Harvard School of Engineering and Applied Sciences — CS 152: Programming Languages

Control Flow Analysis
Section and Practice Problems

Thursday April 28, 2016

1 Control Flow Analysis

Consider the following lambda calculus program.

$$(\lambda f.\,(f\ 76) + (f\ 77))\ (\lambda a.\,a)$$

(a) Add labels to the program. That is, make it an expression in the labeled lambda calculus of Lecture 24, where every label is unique.

Answer:

$$((\lambda f.\,((f^1\ 76^2)^3 + (f^4\ 77^5)^6)^7)^8\ (\lambda a.\,a^9)^{10})^{11}$$

To be clear about what exactly the subexpressions are, here is the result of $\mathit{exprof}(e, i)$, where e is the labeled lambda calculus program.

$$\begin{aligned}
\mathit{exprof}(e,1) &= f^1\\
\mathit{exprof}(e,2) &= 76^2\\
\mathit{exprof}(e,3) &= (f^1\ 76^2)^3\\
\mathit{exprof}(e,4) &= f^4\\
\mathit{exprof}(e,5) &= 77^5\\
\mathit{exprof}(e,6) &= (f^4\ 77^5)^6\\
\mathit{exprof}(e,7) &= ((f^1\ 76^2)^3 + (f^4\ 77^5)^6)^7\\
\mathit{exprof}(e,8) &= (\lambda f.\,((f^1\ 76^2)^3 + (f^4\ 77^5)^6)^7)^8\\
\mathit{exprof}(e,9) &= a^9\\
\mathit{exprof}(e,10) &= (\lambda a.\,a^9)^{10}\\
\mathit{exprof}(e,11) &= ((\lambda f.\,((f^1\ 76^2)^3 + (f^4\ 77^5)^6)^7)^8\ (\lambda a.\,a^9)^{10})^{11}
\end{aligned}$$

(b) Let e be your labeled lambda calculus program. Write out $\mathcal{C}[\![e]\!]_e$, i.e., the set of constraints for the program e. (Hint, you should have 20 constraints in total. In particular, for each of the 3 applications, you should have 4 constraints, 2 for each of the lambda terms in the program.)

Answer: *The constraints are as follows. Make sure you understand where each constraint comes from. We* *have generated the constraints first for labeled expression 1, then for labeled expression 2, and so on.*

$$\begin{aligned}\mathcal{C}[\![e]\!]_e = \{&\mathsf{r}(f) \subseteq \mathsf{C}(1),\\ &2 \in \mathsf{C}(2),\\ &8 \in \mathsf{C}(1) \Rightarrow \mathsf{C}(2) \subseteq \mathsf{r}(f),\\ &10 \in \mathsf{C}(1) \Rightarrow \mathsf{C}(2) \subseteq \mathsf{r}(a),\\ &8 \in \mathsf{C}(1) \Rightarrow \mathsf{C}(7) \subseteq \mathsf{C}(3),\\ &10 \in \mathsf{C}(1) \Rightarrow \mathsf{C}(9) \subseteq \mathsf{C}(3),\\ &\mathsf{r}(f) \subseteq \mathsf{C}(4),\\ &5 \in \mathsf{C}(5),\\ &8 \in \mathsf{C}(4) \Rightarrow \mathsf{C}(5) \subseteq \mathsf{r}(f),\\ &10 \in \mathsf{C}(4) \Rightarrow \mathsf{C}(5) \subseteq \mathsf{r}(a),\\ &8 \in \mathsf{C}(4) \Rightarrow \mathsf{C}(7) \subseteq \mathsf{C}(6),\\ &10 \in \mathsf{C}(4) \Rightarrow \mathsf{C}(9) \subseteq \mathsf{C}(6),\\ &7 \in \mathsf{C}(7),\\ &8 \in \mathsf{C}(8),\\ &\mathsf{r}(a) \subseteq \mathsf{C}(9),\\ &10 \in \mathsf{C}(10),\\ &8 \in \mathsf{C}(8) \Rightarrow \mathsf{C}(10) \subseteq \mathsf{r}(f),\\ &10 \in \mathsf{C}(8) \Rightarrow \mathsf{C}(10) \subseteq \mathsf{r}(a),\\ &8 \in \mathsf{C}(8) \Rightarrow \mathsf{C}(7) \subseteq \mathsf{C}(11),\\ &10 \in \mathsf{C}(8) \Rightarrow \mathsf{C}(9) \subseteq \mathsf{C}(11)\ \}\end{aligned}$$

(c) Find C^* and r^*, the smallest functions that satisfy the constraints you generated in the question above.

Answer: *We show the result of the iterative algorithm described in the lecture notes. Note that iteration 7 is the same as iteration 6, so that is a fixed point.*

i	C(1)	C(2)	C(3)	C(4)	C(5)	C(6)	C(7)	C(8)	C(9)	C(10)	C(11)	r(f)	r(a)
0													
1		2			5		7	8		10			
2		2			5		7	8		10	7	10	
3	10	2		10	5		7	8		10	7	10	
4	10	2		10	5		7	8		10	7	10	2, 5
5	10	2		10	5		7	8	2, 5	10	7	10	2, 5
6	10	2	2, 5	10	5	2, 5	7	8	2, 5	10	7	10	2, 5
7	10	2	2, 5	10	5	2, 5	7	8	2, 5	10	7	10	2, 5

(d) Check that your functions C^* and r^* make sense. That is, if an expression labeled l can evaluate to an expression labeled l', do you have $l' \in \mathsf{C}^*(l)$?

Answer: *Yes, it does make sense! Consider* $\mathsf{r}^*(f)$*, i.e., what values variable* f *can be bound to.* $\mathsf{r}^*(f) = \{10\}$*, meaning that the only value that* f *can be bound to is the expression labeled 10:* $(\lambda a.\, a^9)^{10}$*. That is indeed the* *case.*

Consider also $\mathsf{r}^*(a) = \{2, 5\}$. *The variable* a *can be bound to expressions* 76^2 *and* 77^5, *which is indeed the case.*

Check some of the other results, and make sure that they correctly reflect the program execution.

(e) Consider adding the expression $(\mathsf{let}\ x = e_1\ \mathsf{in}\ e_2)^l$ to the language. Define $\mathcal{C}[\![(\mathsf{let}\ x = e_1\ \mathsf{in}\ e_2)^l]\!]_e$. Try rewriting the program above using one or more let expressions, and make sure that the constraints you generate for the modified program produce the same solution C^* and r^*.

Answer:

$$\mathcal{C}[\![(\textit{let}\ x = e_1^{l_1}\ \textit{in}\ e_2^{l_2})^l]\!]_e = \mathcal{C}[\![e_1^{l_1}]\!]_e \cup \mathcal{C}[\![e_2^{l_2}]\!]_e \cup \{\mathsf{C}(l_1) \subseteq \mathsf{r}(x), \mathsf{C}(l_2) \subseteq \mathsf{C}(l)\}$$

We leave the rewriting of the program and the generation of constraints as an exercise.