Harvard School of Engineering and Applied Sciences — CS 152: Programming Languages

Self Assessment

Students taking CS 152 should have good programming skills, and be very comfortable with recursion, proofs, basic mathematical ideas and notations, including sets, relations, functions, and induction.

The questions below are designed to help you figure out if you have sufficient mathematical preparation for CS 152. It does not address whether you have appropriate programming skills, but if you've taken CS51 or equivalent, you will be fine.

The questions below are intended to be fairly straightforward. Note that the emphasis is on being able to **prove** the statements, not simply determine whether the statements are true or false. That is, you need to be comfortable with how to perform proofs of logical statements (e.g., how do you prove a universal quantification, an existential quantification, or an implication), proofs by mathematical induction, as well as being comfortable with sets, relations, and functions.

If any of the notation or terminology below is unfamiliar, you can learn about some of this background material in the readings below, and try these questions again. (See `https://www.seas.harvard.edu/courses/cs152/2019sp/resources.html` for bibliographic details of the books).

- Winskel Chapter 1
- Harper Chapter 2 and Appendix
- Pierce Chapter 2

1. A symmetric relation R is a relation such that for all x, y, if $(x, y) \in R$ then $(y, x) \in R$. Given a relation R, define R^+ to be the relation $R \cup \{(x, z) \mid \exists y.\ (x, y) \in R \land\ (y, z) \in R\}$.

Prove that if R is a symmetric relation, then R^+ is symmetric.

2. Prove using mathematical induction that for all $n \in \mathbb{N}$, $0 + 1 + 2 + \cdots + n = \frac{n(n+1)}{2}$.

3. Consider the relation $R = \{(3n, n) \mid n \in \mathbb{N}\}$. Prove that R is a partial function. (i.e., prove $\forall x, y, z \in \mathbb{N}.\ (x, y) \in R \land (x, z) \in R \implies y = z$ and also prove that $\exists a \in \mathbb{N}.\ \forall b \in \mathbb{N}.\ (a, b) \notin R$.)