Lecture 8

Glacial-interglacial variability: phenomenology and dynamical background

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8.7 A brief description of the phenomenology

Information about climate history over the past two million years or so is obtained from two main sources [4]. The first is sediment cored from the ocean's bottom, where the isotopic compositions of buried plankton skeletons and other buried material are used as proxy indicators to past climate, and are related empirically to past ice land volume, paleo temperature, etc (Fig. 3). The second source is ice core records from Antarctica and Greenland, that contain again various isotopic records as well as trapped gas bubbles and atmospheric dust from the past 400,000 years or so (Fig. 37).

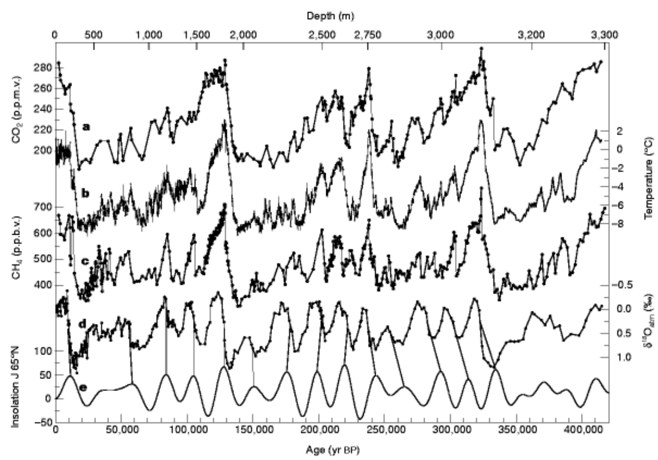

Figure 37: Vostok ice core record for atmospheric gasses.

A brief summary of some of the observed glacial cycle characteristics for the past 2 Myr follows:

1. Roughly a 100 kyr time scale between glaciations during the past 800 kyr.
2. Saw-tooth structure: long glaciations (Order 90,000 yr), short deglaciations (5-10,000 yr).
3. A transition from 41 kyr to 100 kyr glacial cycles about 800 kyr ago.
4. Atmospheric CO_2 variations during the glacial cycles.
5. Some phase locking to Milankovitch forcing (this forcing is explained in section 8.8.5).
6. Global extent of the glacial signal.

Besides the need for a theory that explains these observations, we also need to address the following questions regarding the cycles' dynamics:

1. Are the cycles externally forced? by what? or perhaps internally produced (self sustained) within the climate system?
2. Are the cycles produced by the physical climate components (i.e. excluding CO_2 variations that are likely due to a biogeochemical mechanism)? By the biogeochemical components? Both? Only amplified by CO_2 variations that are, in turn, induced by the cycle in the physical system? Which components of the physical climate system participate in the glacial dynamics and on what time scales?
3. Are the cycles driven from the northern hemisphere, where most of the land ice volume changes occur, or from some other region? what phase lags should we expect between northern & southern hemispheres?

We proceed now with a description of some basic climate feedbacks that are likely to be important in glacial dynamics, and then describe a few of the toy models/ mechanisms for the glacial cycles which have been proposed over the years and that are based on these climate feedbacks.

8.8 Basics and relevant climate feedbacks

8.8.1 Energy balance, and the ice albedo feedback

As a crude simplification, one may write a simple equation for a globally averaged temperature T in which incoming solar radiation $H_{\odot}^{\downarrow}$ is partially reflected by the earth albedo, and partially compensated by long wave modified black body radiation $e\sigma T^4$ (see Lectures 6-7).

$$\frac{dT}{dt} = H_{\odot}^{\downarrow} \times (1 - \text{albedo}) - e\sigma T^4 \tag{31}$$

This equation implies, of course, that a higher albedo results in a cooling effect, which is a feedback that will play a significant role in the followings due to the albedo effect of changing land and sea ice covers, as follows. Given the higher albedo of land and sea ice relative to that of the land or ocean, larger ice covered area results in larger albedo, and based on the above simple energy balance argument, in a lower temperature:

$$\begin{aligned} \text{albedo} &\propto \text{land ice and sea ice area} \\ \text{albedo} &\uparrow \implies \text{temperature} \downarrow \end{aligned}$$

8.8.2 Ice sheet dynamics and geometry

(Ghil and Childress, [15]; or Paterson, [42]). Glaciers flow as non-Newtonian fluids. The flow is governed by the stress-strain or stress-rate of strain relation. (The stress tensor $\tau = \tau_{ij}$, denoting force per unit area, could be shear stress for $i \neq j$ or normal stress for $i = j$. The strain tensor e_{ij} is the displacement or deformation per unit length). Some examples of stress-strain relations are (Fig. 38):

- Elastic materials (Hook's law, metals for low stress): $\tau = Be$
- Plastic materials (metals beyond their critical stress): no deformation below a critical yield stress, and then beyond that point arbitrarily large deformation with no increase in stress.
- Viscous Newtonian fluid: $\tau = \nu(de/dt)$.
- For ice, there is Glenn's law, $de/dt = A(T)\tau^n$, where $A(T)$ is exponential in the temperature (warm ice is softer...) and $n = 3$ is a typical value that fits laboratory data etc. Note that $n \to \infty$ corresponds to plasticity. Extending Glenn's law from the 1d normal stress-strain relation to the relation between the full tensors τ_{ij} and e_{ij} is nontrivial, see [42]...

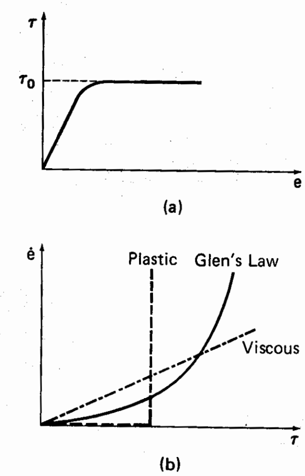

Fig. 11.4. Constitutive relations for a metal (a) and for ice (b) (after Paterson, 1980).

Figure 38: Strain-stress relations for ice and some other materials (Ghil and Childress Fig. 11.4).

Parabolic profile of ice sheets: consider a balance of forces for a glacier that is symmetric in longitude x. The glacier height as function of latitude is $h(y)$. The balance of forces on a slice of the glacier between latitudes $(y, y + dy)$ is between hydrostatic pressure integrated along the face of the slice, and stress applied by bottom friction (Fig. 39)

$$\int_0^{h(y+dy)} \rho_{ice} gzdz - \int_0^{h(y)} \rho_{ice} gzdz = \tau(y, z = 0)dy$$

or simply

$$h(y)\frac{dh}{dy}\rho_{ice}g = \tau(y, z = 0) = \tau_0$$

where we assume that the bottom is at the yield stress τ_0 (i.e. glacier in a "critical" state). In other words, we assume perfect plasticity: glacier yields to the hydrostatic-induced stress at the above critical stress. The solution to the last equation gives the desired parabolic profile that is not a bad fit to observations (Fig. 40).

$$\frac{1}{2}h(y)^2 = \frac{\tau_0}{g\rho_{ice}}(y - y_0).$$

Accumulation/ ablation: The area of an ice sheet is divided into an accumulation zone and an ablation zone. The net accumulation minus ablation depends on both the latitude and the height of the ice sheet surface. The interaction between the mass balance and the elevation is complex... On the one hand, there is the Elevation-desert effect: as the ice sheet surface reaches higher elevations, the amount of precipitation on it is reduced due to the decreased humidity content of the atmosphere with height. However, increased elevation also means colder temperatures and therefore decreased ablation. This effect is often assumed dominant. The line at which the net accumulation minus ablation is zero (equilibrium line, $E(y)$) starts at sea level at some northern latitude, and increases in elevation southward, to compensate for the increased surface temperature.

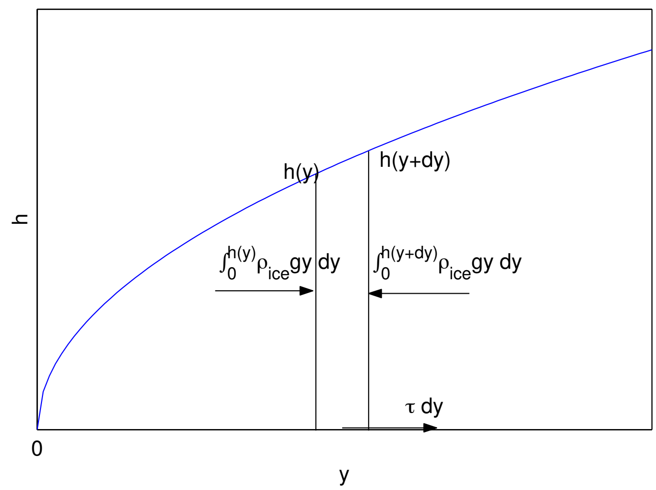

Figure 39: Force balance on a slice of an ice sheet, used to deduce the parabolic profile.

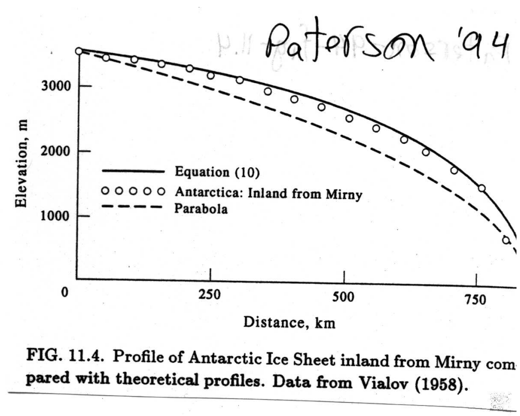

Figure 40: Fit of parabolic profile for ice sheet geometry to observations. From Paterson [42].

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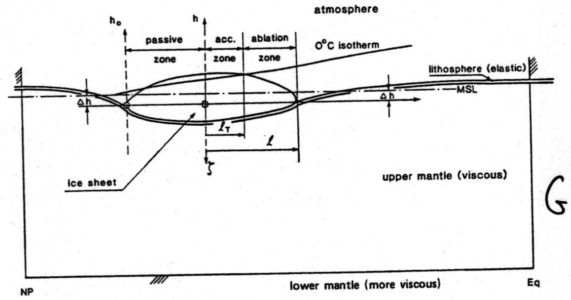


Ghil '94

Fig. 7. Meridional cross section through the Earth's upper strata, Eq. (4.6b) (after [4]). Modeling is restricted to a single ice-sheet, in the Northern Hemisphere: while the Antarctic ice sheet is currently much larger than the Greenland one, the former ice sheet has changed little in size over the Quaternary [4,6,7, and references therein].

Figure 41: ice sheet geometry and the equilibrium line separating accumulation and ablation zones (Figure 7 from Ghil [14]).

Given these considerations, the source term for an ice sheet mass balance is written as (Oerlemans, Pollard)

$$G = \begin{cases} a(h + h' - E(y)) - b(h + h' - E(y))^2 & h + h' - E < 1500m \\ c & h + h' - E > 1500m \end{cases} \tag{32}$$

Ice streams: The ice in ice sheets flows from the accumulation to the ablation zones at an averaged velocity of meters to tens of meters per year (Fig. 42). However, some 90% of the discharge flow in glaciers actually occurs in rapid and narrow ice streams (velocities can reach 4 km/yr, which is 100-1000 times that of a laminar ice sheet flow) that occupy only a small area of the ice sheet. These ice streams are also transient in time rather than in a slow uniform and steady flow (Figs. 43, 44). The dynamics of ice streams are complex and not fully understood. Among the relevant feedbacks are some that involve the melting and deformation of the till below the ice stream. Another feedback is induced by the bottom topography and the internal heat of deformation: a flow of a glacier over a bump induces larger ice velocities, and therefore in increased heating due to internal glacier deformation; this softens the ice and affects the coefficient in Glenn's law ($A(T)$), therefore increasing the ice velocity again.

Calving processes: (Pollard [50], Fig. 45) When the ice sheets are sufficiently large and heavy, they deform the earth crust (see section 8.8.4), sink below sea level, and may be floated by incoming sea water. This detaches them from the bedrock and may cause a rapid ice flow/ sliding to the ocean. A simple parameterization of this calving process in the framework of the above 1D ice sheet model is to add the following term to the ablation parameterization [50, 48, 49]

$$G(x_{i+1}) = -20\, m\ yr^{-1} \qquad if\ \rho_{ice} h(x_i) < \rho_w (S - h'(x_i))\ and\ h'(x_{i+1}) < S \tag{33}$$

where S is the sea level, the first above conditions requires that sea level is large enough to be able to float the ice, and the second condition is that sea ice can reach point x_i (Fig. 45). The calving process may be triggered as follows: suppose that the ice sheet has reached a maximum size that causes a significant bedrock depression, and that at that stage an increased summer radiation due to Milankovitch cycles caused some retreat of the southern ice tip into the depression formed by the isostatic adjustment (both Milankovitch cycles and isostatic adjustment are discussed below). Because the bedrock takes some time to respond to the new position of the ice sheet, the gap that is formed between the ice sheet and the depressed bedrock allows sea water to penetrate the empty depression and float the ice sheet, causing the calving process. There is also a positive feedback involved with the

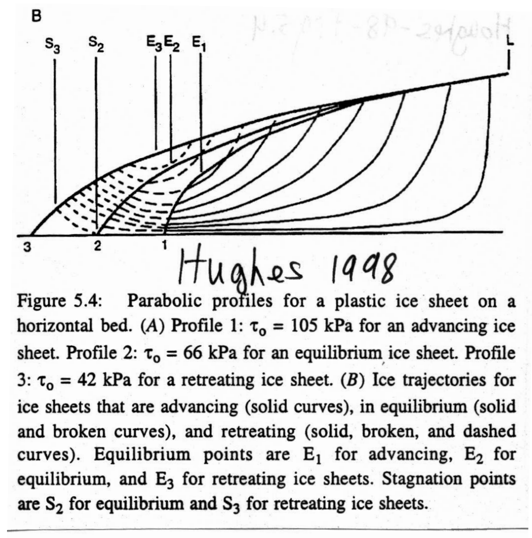


Figure 42: Schematic ice flow in an ice sheet [24].

Figure 43: Ice stream locations in Antarctica (http://nsidc.org/NASA/RAMP/icestreamb_mapw.html, http://web.mit.edu/dabrams/www/).

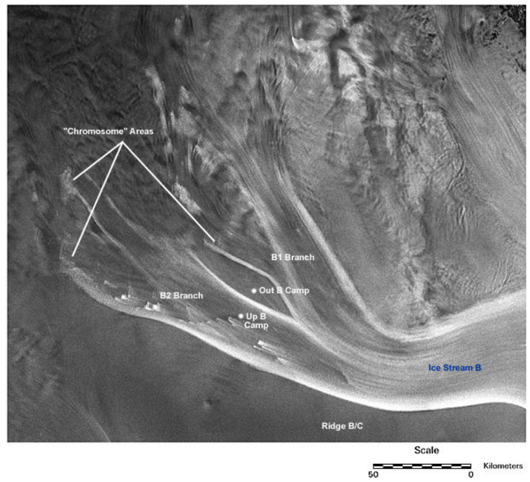


Figure 44: Ice stream b in Antarctica (http://nsidc.org/NASA/RAMP/icestreamb_mapw.html, http://-web.mit.edu/dabrams/www/).

calving process (Watts and Hayder [65]): once some calving occurs, it raises the sea level, and therefore reinforces the floating of more ice and induces yet more calving. Presently, 80% of the ablation in Greenland is due to calving, although Peltier and Marshal [44] find that this process is not sufficient to eliminate the Laurentide ice sheet during the deglaciation stage in their model.

There are other instability processes that were used in various models as part of the ablation parameterization, such as specifying that when ice sheets get too large they collapse due to gravitational instability, etc...

Dust loading and enhanced ablation: (Peltier and Marshal [44]) The atmosphere during glacial periods, being drier, more windy (due to the larger meridional temperature gradient), and having larger exposed continental shelves, contains larger amounts of aerosols and dust. Continental dust is up to 10-30 times that during interglacials, and marine aerosols (salt) up to 3-4 times more. Dust loading could affect the albedo of ice sheets in ablation areas, which are relatively narrow strips along the boundary of the ice sheet where ablation is larger than accumulation. The dust cover may reduce the surface albedo there, therefore causing the the ice sheet to absorb more solar radiation and enhancing melting. A reduction of the dirty snow/ ice albedo from 0.7 to 0.1-0.4 results in the surface absorbing 2-3 times more radiation, much of which causes enhanced ice melting. Peltier and Marshal [44] find that the dust loading effect is critical for getting rapid terminations in their ice sheet model. However, this is only an indirect indication of the importance of dust loading, as they parameterize the effects of dust on ablation directly, rather than deal with the effect of dust on the albedo, which then affects the radiation absorption and eventually the melting.

8.8.3 Temperature-precipitation feedback

Ice core proxy observations show that the rate of accumulation of snow over land ice sheets is significantly higher during relatively warm periods (Fig. 46). GCM experiments indicate a similar trend. In particular, greenhouse scenarios show that as the temperature increases, accumulation increases initially faster than ablation, so that the net accumulation is larger for warmer temperatures. Once the warming passes some threshold, the increase in ablation surpasses that in precipitation, so that the net accumulation finally decreases with temperature (Figs. 47, 48). The increase in net accumulation over land ice with increased temperature seems to have been quite robust during glacial-interglacial cycles, and has been termed the temperature-precipitation feedback [14].

There are several mechanisms that could be responsible for this feedback. First, higher temperature implies

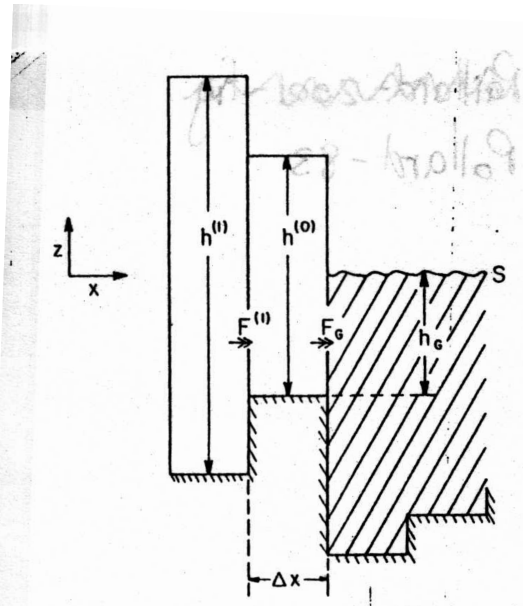


FIG. 6. Finite-difference representation of the ice-shelf parameterization at the grounding line (Eq. (7)). Only the outermost two grid boxes of the ice sheet are shown, as are the first two grid points occupied by the water body (hatched). $h^{(0)}$ and $h^{(1)}$ are the ice thicknesses in each box, h_G is taken as the ice thickness at the grounding line, $F^{(1)}$ and F_G are fluxes of ice between grid boxes, S is sea level, and Δx is the horizontal grid size. Pollard '83

Figure 45: Calving parameterization, from Pollard 1983.

larger moisture content of the atmosphere based on the Clausius Clapeyron relation, and therefore a stronger hydrological cycle. Second, at least some of the precipitation falling on northern hemisphere land ice sheets is due to local evaporation from the polar and high latitude ocean. During sufficiently cold periods, the high latitude ocean is covered by (perennial and seasonal) sea ice which significantly reduces evaporation from the ocean, and therefore limits the precipitation of snow over the land ice. Finally, the presence of even seasonal sea ice may shift the storm track away from the land ice sheets, thus again reducing the precipitation brought by winter storms from falling on the ice sheets.

The temperature-precipitation feedback plays quite an important role in a number of glacial cycle theories as we shall see below.

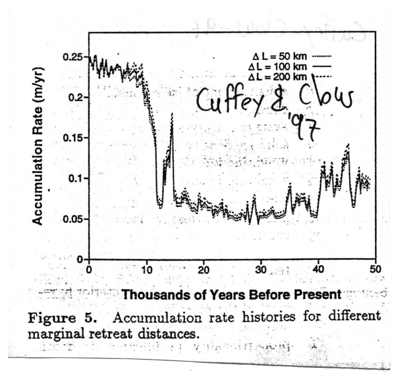


Figure 46: accumulation rate for warm and cold periods, showing the temperature-precipitation feedback. Fig. 5 from Cuffey and Clow [8].

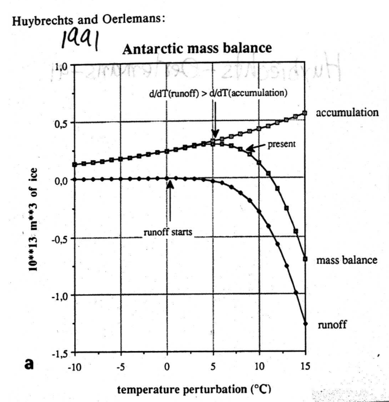
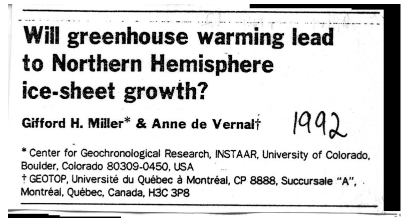


Figure 47: Mass balance of ice sheet as function of temperature, showing temperature-precipitation feedback for small temperature increases, and increased ablation dominating for a larger temperature increase. Fig. 5a from Huybrechts and Oerlemans [25].



Will greenhouse warming lead to Northern Hemisphere ice-sheet growth?

Gifford H. Miller* & Anne de Vernal† 1992

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Figure 48: Temperature-precipitation feedback in the news... [35]

8.8.4 Isostatic adjustment

Ice density is roughly a $\frac{1}{3}$ of the earth interior density. Ice sheets therefore sink into the crust roughly a $\frac{1}{3}$ of their height, and this process is referred to as the "isostatic adjustment" (Fig. 49).

This adjustment process is not immediate and there is a time scale of a few thousands of years involved. Let us derive an equation for an ice sheet evolution including the isostatic adjustment effect (Oerlemans [39]; Pollard [50, 48, 49]). Start with a simple relation of a Glenn's law type between the vertical average velocity of the ice sheet and the shear stress at the bottom

$$\mathbf{u} = B\vec{\tau}_b^m$$

In principle, B is a function of the temperature, and m might change depending on sliding conditions at the base of the glacier (frozen/ melted), but let us assume they are both constant. We can show, based on similar arguments to those used for deriving the parabolic glacier geometry, that

$$\vec{\tau}_b = \rho_{ice} g h \frac{\partial h^*}{\partial y}$$

where h is the ice thickness and $h^* = h + h'$ is the ice surface elevation, and where h' is the elevation of the bedrock above some reference level (Fig. 50).

Now, the (1 dimensional) mass continuity of the glacier is simply

$$\frac{\partial h}{\partial t} = \frac{\partial}{\partial y}(hu) + G(h, y, t)$$

where $G(h, y, t)$ is the net accumulation-ablation. Substituting the above expression for the velocity

$$\begin{aligned}\frac{\partial h}{\partial t} &= \frac{\partial}{\partial y}(hB(\rho_{ice} g h \frac{\partial}{\partial y}(h + h'))^m) + G \\ &= A \quad \frac{\partial}{\partial y}(h^{m+1}(\frac{\partial}{\partial y}(h + h'))^m) + G\end{aligned}$$

which may be written as a nonlinear diffusion process

$$\begin{aligned}\frac{\partial h}{\partial t} &= \frac{\partial}{\partial y}(D(h)\frac{\partial}{\partial y}(h + h')) + G(h, y, t) \\ D(h) &= h^{m+1}(\frac{\partial}{\partial y}(h + h'))^{m-1} \qquad (34)\end{aligned}$$

Now, the flow within the upper part of the earth interior (lithosphere...) can be modeled as an adjustment to perturbations introduced due to the ice sheet load that penetrates down into the lithosphere to a depth rh where

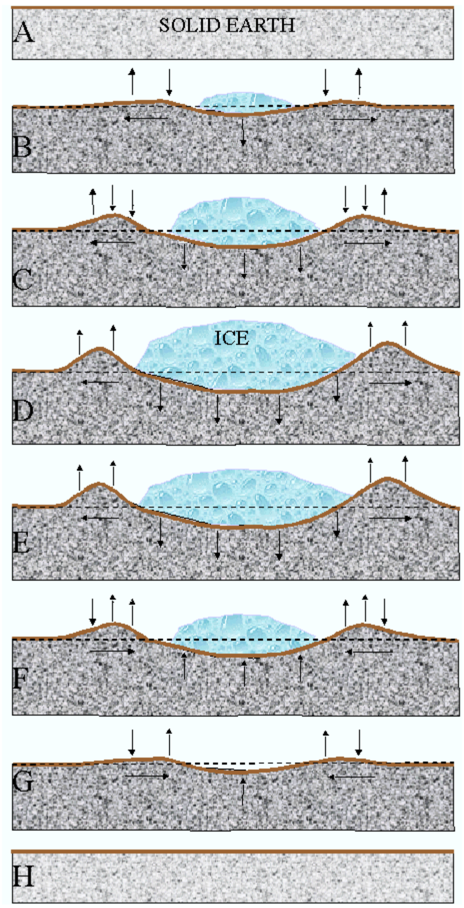

Figure 49: Schematic plot of isostatic adjustment for changing ice sheet volume, from http://rgalp6. harvard.edu/background.html.

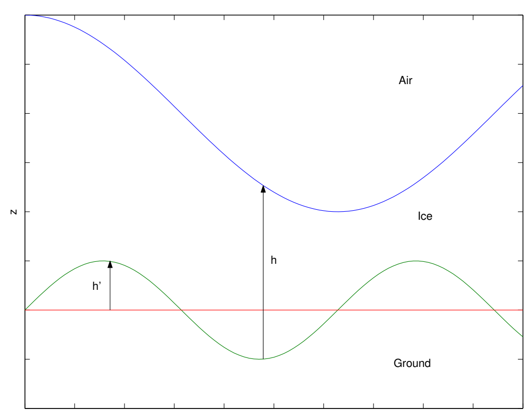

Figure 50: Ice sheet geometry and variables for the isostatic adjustment model.

$r = \frac{1}{3}$, plus an inherent topography of the crust $h'_0(y)$. The adjustment could be modeled using a simple time scale $T_{isostatic}$,

$$\frac{\partial h'}{\partial t} = (h' - h'_0(y) + rh)/T_{isostatic}$$

or using a scale selective adjustment using a simple diffusion law

$$\frac{\partial h'}{\partial t} = \nu \frac{\partial^2}{\partial y^2}(h' - h'_0(y) + rh) \tag{35}$$

The typical time scale for the isostatic adjustment seems to be about 3000 yr. Equations (34, 35) provide us with an ice sheet model based on the isostatic adjustment feedback, where in order to solve for the glacier distribution history we still need to specify the ice source/ sink (accumulation/ ablation) G, as in (32) and (33), for example.

8.8.5 Milankovitch forcing

One of the main ingredients for many glacial cycle theories is the temporal changes in the solar radiation arriving to the earth surface due to changes in orbital parameters of the earth around the sun. A useful recent review is given by Paillard [41]. The commonly used orbital parameters and the corresponding time scales at which they change are as follows, following Paillard [41]. (Figs. 51, 52). **Eccentricity**, e, with a time scale of about 100kyr: corresponds to changes in the elongation of the ellipse along which earth circles the sun; affects the annual mean, global mean radiation (though only by a very small factor of about, has a negligibly weak climatic effect. Note that Earth's trajectory was nearly spherical 400 kyr ago. **Obliquity**, ε, 41 kyr: the tilt of earth's axis, varies due to the torque acting on the earth by the sun and moon because of its equatorial bulge, that is, because the earth is not perfectly spherical; the corresponding amplitude in solar radiation changes is a few (5-15) $watt/m^2$, and the effect is on the annual mean contrast between the poles and the equator, as well as on the contrast between the seasons, and is of the same magnitude in both hemispheres. **Precession**, γ, has time scales of 19 kyr and 23 kyr. It corresponds to the circular motion of the earth's rotation axis in space, and has a climatic effect only when the earth orbit is not exactly spherical. The amplitude of changes in solar radiation is of the order of 20% ($O(100)\, watt/m^2$), and the effect is antisymmetric with respect to seasons and hemispheres. Because it does not have any climatic effect when earth's trajectory is exactly spherical, the climatic precession parameter is defined to be proportional to e. NO effect on annual mean radiation or globally mean radiation.

The main effect of Milankovitch forcing is not due to the direct effect of changes in the solar radiation on the atmospheric energy balance and therefore on the atmospheric temperature, but rather due to its effect on glacier ablation (Held, [22]): ice is a very poor heat conductor, and the heating by solar radiation therefore remains near the ice sheet surface. Thus a 25% variation in the amplitude of summer radiation can cause a significant change in surface ice temperature, which can therefore also strongly control summer melting. Note that most of the melting occurs within a few weeks during the summer. A change in the summer solar radiation can bring the ice surface temperature to above or below the melting temperature, while in the winter the temperature is too low for the Milankovitch variations to be able to bring the temperature to the melting point. This is why the Milankovitch summer radiation is what counts, and not the annual average nor the winter solar radiation.

8.8.6 More feedbacks

The above list of climate feedbacks that may participate in glacial-interglacial dynamics is far from complete. In particular, we have concentrated only on feedbacks of the physical climate system, ignoring biogeochemical feedbacks that will be briefly mentioned below. There are certainly some additional physical feedbacks that have been neglected, such as the geothermal heating at the base of ice sheets, and numerous others... In any case, the above ingredients already allow us to describe some of the existing theories for the glacial cycles, which are the subject of the next lecture.

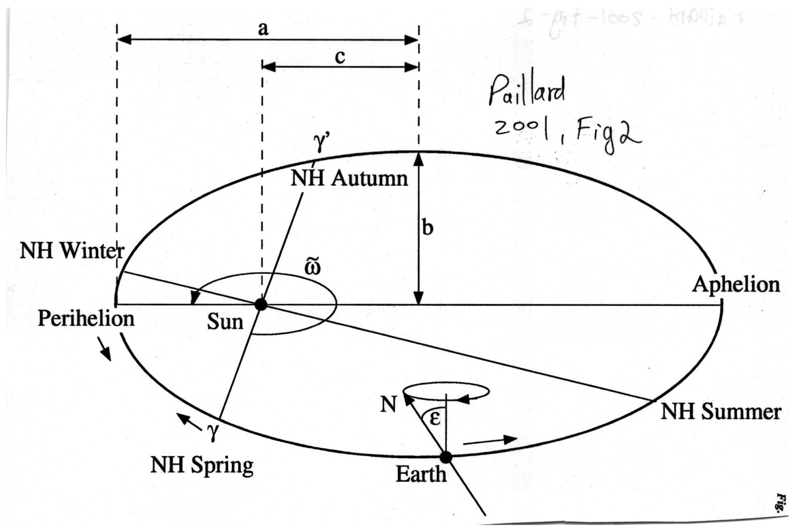

Figure 51: Orbital parameters (Paillard 2001, Fig. 2).

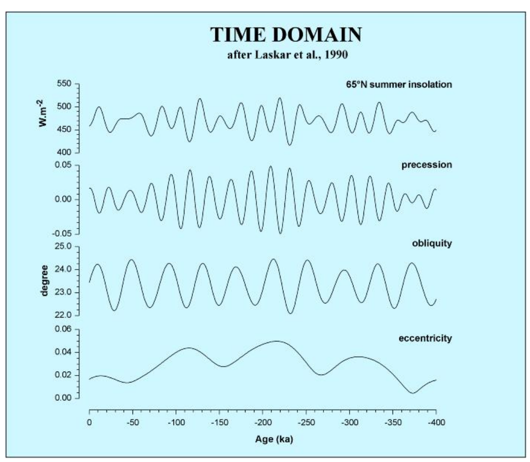

Figure 52: Milankovitch radiation in the time domain.

References for this lecture are at the end of Lecture 9.