cs124 section 8

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April 16, 1997

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In all the network flow problems, there are three principle constraints that define the problem:

1.

2.

3.

In order to solve this problem, lets define some terminology:

- Residual network
- augmenting flow
- cut
- capacity

Theorem 1 *Max-Flow Min-Cut*

Given a graph, $G = \{V, E\}$ with source and sink, s, t and flow, f, the following statements are equivalent:

1. f is a maximum flow
2. G has no more augmenting paths
3. $|f| = c(S, T)$ for some cut (S, T) of a G

Now (1) $\rightarrow$ (2) is obvious. If we have a maximum flow, then there is no way to find an augmenting path!

For (2) $\rightarrow$ (3), we know that G has no augmenting paths, so we can define a nontrivial set S that consists of all nodes that are reachable from the source. Let $T = V - S$, be the set of leftovers. Since there are no augmenting paths, we know that all the edges between the nodes in S and the nodes in T are *full* (i.e, the flows on those edges are the same as the capacities of those edges). We know the flow in this network is precisely the flow across this cut. However, we also know that the flow is the capacity of the cut since there are no augmenting paths, hence we have $|f| = c(S, T)$.

For (3) $\rightarrow$ (1), we need a lemma:

Lemma 1 *Bottleneck Lemma*

Any flow over G must be less than or equal to the capacity of any cut of G.

The proof of this is in CLR on page 592. Intuitively, we are saying that any flow that goes through the network is constrained by the smallest bottleneck in the network, i.e, the strength of a chain is precisely its weakest link.

Hence, if $|f| = c(S, T)$, then this means that f must be the maximum flow, because all flows must be less than any cut (that is, the capacity of any cut), and hence, there can be no other flow that is larger than f.

1.1 example

Here is yet another simple example:

There is a famine in Moscow. Airlift 20 tons of food from Austin Texas to Moscow by tomorrow evening!

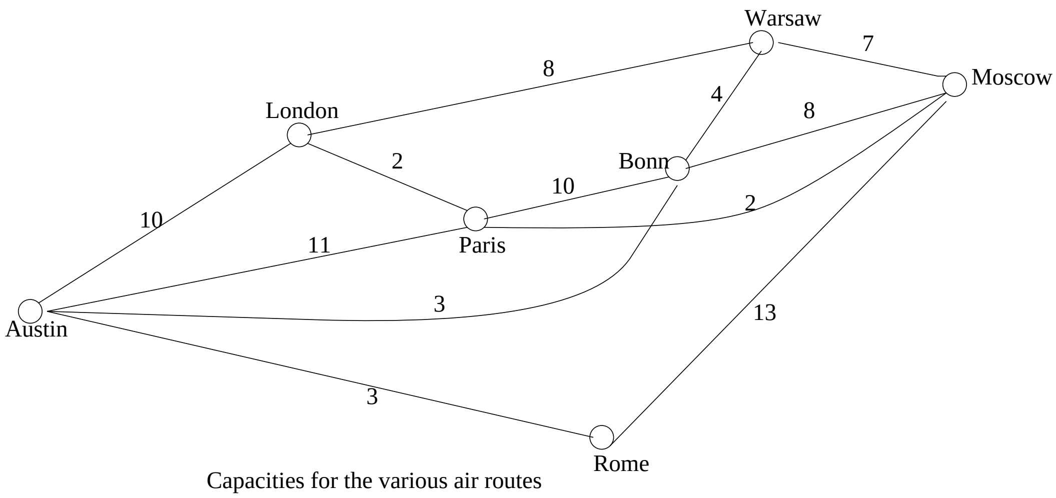

Figure 1:

1.2 questions

How do you find the minimum cut? Now suppose you incremented the capacity of one edge by 1. How would you find the new max flow?