CS 124 Challenge Problem 10

April 23, 1997

Tourist Attractions

Suppose you are given a map of an abritrary country that is labelled with tourist attractions. Assume that you have a weighted-graph representation for this map.

We have studied graph algorithms that take as input an arbitrary graph with weighted edges, and two nodes, and discover the shortest path between the two nodes. However, a shortest-path need not be a unique occurence in a graph. For example, the graph in Figure 1 exhibits two paths of weight 13 between A and B, both of which are shortest paths.

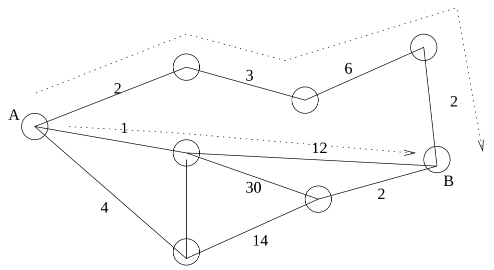

Figure 1: Two different shortest-paths between A and B.

Being the adventurous traveller that you are, you are not interested in just the shortest path– you would like to take a path from A to B that crosses through as many nodes as possible, but is no *longer* than the shortest path between A and B. The length of a path is measured by the sum of the individual edge weights of the path. So in other words, you want to see as many tourist attractions as possible, without making any sacrifice in terms of the distance you travel.

1. First, devise an algorithm that accepts $\{G, A, B, k\}$ and returns YES if and only if there exists a shortest-path between A and B that crosses exactly k nodes, and NO otherwise.

2. How would you use the above algorithm to solve the original problem?

3. As a caveat, we request that you formulate this problem with a matrix representation. As a hint, does *repeated matrix squaring* reduce the running-time?

Be sure to analyze your algorithm and prove its correctness, including perhaps, a proof of associativity in step 3.

Rules

Please limit the text of your solution to one typewritten page (you can add relevant diagrams on additional pages if you choose). You need not describe every detail of this problem in your writeup, since you will also be giving a presentation of your solution later on. Rather, we ask that you outline the major steps or insights of your approach. Place the names of the people in your group on the top of your written submission. Please limit presentations to 7 minutes to allow for a few minutes of questions. Each group can structure their presentation in anyway that they choose. We will post a schedule for presentations later next week.