. The SST heating further weakens the central Pacific winds and the event is therefore amplified by ocean-atmosphere instability. The original wind weakening also creates off-equatorial cold (upwelling) Rossby waves (due to the induced changes to the wind curl, as will be shown below) (3) that are reflected from the western boundary as cold Kelvin waves (4), arrive at the eastern boundary about 6 months later and terminate the event.

Note that we ignored reflection at the eastern boundary. Much of the energy of an eastward traveling equatorial Kelvin wave incident on the eastern boundary will be reflected as poleward traveling coastal Kelvin waves and will escape from the equatorial domain. In contrast, westward traveling Rossby waves incidenting on the western boundary are reflected as equatorward traveling coastal Kelvin waves. These, in turn, are reflected eastward at the equator as equatorial Kelvin waves, hence creating an efficient reflection process in which the wave energy ermains in the equatorial strip.

2.3.1 Analysis of the delayed oscillator equation

Battisti [2] and Suarez and Schopf [56] (see also Dijkstra [9] section 7.5.4.2) have used a slightly different delayed oscillator equation, basically ignoring the shorter Kelvin wave delay, which in a nondimensional form is

$$\frac{dT(t)}{dt} = T(t) - \alpha T(t-\delta_T) - T^3(t). \tag{10}$$

Note first that a delayed equation formally has an infinite number of degrees of freedom (it requires an infinite number of initial conditions corresponding to the times from $t = -\delta_T$ to $t = 0$, and is thus equivalent to an infinite number of ODEs). So formally this is not a "simple" equation. Only a few of these degrees of freedom are actually activated in reasonable parameter regimes (as measured by the dimension of the attractor in phase space). The various delayed oscillator equations result in El-Nino like oscillations whose periods may be tuned, by changing the coefficients, to about 4 years (Fig. 22).

Let us analyze the linearized stability behavior of 10. The equilibria of the above delayed oscillator equation are the zero solution, and then one warm solution and one cold solution

$$\bar{T} = 0, \pm\sqrt{1-\alpha}.$$

Considering a perturbation about these steady states by setting $T = \bar{T} + \tilde{T}$ and linearizing, we have

$$\frac{d\tilde{T}(t)}{dt} = \tilde{T}(t)(1 - 3\bar{T}^2) - \alpha\tilde{T}(t-\delta_T).$$

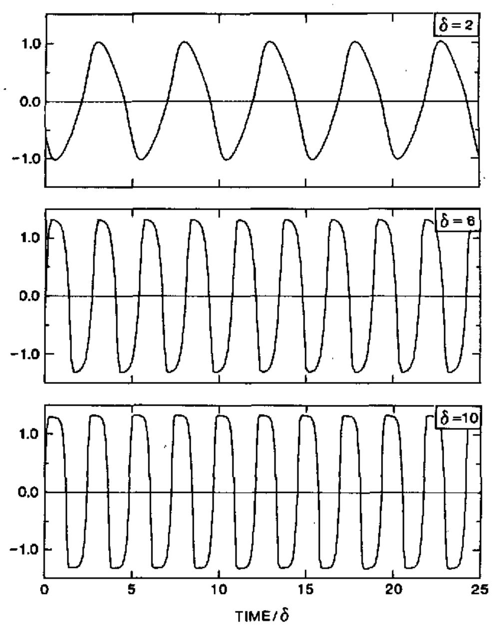

FIG. 4. Behavior of the nonlinear oscillator. (a) $\alpha = 0.75$, $\delta = 2$, (b) $\alpha = 0.75$, $\delta = 6$, and (c) $\alpha = 0.75$, $\delta = 10$. The time axis is scaled in units of the delay.

Figure 22: Results of the delayed oscillator of equation 10, from [56].

Letting $\tilde{T} = e^{\sigma t}$ where $\sigma = \sigma_r + i\sigma_i$, results in the linearized eigenvalue problem

$$\sigma = 1 - 3\bar{T}^2 - \alpha e^{-\sigma\delta_T}$$

(note that this is a complex transcendental equation, with the real and imaginary parts of σ satisfying equations that involve sine and cosine functions) which can be solved for the frequency σ as function of the two nondimensional parameters α and δ_T. It turns out that the zero solution is unstable, with a non oscillatory exponential growth. The two other (warm and cold) equilibria may become oscillatory unstable, as shown in Fig. 23.

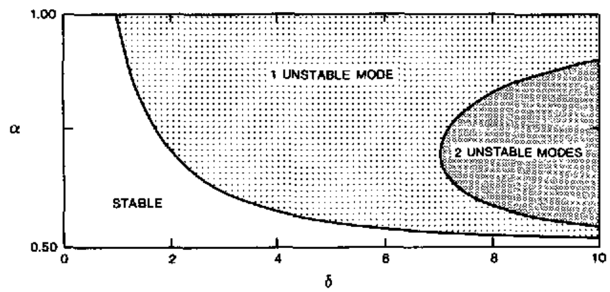

FIG. 2. Neutral stability curves of the outer stationary solution. Parameters lying below the lower line are stable. An infinite number of additional neutral curves exist to the right of the lines shown, but are only found for large δ.

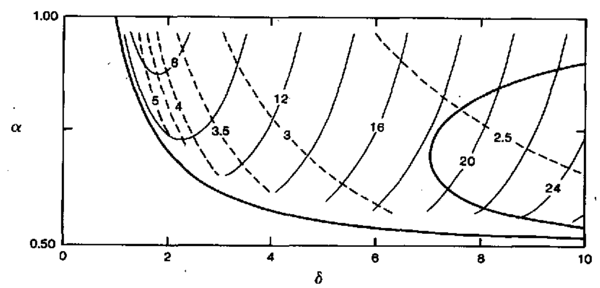

FIG. 5. The fundamental period of the nonlinear oscillator found numerically. The heavy solid lines are the neutral curves of the linear problem, reproduced from Fig. 2. The light solid contours give the period of the oscillation ($2\pi/\sigma_i$), while the dashed contours present the period in multiples of the delay.

Figure 23: Stability and period of the delayed oscillator of equation 10; Suarez & Schopf [56].

The behavior of the unstable modes is not completely simple nor intuitive: the unstable modes appear for larger values of the negative feedback (Rossby term) α, and for larger values of the delay time δ... The period of the oscillatory solutions in the delay model is shown by the light solid lines in Fig. 23, while the dashed contours give the period in multiples of the delay time. The period of the unstable modes is in the range of up to 2-3 times the Rossby delay time. Taking that delay time to be some 8 months, we get a 16-24 months period, which is significantly smaller than the observed period of 48 months. Clearly the period is not a well determined part of the picture, as it is not a robust outcome of this model, and has reasonable values for a fairly small range of model parameters. Other studies [36] also found that the period of ENSO may not be well determined by linearized theories, and may be due to some not understood nonlinear effects.

While the delayed oscillator model above is useful in providing us with a feeling of what the mechanism of ENSO is, it actually represents only a specific limit of the fuller dynamics. It assumes that once the waves arrive to the East Pacific, they immediately influence the SST. In reality, there is another time scale (delay) that accounts for the time it takes the sub-surface thermocline depth anomalies in the eastern Pacific to affect the eastern Pacific SST. To introduce this and other processes, it is useful to go through a more rigorous derivation, starting from the β plane equations.

2.4 Fast SST, fast wave and mixed mode ENSO regimes

2.4.1 Ocean dynamics

Let us represent the equatorial dynamics using the two-strip approximation of Jin [27, 28] with an equatorial strip and an off-equatorial strip. The equations for the ocean wave dynamics for each strip are then solved by integrating them along wave characteristics following Galanti and Tziperman [12]. Further simplification is achieved by neglecting the meridional damping ($-\varepsilon_m v$) and the meridional wind stress ($\tau_y/\rho H$) terms. Yet another simplification is obtained by taking the long wave approximation, which results in dropping the time derivative from the y momentum equation. This occurs because the meridional velocity v scales like $C_o\frac{\lambda}{L}$ while the zonal velocity scales like C_o, where C_o, λ, L are the gravity wave speed, meridional scale (equatorial Rossby radius) and the long zonal scale of the wave, respectively, and because $\frac{\lambda}{L} \ll 1$. The resulting set of equations is

$$\begin{aligned} \frac{\partial u}{\partial t} - \beta y v + g'\frac{\partial h}{\partial x} &= -\varepsilon_m u + \frac{\tau_x}{\rho H}, \\ \beta y u + g'\frac{\partial h}{\partial y} &= 0, \\ \frac{\partial h}{\partial t} + H\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right] &= -\varepsilon_m h, \end{aligned} \tag{11}$$

where ε_m is the oceanic damping coefficient. Eliminating u and v from (11), a single equation for h may be obtained,

$$\beta y^2(\partial_t + \varepsilon_m)h + \frac{g'H}{\beta}\left[\frac{2}{y}\partial_y - \partial_{yy}\right](\partial_t + \varepsilon_m)h - g'H\partial_x h + \frac{1}{\rho}(\tau_x - y\partial_y\tau_x) = 0. \tag{12}$$

Next, evaluate this equation at the equator ($y = 0$), and at a zonal band off the equator ($y = y_n$). This "two-strip" approximation assumes that the ocean dynamics in the equatorial region is well represented by a combination of equatorial Kelvin waves and off-equatorial long Rossby waves, both well represented by the two strips at latitudes $y = 0$ and $y = y_n$.

A Kelvin wave solution of the form

$$h(x, y, t) = h_e(x, t)\exp(-\frac{\beta}{2C_o}y^2), \tag{13}$$

satisfies equation (12), and therefore, taking advantage of the known meridional structure, we can get an equation for a forced and dissipated Kelvin wave at $y = 0$ of the form

$$(\partial_t + C_o\partial_x + \varepsilon_m)h_e = \frac{1}{C_o\rho}\tau_{ex}, \tag{14}$$

where τ_{ex} is the wind stress at the equator and $C_0 = \sqrt{g'H}$. The rhs forcing for the Kelvin waves is proportional to the wind stress, and we shall see below that this implies that a weakening of the easterlies results in the excitation of warm Kelvin waves. Next, integrate (14) over the trajectory of an eastward propagating Kelvin wave that starts from the western boundary at a time $t - \tau_2$ and reaches the eastern boundary at a time t, where

$\tau_2 = L/C_o$ is the Kelvin crossing time of a basin of length L. The wave is assumed to be excited by the wind stress in the central part of the basin, from $x = x_W + .25L$ to $x = x_W + .75L$. The wind stress is evaluated at the middle of the basin, $x = x_w + L/2$, at a time $t - \tau_2/2$, which is the time when the Kelvin wave crosses the middle of the basin. We denote the equatorial thermocline depth anomaly at the western (eastern) edge of the basin by h_{eW} (h_{eE}), and the solution to (14) obtained by integrating along characteristics is then

$$h_{eE}(t) = h_{eW}(t-\tau_2)e^{-\varepsilon_m \tau_2} + \frac{1}{\rho C_o} dt \tau_2 \tau_{ex}(\frac{L}{2}, t - \frac{\tau_2}{2}) e^{-\varepsilon_m \frac{\tau_2}{2}}, \tag{15}$$

where $dt = 0.5$ is the fraction of crossing time during which the wind stress affects the oceanic waves.

Next, we wish to solve (12) at the off-equatorial band ($y = y_n$), in order to include the Rossby wave dynamics in the model. It can be shown that at $y_n \geq 2L_o$ (where L_o is the oceanic Rossby radius of deformation) the second term in (12) is negligible [27, 28], resulting in the off-equatorial equation for a forced and dissipated Rossby wave

$$(\partial_t - \frac{C_0^2}{\beta y_n^2}\partial_x + \varepsilon_m) h_n = \frac{1}{\beta\rho}\left[\frac{\partial}{\partial y}(\frac{\tau_x}{y})|_{y=y_n}\right]. \tag{16}$$

Note that the rhs forcing for the Rossby waves is the curl of the wind this time, and we shall use this below to show that a weakening of the easterlies results in the excitation of cold Rossby waves. Solving (16) again along characteristics, for a Rossby wave that starts from the eastern boundary at time $t - \tau_1$, where $\tau_1 = L y_n^2 \beta / c^2$ is the Rossby crossing time of a basin length L, at a latitude y_n, we find

$$h_{nW}(t) = h_{nE}(t-\tau_1)e^{-\varepsilon_m \tau_1} - \frac{1}{\beta\rho} dt \tau_1 \left[\frac{\partial}{\partial y}(\frac{\tau_x}{y})|_{(y_n, \frac{L}{2}, t - \frac{\tau_1}{2})}\right] e^{-\varepsilon_m \frac{\tau_1}{2}}. \tag{17}$$

The eastern and western boundary conditions represent the reflection of Kelvin waves into Rossby waves at the east, and the reflection of Rossby waves into Kelvin waves at the west. In terms of the thermocline depth at the boundaries, these boundary conditions are

$$h_{eW} = r_W h_{nW}, \qquad h_{nE} = r_E h_{eE}, \tag{18}$$

where r_W and r_E are reflection coefficients at the western and eastern boundaries, respectively. Using the above boundary conditions, (15) and (17) may be joined to give an expression for the equatorial thermocline depth anomaly at the eastern Pacific,

$$\begin{aligned} h_{eE}(t) &= r_W r_E h_{eE}(t - \tau_1 - \tau_2) e^{-\varepsilon_m(\tau_1+\tau_2)} && \text{free RW reflected as KW} \\ &- \frac{1}{\beta\rho} dt \tau_1 [\frac{\partial}{\partial y}(\frac{\tau_x}{y})|_{(y_n, \frac{L}{2}, t - \tau_2 - \frac{\tau_1}{2})}] e^{-\varepsilon_m \frac{\tau_1}{2}}] && \text{forced RW reflected as KW} \\ &+ \frac{1}{\rho C_o} dt \tau_2 \tau_{ex}(\frac{L}{2}, t - \frac{\tau_2}{2}) e^{-\varepsilon_m \frac{\tau_2}{2}} && \text{forced KW} \end{aligned} \tag{19}$$

This form of equation manifests clearly the delayed dependence of $h_{eE}(t)$ on the wave dynamics. The first term represents the effects of a thermocline depth anomaly at the eastern boundary at a time $t - \tau_1 - \tau_2$. This anomaly is reflected poleward and then propagates as a free Rossby wave. This wave in turn, is reflected at the western boundary as a Kelvin wave at time $t - \tau_2$ and arrived at the eastern Pacific at time t. The second term represents the Rossby waves excited at a time $t - \tau_2 - \tau_1/2$ in the central Pacific, and the third represents the Kelvin waves excited at a time $t - \tau_2/2$. To calculate the forced RW terms explicitly, we need to discuss the SST and atmospheric dynamics now.

2.4.2 SST response to thermocline movements

The equation describing SST changes at the equator is based on that of Zebiak and Cane [66]. Following Jin [27, 28], we only keep the time rate of change, the advection by the mean upwelling $\overline{w}\frac{\partial T}{\partial z}$, and the damping terms,

$$\partial_t T = -\varepsilon_T T - \gamma \frac{\overline{w}}{H_1}(T - T_{sub}(h)), \tag{20}$$

where ε_T is a thermal damping coefficient, $T_{sub}(h)$ is the temperature anomaly at some specified constant depth H_1 (not to be confused with other H_1s appearing above...), and is a function of the thermocline depth anomaly h, typically taken as some hyperbolic tangent [66]. The parameter $0 < \gamma < 1$ relates the temperature anomalies entrained into the surface layer to the non local deeper temperature variations due to $T_{sub}(h)$.

2.4.3 Wind response to SST forcing

Based on the solution to Gill's model above (8), we take the wind stress to be a function of the SST at the equator (T_e) which decays in latitude according to the *atmospheric* Rossby radius of deformation L_a

$$\tau_x(x, y, t) = \mu A(T_e, x) \exp(-\frac{y^2\alpha}{2L_o^2}). \tag{21}$$

In this last formula, $\alpha = (\frac{L_o}{L_a})^2$, $A(T_e, x)$ is a non local function that relates the equatorial SST to the wind stress, and μ serves as a relative coupling coefficient. The wind stress terms in (19) may thus be expressed as

$$\begin{aligned} \tau_{xe} &= \mu A(T_e, x), \\ \partial_y(\tau_x/y)|_{y=y_n} &= -\mu A^* A(T_e, x), \end{aligned}$$

where

$$A^* = \left[\frac{L_o^2 + \alpha y_n^2}{(y_n L_o)^2}\right] e^{-\frac{y_n^2\alpha}{2L_o^2}},$$

$A(T_e, x)$ is obtained by solving a Gill-type atmospheric model [19] using a long wave approximation (see section 1.2.5above, or Hao et al. [21]), resulting in a linear relation between the wind stress and the equatorial SST. As derived in (8), the wind stress in the central Pacific may be assumed to be proportional to the temperature anomaly in the East Pacific (this implies that the information about the East Pacific heating is propagated in the atmosphere by atmospheric Rossby waves to affect the wind stress in the central Pacific)

$$A(T_e, x = x_w + L/2) = b_0 T_{eE}(t), \tag{22}$$

where b_0 is the annual mean coupling strength. The assumption embedded in (22) is that most of the SST variability and thus atmospheric heating is in the eastern part of the equatorial Pacific. The resulting wind stress anomaly, according to the Gill model, will reach the central Pacific where it affects the ocean wave dynamics [27, 28].

2.4.4 Mixed mode ENSO model

The expression (19) for the East Pacific thermocline depth may now be written more explicitly, using the above equations as

$$\begin{aligned} h_{eE}(t) &= r_W r_E h_{eE}(t - \tau_1 - \tau_2)e^{-\varepsilon_m(\tau_1+\tau_2)} \\ &- r_W \frac{1}{\beta\rho} A^* dt\tau_1 \mu b_0 T_{eE}(t - \tau_2 - \frac{\tau_1}{2})e^{-\varepsilon_m(\frac{\tau_1}{2}+\tau_2)} \\ &+ \frac{1}{\rho C_o} dt\tau_2 \mu b_0 T_{eE}(t - \frac{\tau_2}{2})e^{-\varepsilon_m\frac{\tau_2}{2}}, \end{aligned} \tag{23}$$

expressing h_{eE} at time t as function of h_{eE} and T_{eE} at previous times. As before, the first term represents the free Rossby and Kevin waves, the second represents the excited Rossby wave, and the third represents the excited Kelvin wave. The thermodynamic equation (20) evaluated at the eastern side of the basin gives the dynamical equation in which the above $h_{eE}(t)$ is used

$$\partial_t T_{eE} = -\varepsilon_T T_{eE} - \gamma \frac{\overline{w}}{H_1}(T_{eE} - T_{sub}(h_{eE})). \tag{24}$$

Equations (23) and (24), together with an explicit expression for T_{sub}, form the mixed mode model originally derived in a slightly different format (no explicit delays) by Jin [27, 28], and then re-derived in the present form by [12]. The mixed mode dynamics and its fast wave and fast SST limits were originally investigated by Neelin and Jin [29, 30, 38]. Hereafter we denote T_{eE} by T and h_{eE} by h. Note that the nonlinearity in the model is due to the nonlinear function $T_{sub}(h)$.

The mechanism of the oscillation in this mixed mode model is similar to that of the above heuristic delayed oscillator, except that there is an additional explicit delay time due to the time it takes the SST in the East Pacific to adjust to changes in the thermocline depth there. An alternative description of the mechanism has been used by Jin [27, 28], emphasizing water transport rather than wave propagation, and is shown in Fig. 24.

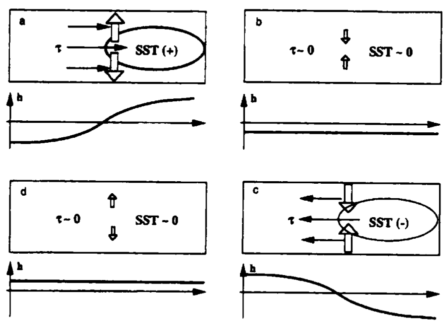

Figure 7.37. Sketch of the different stages of the recharge oscillator (Jin, 1997a).

Figure 24: The recharge oscillator mechanism (Jin, 1997).

2.4.5 The fast SST Limit

In the fast SST limit, the SST adjustment time is assumed to be much shorter than the ocean dynamics adjustment time, or in other words SST is assumed to respond instantaneously to thermocline depth changes [37, 29, 30, 38]. We obtain our model equations for this limit by taking the term $\partial_t T_{eE}$ in (24) to be zero, so that the SST equation becomes a diagnostic equation balancing the Newtonian cooling and the upwelling terms, and giving an instantaneous relation between the thermocline depth anomaly $h(t)$ and the SST $T(t)$

$$T(t) = \gamma \frac{\overline{w}}{H_1} \left(\varepsilon_T + \gamma \frac{\overline{w}}{H_1} \right)^{-1} T_{sub}(h). \tag{25}$$

The oscillation mechanism in this case is pretty much the same as of the heuristic delayed oscillator described above.

2.4.6 The fast wave limit

In the fast wave limit, the Rossby and Kelvin wave propagation times are assumed to be much shorter than the SST adjustment time of the SST to thermocline perturbations. The wave speeds are actually assumed to be infinite, resulting in an instantaneous adjustment of ocean thermocline depth and current velocities to wind stress anomalies [21]. Hence, the SST adjustment time to thermocline depth changes is the only delay and plays the central role in the physical mechanism of the oscillations obtained in this parameter regime. The fast wave limit results in somewhat unrealistic oscillations, in comparison to both ENSO's time scale and amplitude, as this is not a realistic ENSO regime. Nevertheless, it is still useful to analyze this regime, since it reveals some new aspects that are not considered in the previous two regimes.

The fast wave limit can be derived by taking the time derivatives in the ocean momentum equation to be zero. In the fast wave limit, the dynamics crucially depend on the east-west tilt of the thermocline. One variant of the fast wave limit is obtained by dividing the basin into two boxes, one for the East Pacific and one for the central Pacific. The full derivation of the model equations may be found in [12], and it is a simplification into a system of ODEs based on the PDE model of Hao et al. [21]. The two SST tendency equations for the two regions are

$$\partial_t T_c = -\varepsilon_T T_c - \gamma \frac{\overline{w}}{H_1} T_c + \gamma \frac{\overline{w}}{H_1} T_{sub} \left(h_c(T_c, T_e) \right), \quad (26)$$

$$\partial_t T_e = -\varepsilon_T T_e - \gamma \frac{\overline{w}}{H_1} T_e + \gamma \frac{\overline{w}}{H_1} T_{sub} \left(h_e(T_c, T_e) \right), \quad (27)$$

where T_c and T_e are the SST in the central Pacific and the East Pacific respectively, and the dependence of T_{sub} on T_c and T_e is via the thermocline depth anomalies h_c and h_e. The oscillatory mechanism of the eastward propagating fast wave oscillations is explained in Hao et al. [21]. Given the lack of wave delay time, the coupled system memory required for an oscillation resides in the different response rates of the SST to thermocline displacements at different longitudes. This may result in either westward propagation or eastward propagation (not in the above two box model, but in a continuous representation of the fast wave regime [21]). It is possible to obtain different time scales from 2 yr to much longer, as well as relaxation oscillations, and an example from Jin and Neelin [29, 30, 38] is shown in Fig. 25.

References for this lecture are at the end of Lecture 9.

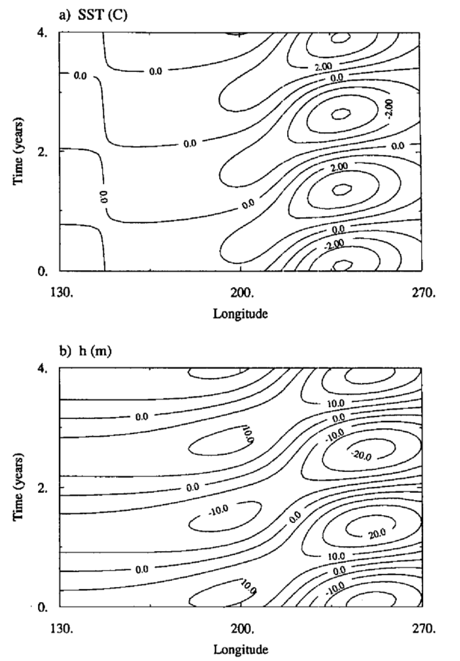

FIG. 8. Time–longitude plot of an eastward-propagating SST mode in the fast-wave limit: (a) SST, (b) thermocline depth anomalies along the equator constructed from the eigenvector for $\mu = 0.9$, $\delta_s = 0$, and $\epsilon_a = 4.0$ (units: °C and m, respectively, up to a normalization factor).

Figure 25: An oscillation of the equatorial Pacific in a model of the fast wave regime (Jin and Neelin, 1993)