Curry-Howard Isomorphism; Existential Types
CS 152 (Spring 2021)

Harvard University

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Today, we will learn about

- Curry-Howard Isomorphism
- Existential types
Curry-Howard Isomorphism
Conjunction = Product
Disjunction = Sum
Function Types?
Parametric Polymorphism
What about False?
Example 1: From Formula to Type
Example 2: From Type to Formula
Negation and Continuations
Existential Types
Syntax

\[ e ::= x \mid \lambda x: \tau. \, e \mid e_1 \, e_2 \mid n \mid e_1 + e_2 \]
\[ \mid \{ \, l_1 = e_1, \ldots, l_n = e_n \, \} \mid e.\!l \]
\[ \mid \text{pack} \, \{ \tau_1, e \} \, \text{as} \, \exists X. \, \tau_2 \]
\[ \mid \text{unpack} \, \{ X, x \} = e_1 \, \text{in} \, e_2 \]

\[ v ::= n \mid \lambda x: \tau. \, e \mid \{ \, l_1 = v_1, \ldots, l_n = v_n \, \} \]
\[ \mid \text{pack} \, \{ \tau_1, v \} \, \text{as} \, \exists X. \, \tau_2 \]

\[ \tau ::= \text{int} \mid \tau_1 \rightarrow \tau_2 \mid \{ \, l_1: \tau_1, \ldots, l_n: \tau_n \, \} \mid X \mid \exists X. \, \tau \]
Example: Counter ADT

let \( \text{counterADT} = \)
  pack
  \{ \text{int}, \{ \text{new} = 0, \}
      \text{get} = \lambda i: \text{int}. i,
      \text{inc} = \lambda i: \text{int}. i + 1 \} \}

as

\( \exists \text{Counter}. \{ \text{new} : \text{Counter}, \)
    \text{get} : \text{Counter} \rightarrow \text{int},
    \text{inc} : \text{Counter} \rightarrow \text{Counter} \} \)

in \ldots
Example: Counter ADT, ctd

\[
\text{unpack } \{C, x\} = \text{counterADT} \text{ in let } y : C = x.\text{new in} \\
x.\text{get} \ (x.\text{inc} \ (x.\text{inc} \ y))
\]
Operational Semantics

\[ E ::= \cdots \mid \text{pack } \{ \tau_1, E \} \text{ as } \exists X. \, \tau_2 \]
\[ \mid \text{unpack } \{ X, x \} = E \text{ in } e \]

unpack \( \{ X, x \} = (\text{pack } \{ \tau_1, v \} \text{ as } \exists Y. \, \tau_2) \text{ in } e \rightarrow e\{v/x\}\{\tau_1/X\} \)
Typing rules

\[ \Delta, \Gamma \vdash e : \tau_2 \{ \tau_1 / X \} \]

\[ \Delta, \Gamma \vdash \text{pack} \{ \tau_1, e \} \text{ as } \exists X. \, \tau_2 : \exists X. \, \tau_2 \]

\[ \Delta, \Gamma \vdash e_1 : \exists X. \, \tau_1 \quad X \notin \Delta \]

\[ \Delta \cup \{ X \}, \Gamma, x : \tau_1 \vdash e_2 : \tau_2 \quad \Delta \vdash \tau_2 \text{ ok} \]

\[ \Delta, \Gamma \vdash \text{unpack} \{ X, x \} = e_1 \text{ in } e_2 : \tau_2 \]

\[ \Delta \cup \{ X \} \vdash \tau \text{ ok} \]

\[ \Delta \vdash \exists X. \, \tau \text{ ok} \]