**Harvard School of Engineering and Applied Sciences — CS 152: Programming Languages**

# Control Flow Analysis
# Section and Practice Problems

Thursday April 28, 2016

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## 1 Control Flow Analysis

Consider the following lambda calculus program.

$$(\lambda f.\,(f\ 76) + (f\ 77))\ (\lambda a.\,a)$$

(a) Add labels to the program. That is, make it an expression in the labeled lambda calculus of Lecture 24, where every label is unique.

(b) Let $e$ be your labeled lambda calculus program. Write out $\mathcal{C}[\![e]\!]_e$, i.e., the set of constraints for the program $e$. (Hint, you should have 20 constraints in total. In particular, for each of the 3 applications, you should have 4 constraints, 2 for each of the lambda terms in the program.)

(c) Find $\mathsf{C}^*$ and $\mathsf{r}^*$, the smallest functions that satisfy the constraints you generated in the question above.

(d) Check that your functions $\mathsf{C}^*$ and $\mathsf{r}^*$ make sense. That is, if an expression labeled $l$ can evaluate to an expression labeled $l'$, do you have $l' \in \mathsf{C}^*(l)$?

(e) Consider adding the expression $(\mathsf{let}\ x = e_1\ \mathsf{in}\ e_2)^l$ to the language. Define $\mathcal{C}[\![(\mathsf{let}\ x = e_1\ \mathsf{in}\ e_2)^l]\!]_e$. Try rewriting the program above using one or more let expressions, and make sure that the constraints you generate for the modified program produce the same solution $\mathsf{C}^*$ and $\mathsf{r}^*$.