Name:______________________________________

CS152, Spring 2011, Final Examination
May 7, 2011

Rules:

- The exam is closed-book, closed-notes, except for one side of one 8.5x11in piece of paper.
- **Please stop promptly at Noon.**
- You can rip apart the pages.
- There are **135 points** total, distributed **unevenly** among **11** questions, most of which have multiple parts.

Advice:

- Read questions carefully. Understand a question before you start writing.
- Write down thoughts and intermediate steps so you can get partial credit.
- The questions are not necessarily in order of difficulty. **Skip around.** In particular, make sure you get to all the problems.
- If you have questions, ask.
- Relax. You are here to learn.

For your reference (page 1 of 2):

$$
\begin{array}{lcl}
e & ::= & \lambda x.\ e \mid x \mid e\ e \mid c \mid \{l_1 = e_1, \ldots, l_n = e_n\} \mid e.l_i \mid \mathsf{fix}\ e \\
v & ::= & \lambda x.\ e \mid c \mid \{l_1 = v_1, \ldots, l_n = v_n\} \\
\tau & ::= & \mathsf{int} \mid \tau \to \tau \mid \{l_1 : \tau_1, \ldots, l_n : \tau_n\}
\end{array}
$$

$\boxed{e \to e'}$

$$
\frac{}{(\lambda x.\ e)\ v \to e[v/x]} \qquad \frac{e_1 \to e_1'}{e_1\ e_2 \to e_1'\ e_2} \qquad \frac{e_2 \to e_2'}{v\ e_2 \to v\ e_2'} \qquad \frac{e \to e'}{\mathsf{fix}\ e \to \mathsf{fix}\ e'} \qquad \frac{}{\mathsf{fix}\ \lambda x.\ e \to e[(\mathsf{fix}\ \lambda x.\ e)/x]}
$$

$$
\frac{}{\{l_1 = v_1, \ldots, l_n = v_n\}.l_i \to v_i}
$$

$$
\frac{e_i \to e_i'}{\{l_1 = v_1, \ldots, l_{i-1} = v_{i-1}, l_i = e_i, \ldots, l_n = e_n\} \to \{l_1 = v_1, \ldots, l_{i-1} = v_{i-1}, l_i = e_i', \ldots, l_n = e_n\}}
$$

$\boxed{\Gamma \vdash e : \tau}$

$$
\frac{}{\Gamma \vdash c : \mathsf{int}} \qquad \frac{}{\Gamma \vdash x : \Gamma(x)} \qquad \frac{\Gamma, x : \tau_1 \vdash e : \tau_2}{\Gamma \vdash \lambda x.\ e : \tau_1 \to \tau_2} \qquad \frac{\Gamma \vdash e_1 : \tau_2 \to \tau_1 \qquad \Gamma \vdash e_2 : \tau_2}{\Gamma \vdash e_1\ e_2 : \tau_1} \qquad \frac{\Gamma \vdash e : \tau \to \tau}{\Gamma \vdash \mathsf{fix}\ e : \tau}
$$

$$
\frac{\Gamma \vdash e_1 : \tau_1 \qquad \ldots \qquad \Gamma \vdash e_n : \tau_n \qquad \text{labels distinct}}{\Gamma \vdash \{l_1 = e_1, \ldots, l_n = e_n\} : \{l_1 : \tau_1, \ldots, l_n : \tau_n\}} \qquad \frac{\Gamma \vdash e : \{l_1 : \tau_1, \ldots, l_n : \tau_n\} \qquad 1 \leq i \leq n}{\Gamma \vdash e.l_i : \tau_i}
$$

$$
\frac{\Gamma \vdash e : \tau \qquad \tau \leq \tau'}{\Gamma \vdash e : \tau'}
$$

$\boxed{\tau_1 \leq \tau_2}$

$$
\frac{}{\{l_1{:}\tau_1, \ldots, l_{i-1}{:}\tau_{i-1}, l_i{:}\tau_i, \ldots, l_n{:}\tau_n\} \leq \{l_1{:}\tau_1, \ldots, l_i{:}\tau_i, l_{i-1}{:}\tau_{i-1}, \ldots, l_n{:}\tau_n\}}
$$

$$
\frac{}{\{l_1{:}\tau_1, \ldots, l_n{:}\tau_n, l{:}\tau\} \leq \{l_1{:}\tau_1, \ldots, l_n{:}\tau_n\}} \qquad \frac{\tau_i \leq \tau_i'}{\{l_1{:}\tau_1, \ldots, l_i{:}\tau_i, \ldots, l_n{:}\tau_n\} \leq \{l_1{:}\tau_1, \ldots, l_i{:}\tau_i', \ldots, l_n{:}\tau_n\}}
$$

$$
\frac{\tau_3 \leq \tau_1 \qquad \tau_2 \leq \tau_4}{\tau_1 \to \tau_2 \leq \tau_3 \to \tau_4} \qquad \frac{}{\tau \leq \tau} \qquad \frac{\tau_1 \leq \tau_2 \qquad \tau_2 \leq \tau_3}{\tau_1 \leq \tau_3}
$$

$$
\begin{array}{lcl}
e & ::= & c \mid x \mid \lambda x{:}\tau.\ e \mid e\ e \mid \Lambda\alpha.\ e \mid e[\tau] \\
\tau & ::= & \mathsf{int} \mid \tau \to \tau \mid \alpha \mid \forall \alpha.\tau \\
v & ::= & c \mid \lambda x{:}\tau.\ e \mid \Lambda\alpha.\ e
\end{array}
\qquad
\begin{array}{lcl}
\Gamma & ::= & \cdot \mid \Gamma, x{:}\tau \\
\Delta & ::= & \cdot \mid \Delta, \alpha
\end{array}
$$

$\boxed{e \to e'}$

$$
\frac{e_1 \to e_1'}{e_1\ e_2 \to e_1'\ e_2} \qquad \frac{e_2 \to e_2'}{v\ e_2 \to v\ e_2'} \qquad \frac{e \to e'}{e[\tau] \to e'[\tau]} \qquad \frac{}{(\lambda x{:}\tau.\ e)\ v \to e[v/x]} \qquad \frac{}{(\Lambda\alpha.\ e)[\tau] \to e[\tau/\alpha]}
$$

$\boxed{\Delta; \Gamma \vdash e : \tau}$

$$
\frac{}{\Delta; \Gamma \vdash x : \Gamma(x)} \qquad \frac{}{\Delta; \Gamma \vdash c : \mathsf{int}} \qquad \frac{\Delta; \Gamma, x{:}\tau_1 \vdash e : \tau_2 \qquad \Delta \vdash \tau_1}{\Delta; \Gamma \vdash \lambda x{:}\tau_1.\ e : \tau_1 \to \tau_2} \qquad \frac{\Delta, \alpha; \Gamma \vdash e : \tau_1}{\Delta; \Gamma \vdash \Lambda\alpha.\ e : \forall \alpha.\tau_1}
$$

$$
\frac{\Delta; \Gamma \vdash e_1 : \tau_2 \to \tau_1 \quad \Delta; \Gamma \vdash e_2 : \tau_2}{\Delta; \Gamma \vdash e_1\ e_2 : \tau_1} \qquad \frac{\Delta; \Gamma \vdash e : \forall \alpha.\tau_1 \quad \Delta \vdash \tau_2}{\Delta; \Gamma \vdash e[\tau_2] : \tau_1[\tau_2/\alpha]}
$$

For your reference (page 2 of 2):

$$\begin{array}{rcl} e & ::= & \ldots \mid \mathsf{A}(e) \mid \mathsf{B}(e) \mid (\mathsf{match}\ e\ \mathsf{with}\ \mathsf{A}x.\ e \mid \mathsf{B}x.\ e) \mid \mathsf{roll}_\tau\ e \mid \mathsf{unroll}\ e \mid (e, e) \mid e.1 \mid e.2 \\ \tau & ::= & \ldots \mid \tau_1 + \tau_2 \mid \mu\alpha.\tau \mid \tau_1 * \tau_2 \\ v & ::= & \ldots \mid \mathsf{A}(v) \mid \mathsf{B}(v) \mid \mathsf{roll}_\tau\ v \mid (v, v) \end{array}$$

$\boxed{e \to e'}$

$$\frac{e \to e'}{\mathsf{A}(e) \to \mathsf{A}(e')} \qquad \frac{e \to e'}{\mathsf{B}(e) \to \mathsf{B}(e')} \qquad \frac{e \to e'}{\mathsf{match}\ e\ \mathsf{with}\ \mathsf{A}x.\ e_1 \mid \mathsf{B}y.\ e_2 \to \mathsf{match}\ e'\ \mathsf{with}\ \mathsf{A}x.\ e_1 \mid \mathsf{B}y.\ e_2}$$

$$\frac{}{\mathsf{match}\ \mathsf{A}(v)\ \mathsf{with}\ \mathsf{A}x.\ e_1 \mid \mathsf{B}y.\ e_2 \to e_1[v/x]} \qquad \frac{}{\mathsf{match}\ \mathsf{B}(v)\ \mathsf{with}\ \mathsf{A}x.\ e_1 \mid \mathsf{B}y.\ e_2 \to e_2[v/y]}$$

$$\frac{e \to e'}{\mathsf{roll}_{\mu\alpha.\tau}\ e \to \mathsf{roll}_{\mu\alpha\tau}\ e'} \qquad \frac{e \to e'}{\mathsf{unroll}\ e \to \mathsf{unroll}\ e'} \qquad \frac{}{\mathsf{unroll}\ (\mathsf{roll}_{\mu\alpha.\tau}\ v) \to v}$$

$$\frac{e_1 \to e_1'}{(e_1, e_2) \to (e_1', e_2)} \qquad \frac{e_2 \to e_2'}{(v, e_2) \to (v, e_2')} \qquad \frac{e \to e'}{e.1 \to e'.1} \qquad \frac{e \to e'}{e.2 \to e'.2} \qquad \frac{}{(v_1, v_2).1 \to v_1} \qquad \frac{}{(v_1, v_2).2 \to v_2}$$

$\boxed{\Delta; \Gamma \vdash e : \tau}$

$$\frac{\Delta; \Gamma \vdash e : \tau_1 + \tau_2 \qquad \Delta; \Gamma, x{:}\tau_1 \vdash e_1 : \tau \qquad \Delta; \Gamma, y{:}\tau_2 \vdash e_2 : \tau}{\Delta; \Gamma \vdash \mathsf{match}\ e\ \mathsf{with}\ \mathsf{A}x.\ e_1 \mid \mathsf{B}y.\ e_2 : \tau}$$

$$\frac{\Delta; \Gamma \vdash e : \tau_1}{\Delta; \Gamma \vdash \mathsf{A}(e) : \tau_1 + \tau_2} \qquad \frac{\Delta; \Gamma \vdash e : \tau_2}{\Delta; \Gamma \vdash \mathsf{B}(e) : \tau_1 + \tau_2} \qquad \frac{\Delta; \Gamma \vdash e : \tau[(\mu\alpha.\tau)/\alpha]}{\Delta; \Gamma \vdash \mathsf{roll}_{\mu\alpha.\tau}\ e : \mu\alpha.\tau} \qquad \frac{\Delta; \Gamma \vdash e : \mu\alpha.\tau}{\Delta; \Gamma \vdash \mathsf{unroll}\ e : \tau[(\mu\alpha.\tau)/\alpha]}$$

$$\frac{\Delta; \Gamma \vdash e_1 : \tau_1 \qquad \Delta; \Gamma \vdash e_2 : \tau_2}{\Delta; \Gamma \vdash (e_1, e_2) : \tau_1 * \tau_2} \qquad \frac{\Delta; \Gamma \vdash e : \tau_1 * \tau_2}{\Delta; \Gamma \vdash e.1 : \tau_1} \qquad \frac{\Delta; \Gamma \vdash e : \tau_1 * \tau_2}{\Delta; \Gamma \vdash e.2 : \tau_2}$$

Module Thread:

```
type t
val create : ('a -> 'b) -> 'a -> t
val join : t -> unit
```

Module Mutex:

```
type t
val create : unit -> t
val lock : t -> unit
val unlock : t -> unit
```

Module Event:

```
type 'a channel
type 'a event
val new_channel : unit -> 'a channel
val send : 'a channel -> 'a -> unit event
val receive : 'a channel -> 'a event
val choose : 'a event list -> 'a event
val wrap : 'a event -> ('a -> 'b) -> 'b event
val sync : 'a event -> 'a
```

Futures:

```
type 'a promise
val future : (unit -> 'a) -> 'a promise
val force : 'a promise -> 'a
```

Some Haskell primitives (some specialized to the IO Monad):

```
(:) :: a -> [a] -> [a] -- "cons"
(++) :: [a] -> [a] -> [a] -- "append"
undefined :: a -- raises exception if needed
(>>=) :: IO a -> (a -> IO b) -> IO b -- "bind"
return :: a -> IO a -- "return"
```