
A simple language of arithmetic expressions

$$\begin{aligned}x, y, z &\in \mathbf{Var} \\ n, m &\in \mathbf{Int} \\ e &\in \mathbf{Exp}\end{aligned}$$

$$e ::= x \mid n \mid e_1 + e_2 \mid e_1 \times e_2 \mid x := e_1; e_2$$

$$\mathbf{VAR} \frac{}{\langle x, \sigma \rangle \longrightarrow \langle n, \sigma \rangle} \text{ where } n = \sigma(x)$$

$$\mathbf{LADD} \frac{\langle e_1, \sigma \rangle \longrightarrow \langle e'_1, \sigma' \rangle}{\langle e_1 + e_2, \sigma \rangle \longrightarrow \langle e'_1 + e_2, \sigma' \rangle}$$

$$\mathbf{RADD} \frac{\langle e_2, \sigma \rangle \longrightarrow \langle e'_2, \sigma' \rangle}{\langle n + e_2, \sigma \rangle \longrightarrow \langle n + e'_2, \sigma' \rangle}$$

$$\mathbf{ADD} \frac{}{\langle n + m, \sigma \rangle \longrightarrow \langle p, \sigma \rangle} \text{ where } p \text{ is the sum of } n \text{ and } m$$

$$\mathbf{LMUL} \frac{\langle e_1, \sigma \rangle \longrightarrow \langle e'_1, \sigma' \rangle}{\langle e_1 \times e_2, \sigma \rangle \longrightarrow \langle e'_1 \times e_2, \sigma' \rangle}$$

$$\mathbf{RMUL} \frac{\langle e_2, \sigma \rangle \longrightarrow \langle e'_2, \sigma' \rangle}{\langle n \times e_2, \sigma \rangle \longrightarrow \langle n \times e'_2, \sigma' \rangle}$$

$$\mathbf{MUL} \frac{}{\langle n \times m, \sigma \rangle \longrightarrow \langle p, \sigma \rangle} \text{ where } p \text{ is the product of } n \text{ and } m$$

$$\mathbf{ASG1} \frac{\langle e_1, \sigma \rangle \longrightarrow \langle e'_1, \sigma' \rangle}{\langle x := e_1; e_2, \sigma \rangle \longrightarrow \langle x := e'_1; e_2, \sigma' \rangle}$$

$$\mathbf{ASG} \frac{}{\langle x := n; e_2, \sigma \rangle \longrightarrow \langle e_2, \sigma[x \mapsto n] \rangle}$$

We write \longrightarrow^* for the reflexive transitive closure of the relation \longrightarrow .