

**More types
Section and Practice Problems**

Mar 10-11, 2016

1 Products and Sums

For these questions, use the lambda calculus with products and sums (Lecture 12, §1.1).

- (a) Write a program that constructs two values of type $\mathbf{int} + (\mathbf{int} \rightarrow \mathbf{int})$, one using left injection, and one using right injection.
- (b) Write a function that takes a value of type $\mathbf{int} + (\mathbf{int} \rightarrow \mathbf{int})$ and if the value is an integer, it adds 7 to it, and if the value is a function it applies the function to 42.
- (c) Give a typing derivation for the following program.

$$\lambda p: (\mathbf{unit} \rightarrow \mathbf{int}) \times (\mathbf{int} \rightarrow \mathbf{int}). \lambda x: \mathbf{unit} + \mathbf{int}. \text{case } x \text{ of } \#1 p \mid \#2 p$$

- (d) Write a program that uses the term in part (c) above to produce the value 42.

2 Recursion

- (a) Use the $\mu x. e$ expression to write a function that takes a natural number n and returns the sum of all even natural numbers less than or equal to n . (You can assume you have appropriate integer comparison operators, and also a modulus operator.)
- (b) Try executing your program by applying it to the number 5.
- (c) Give a typing derivation for the following program. What happens if you execute the program?

$$\mu p: (\mathbf{int} \rightarrow \mathbf{int}) \times (\mathbf{int} \rightarrow \mathbf{int}). (\lambda n: \mathbf{int}. n + 1, \#1 p)$$

3 References

- (a) Give a typing derivation for the following program.

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let a: int ref = ref 4 in
let b: int ref = ref λx: int. x + 38 in
!b !a
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- (b) Execute the program above for 4 small steps, to get configuration $\langle e, \sigma \rangle$. What is an appropriate Σ such that $\emptyset, \Sigma \vdash e: \tau$ and $\Sigma \vdash \sigma$?