Curry-Howard Isomorphism; Existential Types

CS 152 (Spring 2020)

Harvard University

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Today, we will learn about

▶ Curry-Howard Isomorphism

▶ Existential types

Curry-Howard Isomorphism

Conjunction = Product

Disjunction = Sum

Function Types?

Parametric Polymorphism

What about False?

Example 1: From Formula to Type

Example 2: From Type to Formula

Negation and Continuations

Existential Types

Syntax

$$
\begin{aligned}
e ::= &\ x \mid \lambda x\!:\!\tau.\, e \mid e_1\ e_2 \mid n \mid e_1 + e_2 \\
&\mid \{\ l_1 = e_1, \ldots, l_n = e_n\ \} \mid e.l \\
&\mid \mathsf{pack}\ \{\tau_1, e\}\ \mathsf{as}\ \exists X.\ \tau_2 \\
&\mid \mathsf{unpack}\ \{X, x\} = e_1\ \mathsf{in}\ e_2 \\
v ::= &\ n \mid \lambda x\!:\!\tau.\, e \mid \{\ l_1 = v_1, \ldots, l_n = v_n\ \} \\
&\mid \mathsf{pack}\ \{\tau_1, v\}\ \mathsf{as}\ \exists X.\ \tau_2 \\
\tau ::= &\ \mathbf{int} \mid \tau_1 \rightarrow \tau_2 \mid \{\ l_1\!:\!\tau_1, \ldots, l_n\!:\!\tau_n\ \} \mid X \mid \exists X.\ \tau
\end{aligned}
$$

Example: Counter ADT

Example: Counter ADT, ctd

Operational Semantics

$$E ::= \cdots \mid \text{pack } \{\tau_1, E\} \text{ as } \exists X.\ \tau_2$$
$$\mid \text{unpack } \{X, x\} = E \text{ in } e$$

$$\overline{\text{unpack } \{X, x\} = (\text{pack } \{\tau_1, v\} \text{ as } \exists Y.\ \tau_2) \text{ in } e \longrightarrow e\{v/x\}\{\tau_1/X\}}$$

Typing rules

$$\frac{\Delta, \Gamma \vdash e : \tau_2\{\tau_1/X\}}{\Delta, \Gamma \vdash \text{pack } \{\tau_1, e\} \text{ as } \exists X.\ \tau_2 : \exists X.\ \tau_2}$$

$$\frac{\Delta, \Gamma \vdash e_1 : \exists X.\ \tau_1 \quad X \notin \Delta \qquad \Delta \cup \{X\}, \Gamma, x : \tau_1 \vdash e_2 : \tau_2 \quad \Delta \vdash \tau_2 \text{ ok}}{\Delta, \Gamma \vdash \text{unpack } \{X, x\} = e_1 \text{ in } e_2 : \tau_2}$$

$$\frac{\Delta \cup \{X\} \vdash \tau \text{ ok}}{\Delta \vdash \exists X.\ \tau \text{ ok}}$$