

Name: _____

For your reference:

$s ::= \text{skip} \mid x := e \mid s; s \mid \text{if } e \text{ } s \text{ } s \mid \text{while } e \text{ } s$
 $e ::= c \mid x \mid e + e \mid e * e$
 $(c \in \{\dots, -2, -1, 0, 1, 2, \dots\})$
 $(x \in \{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{y}_1, \mathbf{y}_2, \dots, \mathbf{z}_1, \mathbf{z}_2, \dots, \dots\})$

$H; e \Downarrow c$

$\text{CONST} \quad \frac{}{H; c \Downarrow c}$ $\text{VAR} \quad \frac{}{H; x \Downarrow H(x)}$ $\text{ADD} \quad \frac{H; e_1 \Downarrow c_1 \quad H; e_2 \Downarrow c_2}{H; e_1 + e_2 \Downarrow c_1 + c_2}$ $\text{MULT} \quad \frac{H; e_1 \Downarrow c_1 \quad H; e_2 \Downarrow c_2}{H; e_1 * e_2 \Downarrow c_1 * c_2}$

$H_1; s_1 \rightarrow H_2; s_2$

$\text{ASSIGN} \quad \frac{H; e \Downarrow c}{H; x := e \rightarrow H, x \mapsto c; \text{skip}}$ $\text{SEQ1} \quad \frac{}{H; \text{skip}; s \rightarrow H; s}$ $\text{SEQ2} \quad \frac{H; s_1 \rightarrow H'; s'_1}{H; s_1; s_2 \rightarrow H'; s'_1; s_2}$
 $\text{IF1} \quad \frac{H; e \Downarrow c \quad c > 0}{H; \text{if } e \text{ } s_1 \text{ } s_2 \rightarrow H; s_1}$ $\text{IF2} \quad \frac{H; e \Downarrow c \quad c \leq 0}{H; \text{if } e \text{ } s_1 \text{ } s_2 \rightarrow H; s_2}$ $\text{WHILE} \quad \frac{}{H; \text{while } e \text{ } s \rightarrow H; \text{if } e \text{ } (s; \text{while } e \text{ } s) \text{ skip}}$

$e ::= \lambda x. e \mid x \mid e e \mid c$
 $v ::= \lambda x. e \mid c$
 $\tau ::= \text{int} \mid \tau \rightarrow \tau$

$e \rightarrow e'$

$\frac{}{(\lambda x. e) v \rightarrow e[v/x]}$ $\frac{e_1 \rightarrow e'_1}{e_1 e_2 \rightarrow e'_1 e_2}$ $\frac{e_2 \rightarrow e'_2}{v e_2 \rightarrow v e'_2}$

$e[e'/x] = e''$

$\frac{}{x[e/x] = e}$ $\frac{y \neq x}{y[e/x] = y}$ $\frac{}{c[e/x] = c}$
 $\frac{e_1[e/x] = e'_1 \quad y \neq x \quad y \notin FV(e)}{(\lambda y. e_1)[e/x] = \lambda y. e'_1}$ $\frac{e_1[e/x] = e'_1 \quad e_2[e/x] = e'_2}{(e_1 e_2)[e/x] = e'_1 e'_2}$

$\Gamma \vdash e : \tau$

$\frac{}{\Gamma \vdash c : \text{int}}$ $\frac{}{\Gamma \vdash x : \Gamma(x)}$ $\frac{\Gamma, x : \tau_1 \vdash e : \tau_2}{\Gamma \vdash \lambda x. e : \tau_1 \rightarrow \tau_2}$ $\frac{\Gamma \vdash e_1 : \tau_2 \rightarrow \tau_1 \quad \Gamma \vdash e_2 : \tau_2}{\Gamma \vdash e_1 e_2 : \tau_1}$

- Preservation: If $\cdot \vdash e : \tau$ and $e \rightarrow e'$, then $\cdot \vdash e' : \tau$.
- Progress: If $\cdot \vdash e : \tau$, then e is a value or there exists an e' such that $e \rightarrow e'$.
- Substitution: If $\Gamma, x : \tau' \vdash e : \tau$ and $\Gamma \vdash e' : \tau'$, then $\Gamma \vdash e[e'/x] : \tau$.