

# Semantics for Verified Software-Hardware Stacks

## CS 152

Samuel Gruetter

Reusing/citing slides by Nada Amin

Based on joint work with Joonwon Choi, Andres Erbsen, Clark Wood, Adam Chlipala

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# About me

- Undergrad/masters at EPFL (Switzerland), supervised by then-PhD-student Nada Amin
- Internship at Princeton University with Andrew Appel's group using the Verified Software Toolchain (VST)
- Currently: PhD student at MIT in Adam Chlipala's group

# Styles of Semantics

Operational Semantics  
Denotational Semantics  
Axiomatic Semantics  
Algebraic Semantics



Techniques from CS152  
Our PLDI'21 paper



## Integration Verification Across Software and Hardware for a Simple Embedded System

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### Abstract

The interfaces between layers of a system are susceptible to bugs if developers of adjacent layers proceed under subtly different assumptions. Formal verification of two layers

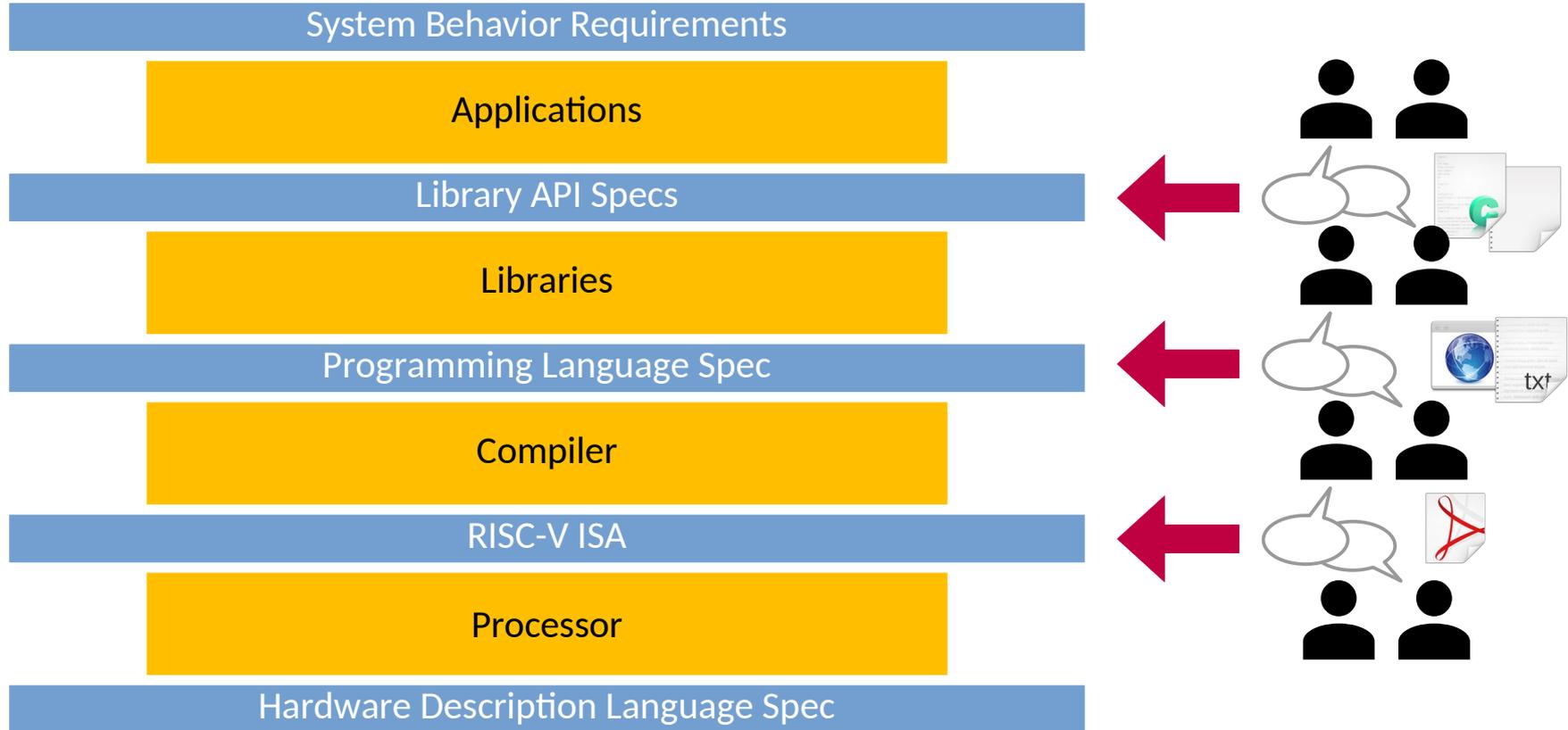
### 1 Introduction

We present the comprehensive and modular verification of functional correctness of a newly realistic but still very simple embedded system, highlighting important challenges

# Software-Hardware Stacks



# Need agreement at each layer boundary



# Problem

No matter

- how the specs are stated
- how often they change
- whether providers and users are the same team or not

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There will always be

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Which lead to

- Bugs
- Security vulnerabilities

# Problem

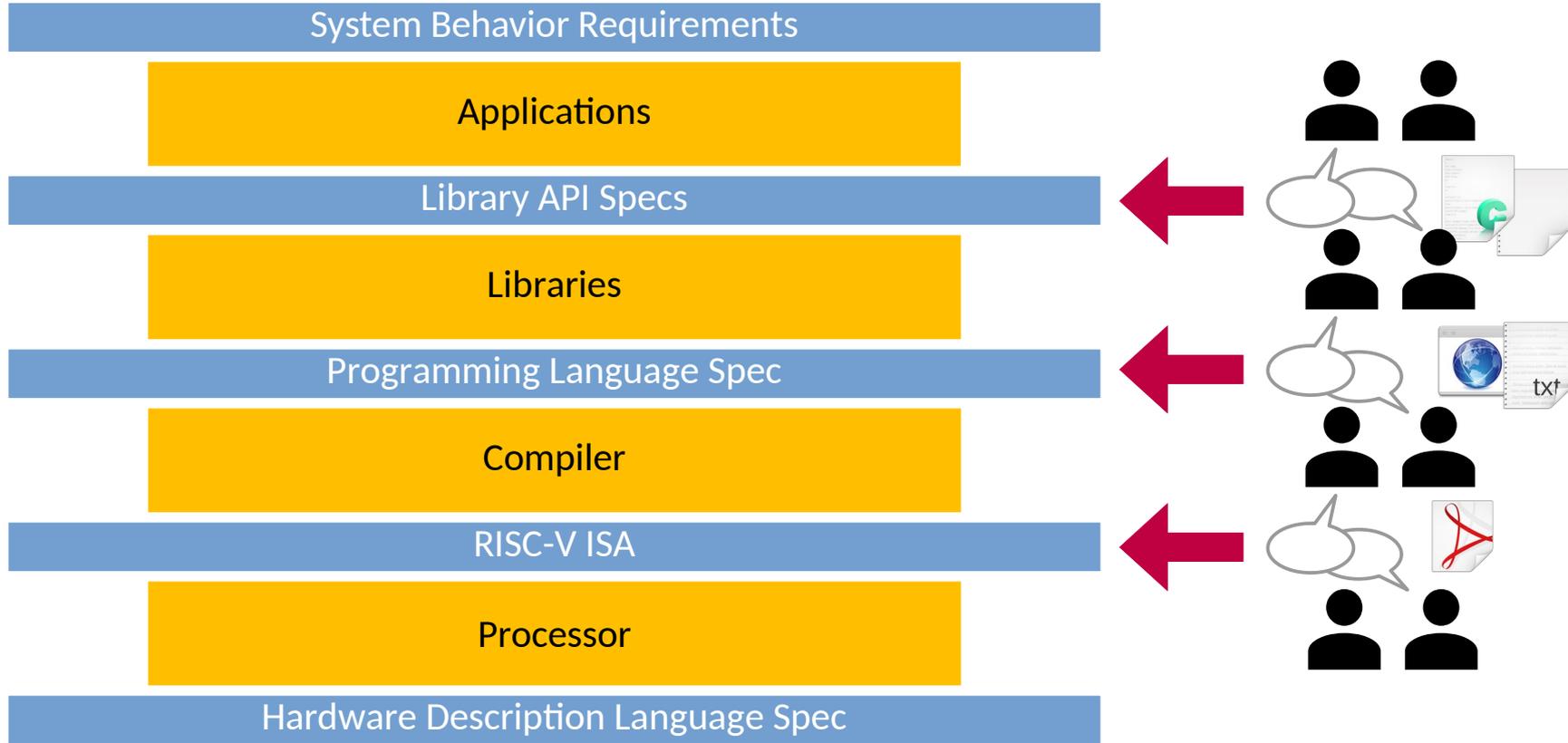
There will always be

Can a computer help us solve this problem?

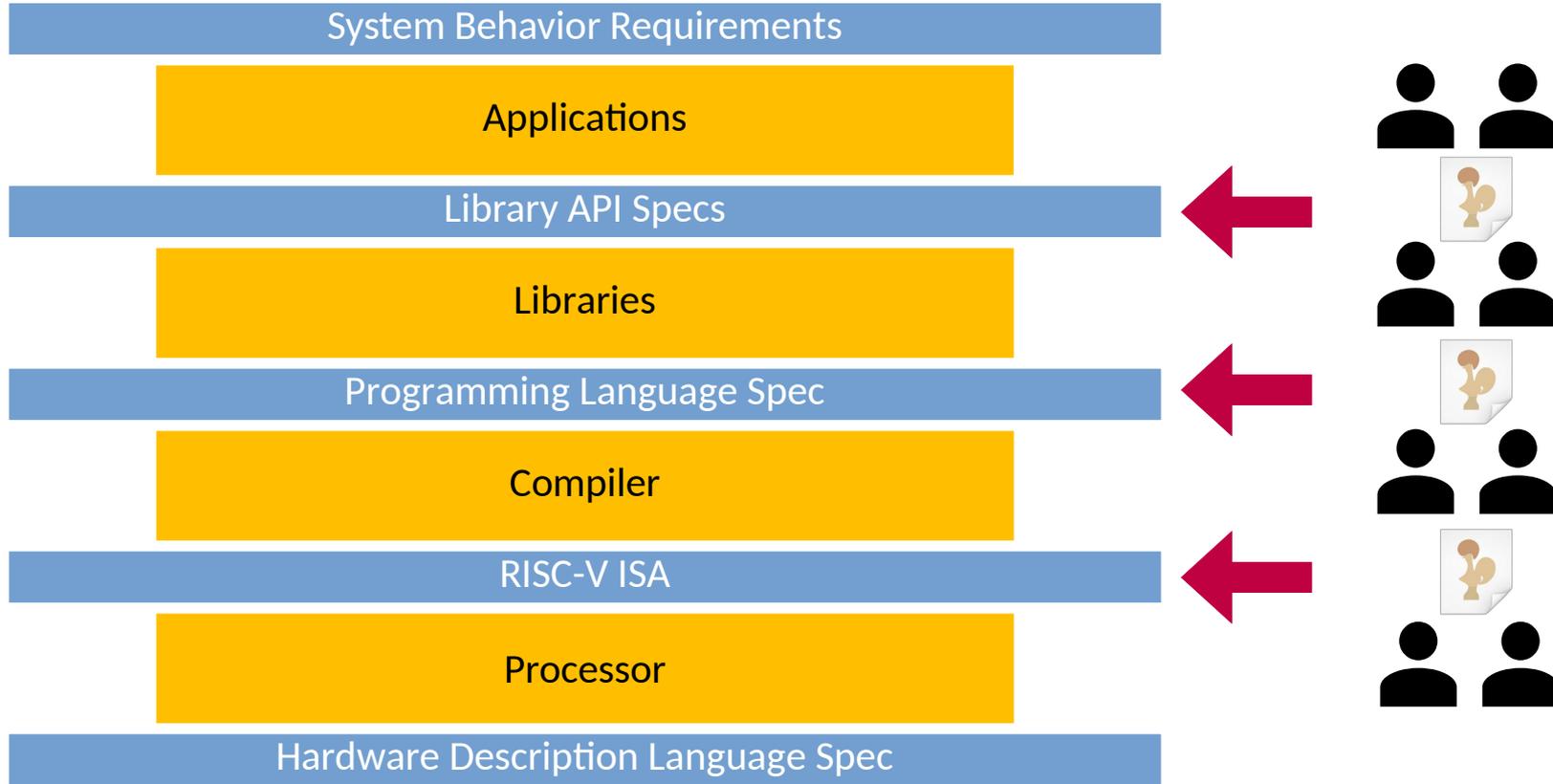
That's our research

- Bugs
- Security vulnerabilities

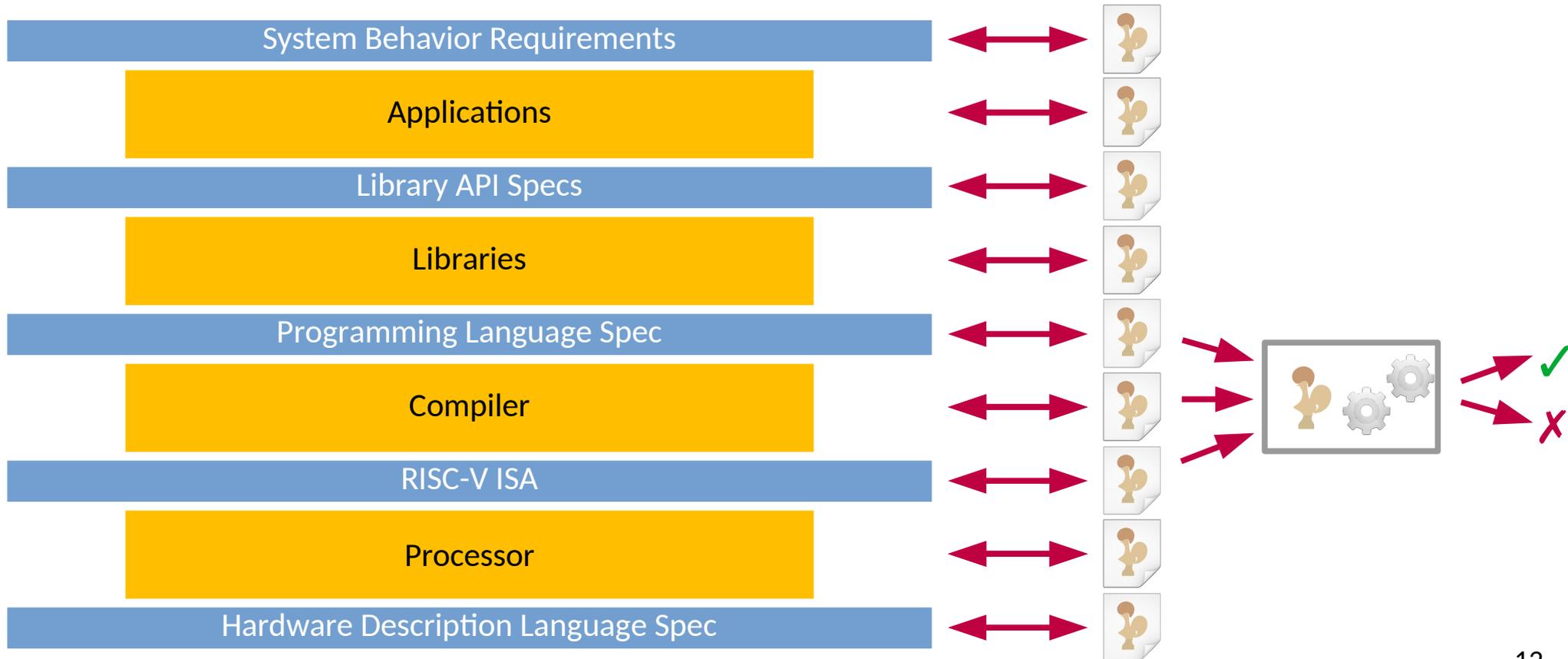
# Systems development using machine-checked specs at all levels



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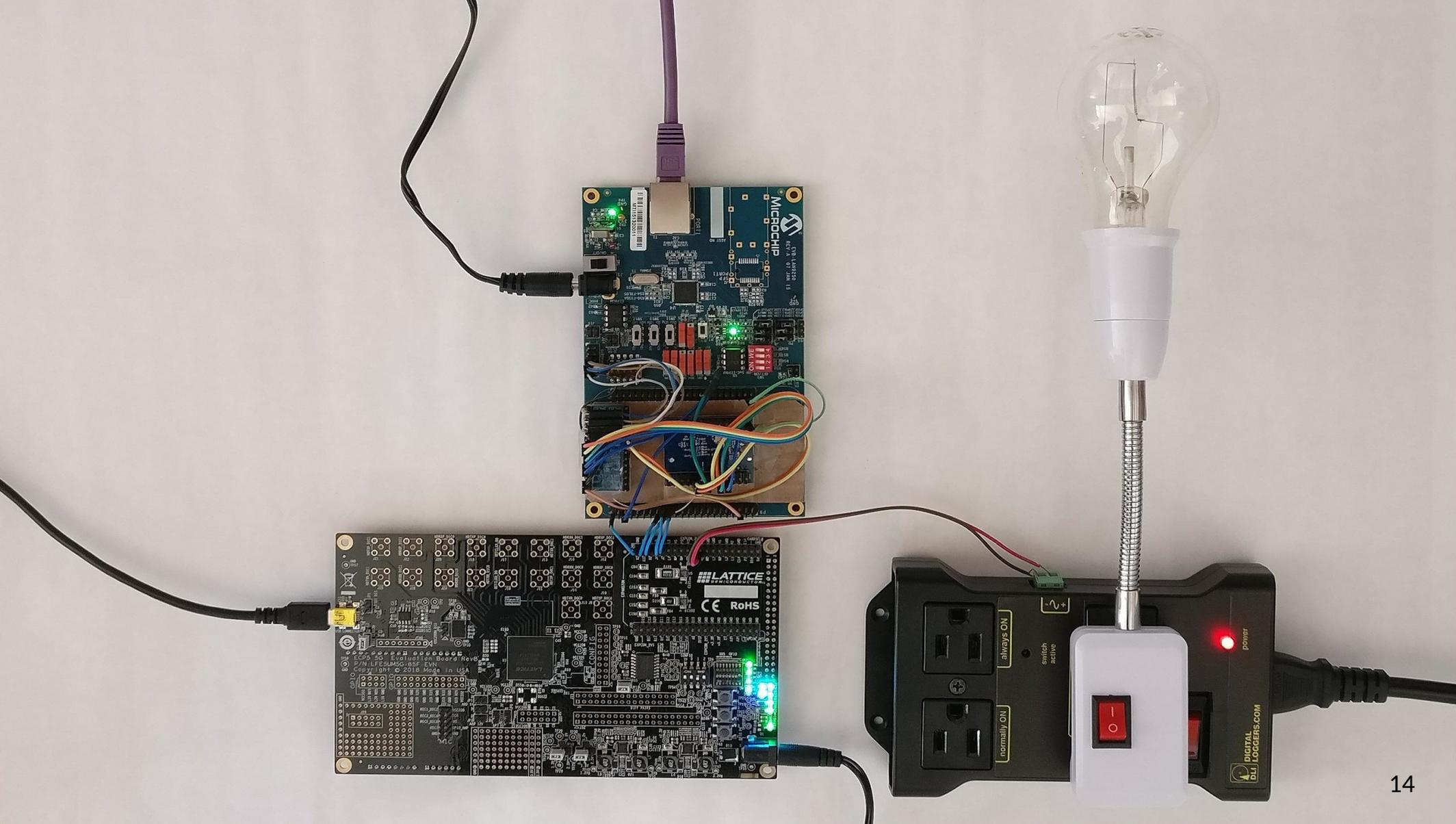
# Systems development using machine-checked specs at all levels



# Prototype: The IoT lightbulb

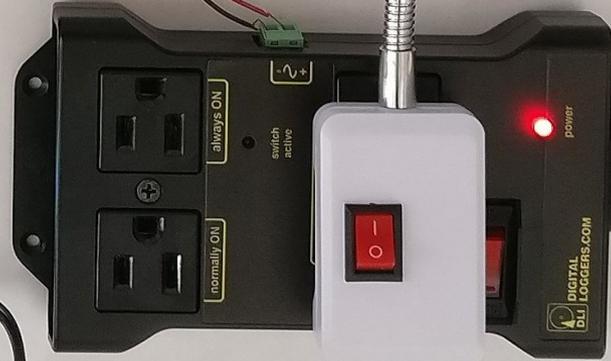
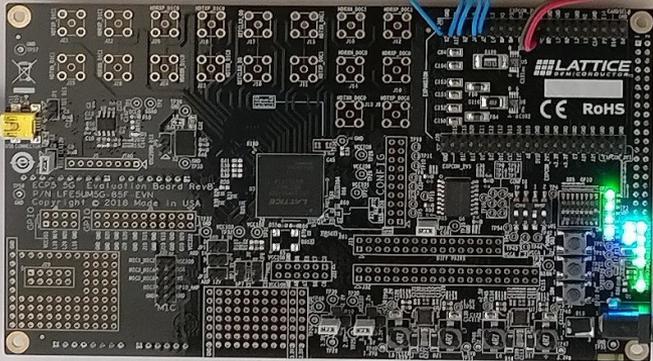
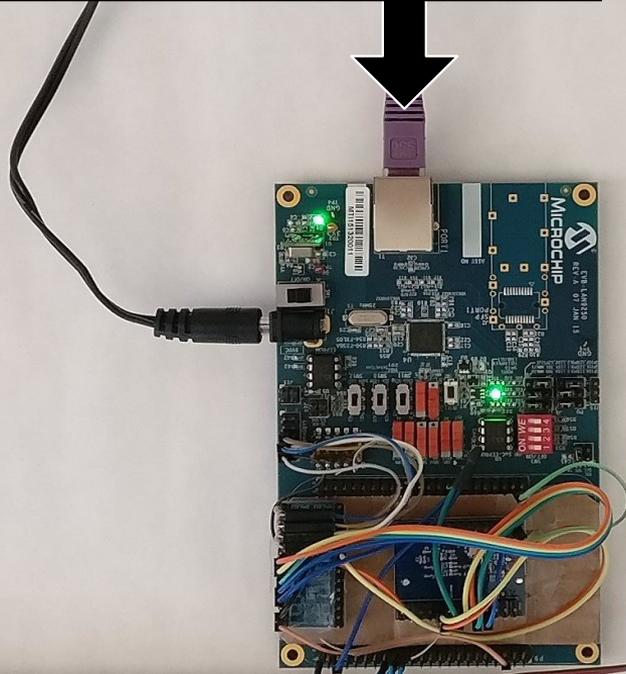
- Wrote all digital parts from scratch
- Using bedrock2
  - a low-level programming language with verification support
- Very simple at the moment
- Goal: Should scale to more realistic applications

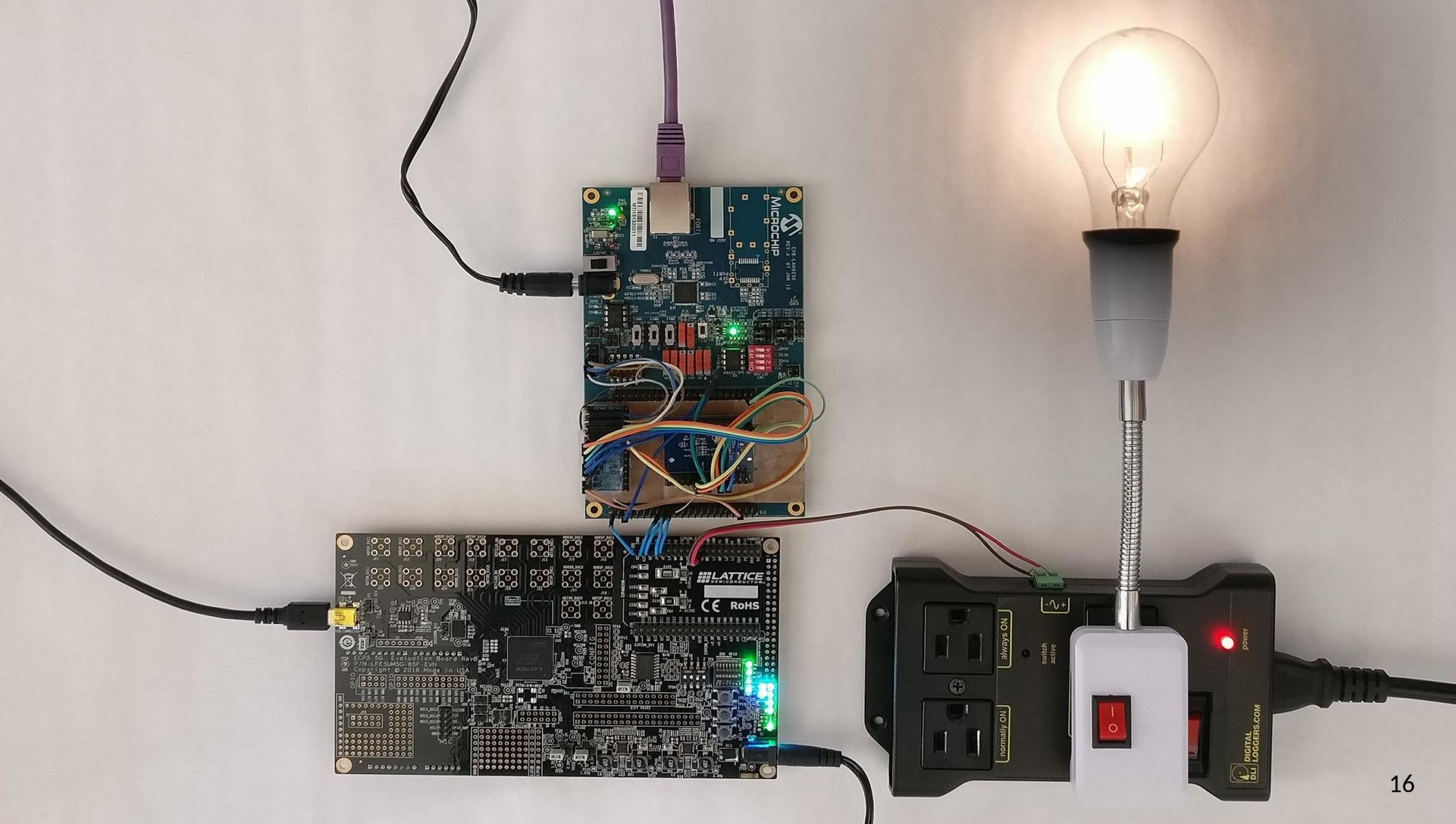






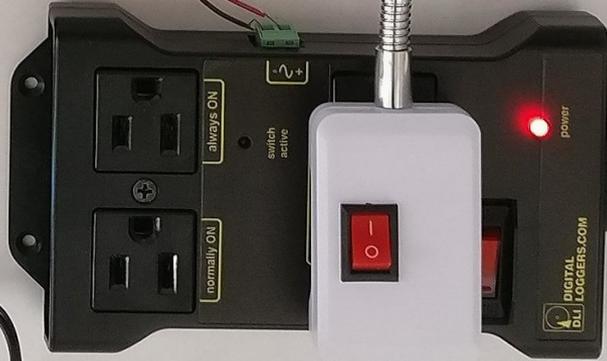
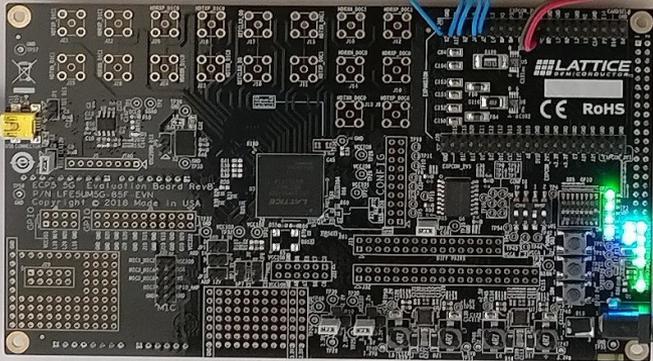
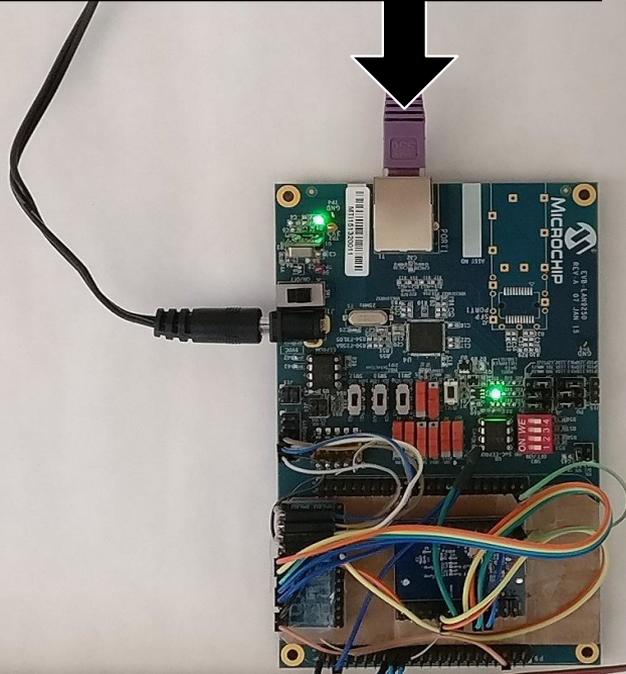
```
printf '1' | nc -w0 -u 192.168.1.123 9999
```

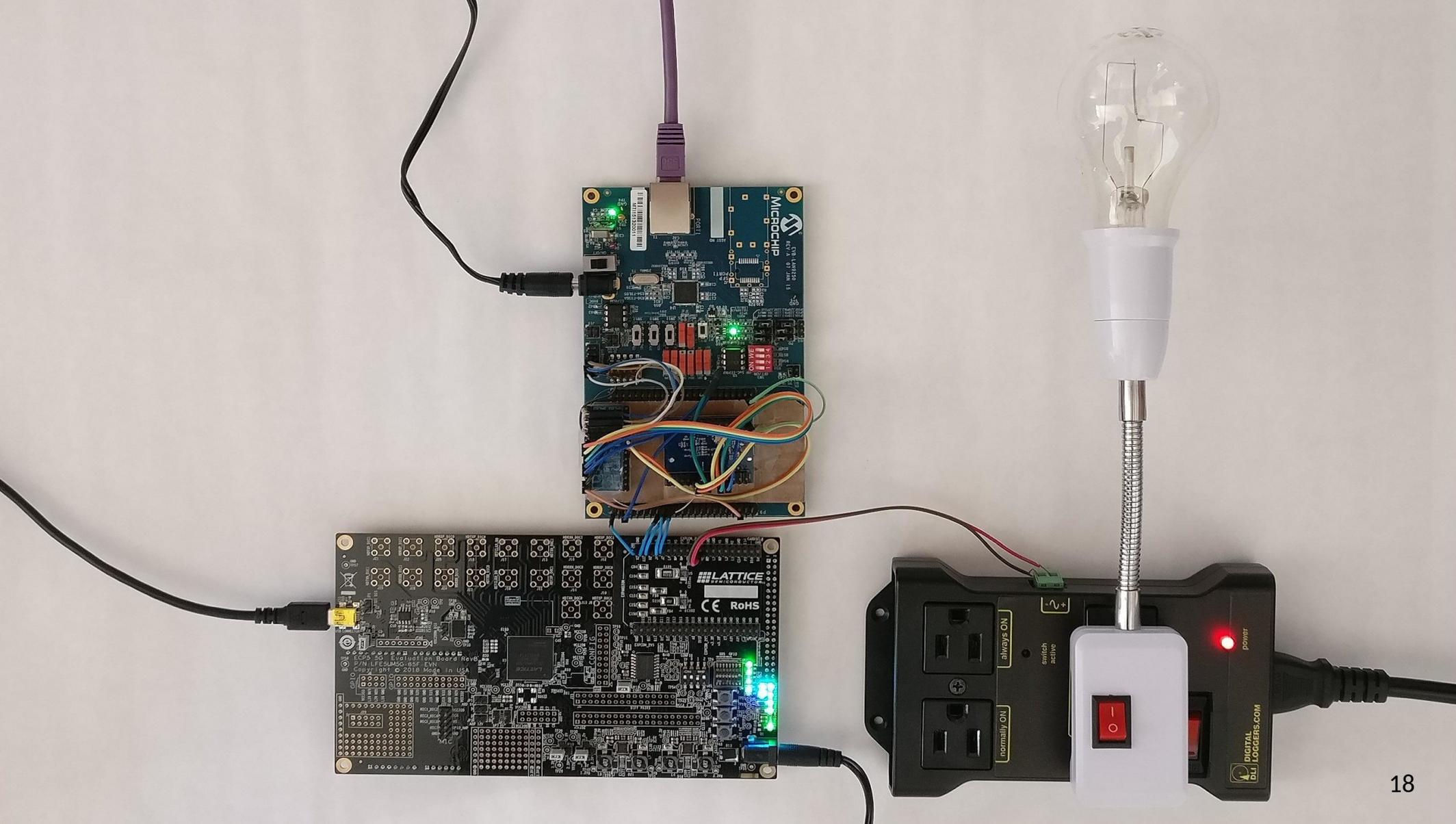


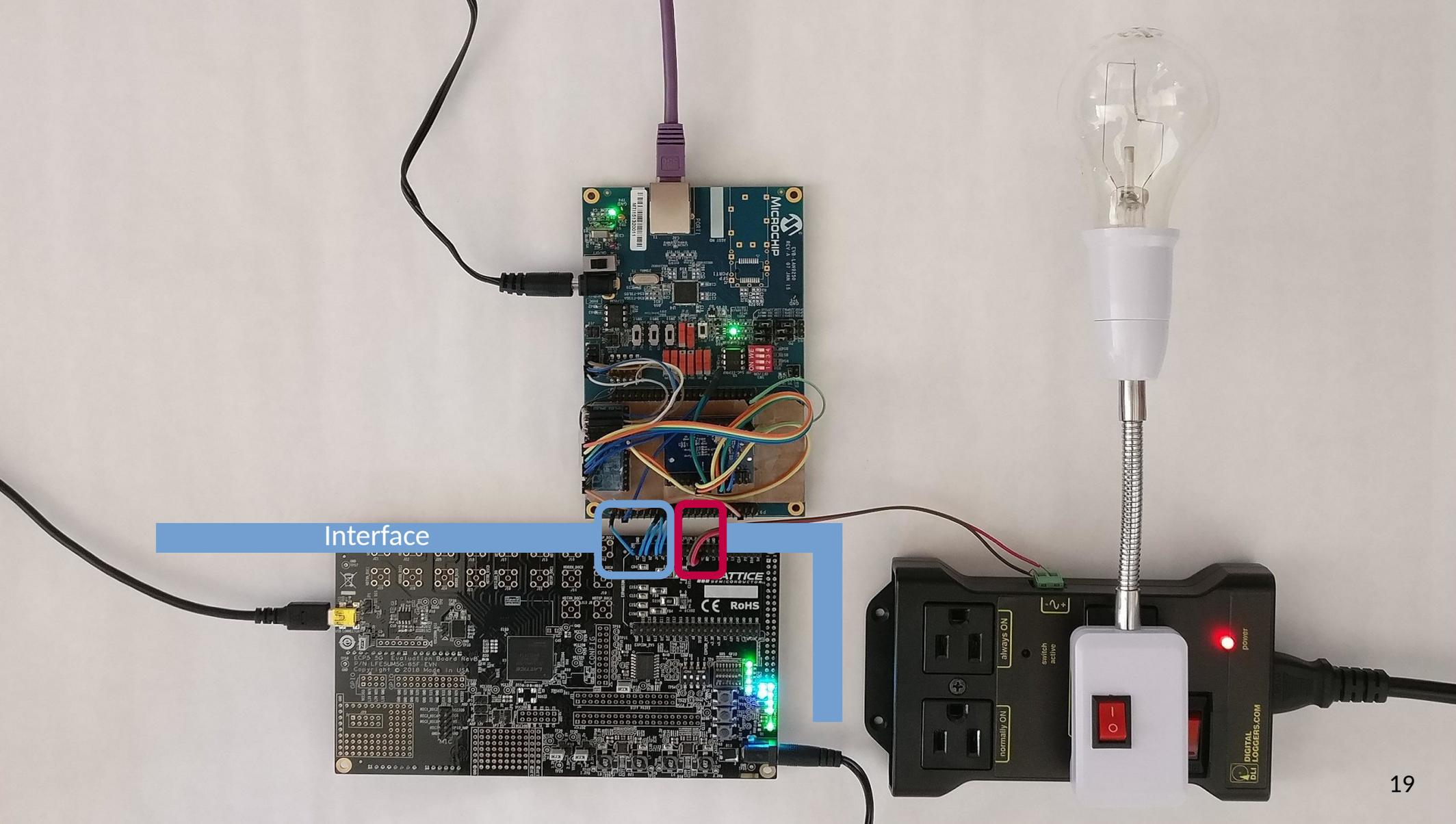




```
printf '0' | nc -w0 -u 192.168.1.123 9999
```



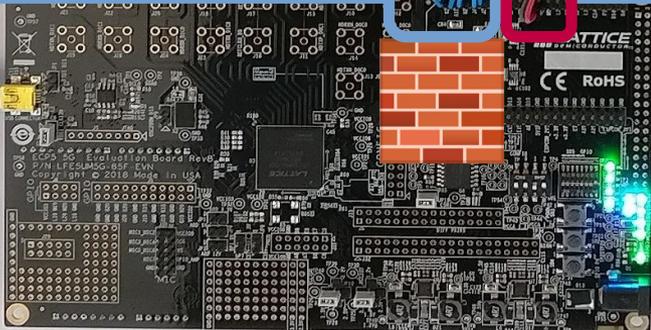
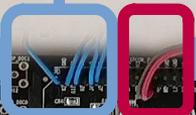




Interface



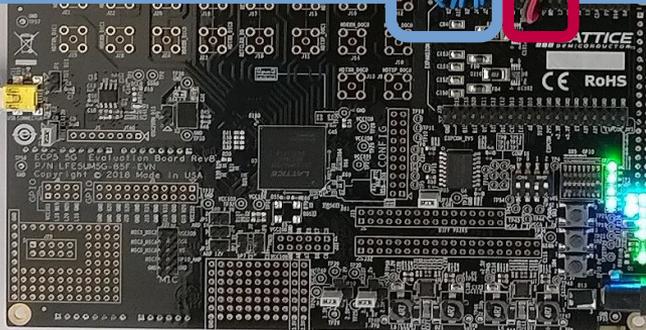
Interface



# Specification using a Log

Recv1  
TurnOn  
PollNone  
Recv0  
TurnOff  
RecvInvalid  
PollNone  
PollNone

Interface



# How to specify the behavior of the processor

# Small-Step Operational Semantics

step from configuration to configuration:

$$c_0 \longrightarrow c_1 \longrightarrow \dots \longrightarrow c_n$$

# Semantic Rules (1/2)

$$\text{LADD} \frac{\langle e_1, \sigma \rangle \longrightarrow \langle e'_1, \sigma' \rangle}{\langle e_1 + e_2, \sigma \rangle \longrightarrow \langle e'_1 + e_2, \sigma' \rangle}$$

$$\text{RADD} \frac{\langle e_2, \sigma \rangle \longrightarrow \langle e'_2, \sigma' \rangle}{\langle n + e_2, \sigma \rangle \longrightarrow \langle n + e'_2, \sigma' \rangle}$$

$$\text{ADD} \frac{}{\langle n + m, \sigma \rangle \longrightarrow \langle p, \sigma \rangle} \text{ where } p \text{ is the sum of } n \text{ and } m$$

$$\text{LMUL} \frac{\langle e_1, \sigma \rangle \longrightarrow \langle e'_1, \sigma' \rangle}{\langle e_1 \times e_2, \sigma \rangle \longrightarrow \langle e'_1 \times e_2, \sigma' \rangle}$$

$$\text{RMUL} \frac{\langle e_2, \sigma \rangle \longrightarrow \langle e'_2, \sigma' \rangle}{\langle n \times e_2, \sigma \rangle \longrightarrow \langle n \times e'_2, \sigma' \rangle}$$

$$\text{MUL} \frac{}{\langle n \times m, \sigma \rangle \longrightarrow \langle p, \sigma \rangle} \text{ where } p \text{ is the product of } n \text{ and } m$$

# Large-step semantics

$$\Downarrow \subseteq \mathbf{Config} \times \mathbf{FinalConfig}$$

where

$$\mathbf{Config} = \mathbf{Exp} \times \mathbf{Store}$$

$$\text{and } \mathbf{Final Config} = \mathbf{Int} \times \mathbf{Store} \subseteq \mathbf{Config}.$$

We write  $\langle e, \sigma \rangle \Downarrow \langle n, \sigma' \rangle$  to mean that  
 $(\langle e, \sigma \rangle, \langle n, \sigma' \rangle) \in \Downarrow$

# Semantic Rules (1)

$$\text{INT}_{\text{LRG}} \frac{}{\langle n, \sigma \rangle \Downarrow \langle n, \sigma \rangle}$$

$$\text{VAR}_{\text{LRG}} \frac{}{\langle x, \sigma \rangle \Downarrow \langle n, \sigma \rangle} \text{ where } n = \sigma(x)$$

$$\text{ADD}_{\text{LRG}} \frac{\begin{array}{l} \langle e_1, \sigma \rangle \Downarrow \langle n_1, \sigma'' \rangle \\ \langle e_2, \sigma'' \rangle \Downarrow \langle n_2, \sigma' \rangle \\ \text{where } n \text{ is the sum of } n_1 \text{ and } n_2 \end{array}}{\langle e_1 + e_2, \sigma \rangle \Downarrow \langle n, \sigma' \rangle}$$

# Arithmetic expressions

$$\mathcal{A}[[n]] = \{(\sigma, n)\}$$

$$\mathcal{A}[[x]] = \{(\sigma, \sigma(x))\}$$

$$\begin{aligned} \mathcal{A}[[a_1 + a_2]] = \{(\sigma, n) \mid & (\sigma, n_1) \in \mathcal{A}[[a_1]] \\ & \wedge (\sigma, n_2) \in \mathcal{A}[[a_2]] \\ & \wedge n = n_1 + n_2\} \end{aligned}$$

$$\begin{aligned} \mathcal{A}[[a_1 \times a_2]] = \{(\sigma, n) \mid & (\sigma, n_1) \in \mathcal{A}[[a_1]] \\ & \wedge (\sigma, n_2) \in \mathcal{A}[[a_2]] \\ & \wedge n = n_1 \times n_2\} \end{aligned}$$

# How to specify the behavior of the processor

- Using the hardware description language Kami
- Embedded in Coq

# Kami Semantics

Expression  $e ::= c \mid x \mid \text{op}(\vec{e}) \mid [\vec{e}] \mid \{\overrightarrow{k = e}\} \mid e[e] \mid e.k$

Action  $a ::= \text{let } x = r \text{ in } a$   
 $\mid r := e; a$   
 $\mid \text{let } x = f(e) \text{ in } a$   
 $\mid \text{let } x = e \text{ in } a$   
 $\mid \text{if } e \text{ then } a \text{ else } a; a$   
 $\mid \text{assert } e; a$   
 $\mid \text{return } e$

Module  $m ::= \langle \langle \overrightarrow{r, c} \rangle, \langle \overrightarrow{s, a} \rangle, \langle \overrightarrow{f, \lambda x : \tau. a} \rangle \rangle$   
 $\mid m + m$

$o$ : old state of register file  
 (mapping from register names  
 $r$  to values  $v$ )

$u$ : updates to register file

$\ell$ : labels (to go in log)

ActionReadReg: 
$$\frac{o \xrightarrow{\ell} (u, v)}{[o(r)/x]a}$$

$$o \xrightarrow{\ell} (u, v)$$
  

$$\text{let } x=r \text{ in } a$$

ActionWriteReg: 
$$\frac{[[e]] = v_r \quad r \notin u \quad o \xrightarrow{\ell} (u, v)}{r := e; a}$$

ActionCall: 
$$\frac{[[e]] = v_a \quad (f(\_) = \_) \notin \ell \quad o \xrightarrow{\ell} (u, v)}{[v_r/x]a}$$

$$o \xrightarrow{\{f(v_a)=v_r\} \cup \ell} (u, v)$$
  

$$\text{let } x=f(e) \text{ in } a$$

Q: Where do you see/expect  
 small-step/big-step/denotational  
 semantics?

More details on Kami: See ICFP'17 paper

# Arithmetic expressions

$$\mathcal{A}[[n]] = \{(\sigma, n)\}$$

$$\mathcal{A}[[x]] = \{(\sigma, \sigma(x))\}$$

$$\begin{aligned}\mathcal{A}[[a_1 + a_2]] &= \{(\sigma, n) \mid (\sigma, n_1) \in \mathcal{A}[[a_1]] \\ &\quad \wedge (\sigma, n_2) \in \mathcal{A}[[a_2]] \\ &\quad \wedge n = n_1 + n_2\}\end{aligned}$$

$$\begin{aligned}\mathcal{A}[[a_1 \times a_2]] &= \{(\sigma, n) \mid (\sigma, n_1) \in \mathcal{A}[[a_1]] \\ &\quad \wedge (\sigma, n_2) \in \mathcal{A}[[a_2]] \\ &\quad \wedge n = n_1 \times n_2\}\end{aligned}$$

## Denotational semantics for Kami expressions

$$[[c]] = c$$

$$[[\text{op}(\vec{e})]] = [[\text{op}]]_{\text{op}}(\overrightarrow{[[e_i]]})$$

$$[[[e_0, \dots, e_{2^n-1}]]] = \lambda i : \text{word}(n). [[e_i]]$$

$$[[\{k_1 = e_{k_1}, \dots, k_n = e_{k_n}\}]] = \lambda k : \{k_1, \dots, k_n\}. [[e_k]]$$

$$[[e_v[e_i]]] = [[e_v]]([[e_i]])$$

$$[[e.k]] = [[e]](k)$$

# Inductive Set $\longrightarrow^*$ (Multi-Step Rel.)

$$\frac{}{\langle e, \sigma \rangle \longrightarrow^* \langle e, \sigma \rangle}$$

$$\frac{\langle e, \sigma \rangle \longrightarrow \langle e', \sigma' \rangle \quad \langle e', \sigma' \rangle \longrightarrow^* \langle e'', \sigma'' \rangle}{\langle e, \sigma \rangle \longrightarrow^* \langle e'', \sigma'' \rangle}$$

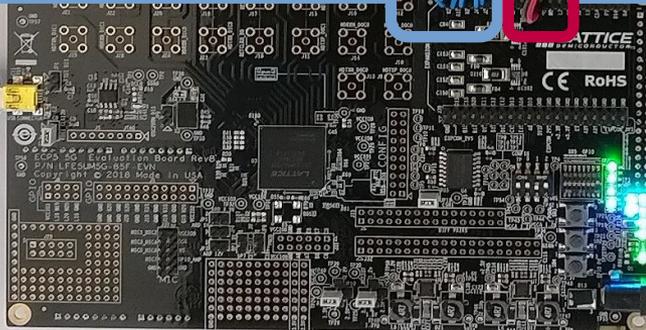
## From single step to multistep in Kami

```
Inductive Multistep: RegsT -> RegsT -> list LabelT -> Prop :=  
| NilMultistep o1 o2: o1 = o2 -> Multistep o1 o2 nil  
| Multi o a n (HMultistep: Multistep o n a)  
  u l (HStep: Step n u l):  
  Multistep o (M.union u n) (l :: a).
```

# Specification using a Log

Recv1  
TurnOn  
PollNone  
Recv0  
TurnOff  
RecvInvalid  
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PollNone

Interface



# The lightbulb spec

*lightbulb\_spec:*

The log of the blue wires and the red wire respects the following regular expression:

BootSeq ((Recv1 On) | (Recv0 Off) | RecvInvalid | PollNone)\*

# The end-to-end theorem

Using Coq, we developed

- a software image (list of bytes)
- a pipelined processor
- a theorem:

*If you put the image at address 0 and set PC to 0 and run our processor, the log of the blue wires and the red wire respects the following regular expression:*

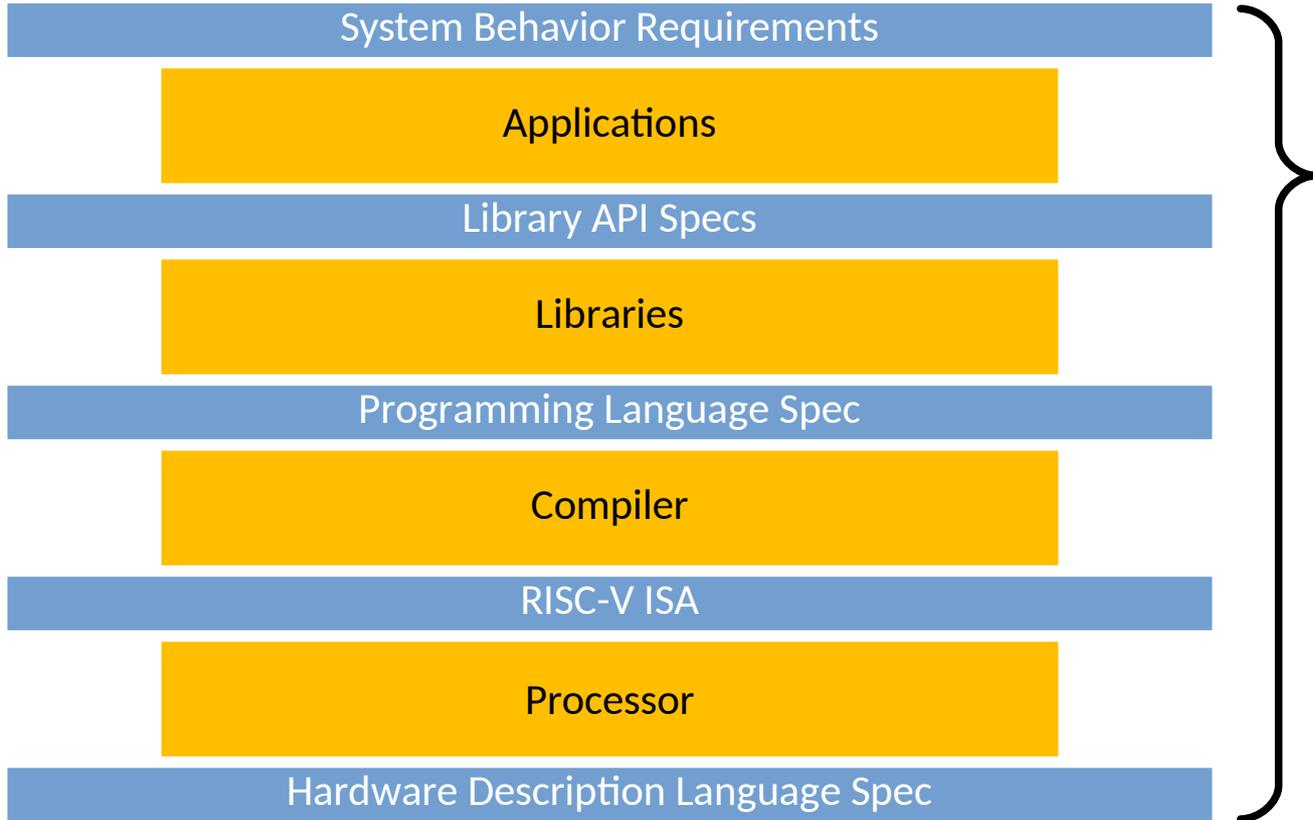
`BootSeq ((Recv1 TurnOn) | (Recv0 TurnOff) | RecvInvalid | PollNone)*`

# End-to-end theorem

**Theorem** end2end\_lightbulb:  $\forall$  mem0 t,  
bytes\_at (instrencode lightbulb\_insts) 0 mem0  $\wedge$   
Trace (p4mm mem0) t  $\rightarrow$   
 $\exists$  t': list (string \* word \* word),  
KamiRiscv.KamiLabelSeqR t t'  $\wedge$   
prefix\_of t' goodHlTrace.

**Definition** goodHlTrace := BootSeq +++  
( (EX b:bool, Recv b +++ LightbulbCmd b)  
||| RecvInvalid ||| PollNone )\*.

# The end-to-end theorem



The end-to-end theorem provides

- a concise description of the behavior of the whole stack
- high assurance of correctness

# The source language

# IMP syntax

$$\begin{aligned} a ::= & x \mid n \mid a_1 + a_2 \mid a_1 \times a_2 \\ b ::= & \mathbf{true} \mid \mathbf{false} \mid a_1 < a_2 \\ c ::= & \mathbf{skip} \mid x := a \mid c_1; c_2 \\ & \mid \mathbf{if } b \mathbf{ then } c_1 \mathbf{ else } c_2 \\ & \mid \mathbf{while } b \mathbf{ do } c \end{aligned}$$

# Bedrock2 Grammar

$e ::=$	
$v$	integer literal
$x$	local variable
$\text{load}N(e)$	load $N$ bytes from address $e$ , $N = 1, 2$ , or $4$
$\text{load}W(e)$	load 4 or 8 bytes, depending on bitwidth
$e_1 \text{ op } e_2$	binary operation, $op = +, -, *, \&, \dots$

$c ::=$	
skip	do nothing
$x = e$	assignment to local variable
$\text{store}N(e_1, e_2)$	store the lower $N$ bytes of $e_2$ at address $e_1$ , $N = 1, 2$ , or $4$
$\text{store}W(e_1, e_2)$	store 4 or 8 bytes, depending on bitwidth, at address $e_1$
if ( $e$ ) $c_1$ else $c_2$	if-then-else
$c_1; c_2$	sequence
while ( $e$ ) $c$	while loop
$x_1, \dots, x_n = f(e_1, \dots, e_m)$	call procedure $f$ and assign return values to $x_1, \dots, x_n$

# Bedrock2 Semantics

$\text{vcgen skip } t \ m \ l \ P :=$

$P \ t \ m \ l$

$\text{vcgen } (x = e) \ t \ m \ l \ P :=$

$\exists v, \text{vcexpr } m \ l \ e \ v \wedge P \ t \ m \ l[x:=v]$

$\text{vcgen } (c1; c2) \ t \ m \ l \ P :=$

$\text{vcgen } c1 \ t \ m \ l \ (\lambda t' \ m' \ l' \Rightarrow \text{vcgen } c2 \ t' \ m' \ l' \ P)$

$\text{vcgen } (\text{store1}(e1, e2)) \ t \ m \ l \ P :=$

$\exists \text{addr}, \text{vcexpr } m \ l \ e1 \ \text{addr} \wedge$

$\exists \text{value}, \text{vcexpr } m \ l \ e2 \ \text{value} \wedge$

$\text{addr} \in \text{dom}(m) \wedge$

$P \ t \ m[\text{addr}:=\text{value}] \ l$

$\text{vcgen } (x = f_{\text{ext}}(e)) \ t \ m \ l \ P :=$

$\exists v, \text{vcexpr } m \ l \ e \ v \wedge$

$\text{vcextern } f_{\text{ext}} \ t \ [v]$

$(\lambda r \Rightarrow P \ ((f, [v], [r]) :: t) \ m \ l[x:=r])$

$\text{vcgen } c \ t \ m \ l \ P :=$

“what do I need to prove to know that executing command  $c$  from a state  $(t, m, l)$  always terminates, and that the final state satisfies  $P$ ?”

# Partial Correctness and Total Correctness

$$\{Pre\} c \{Post\}$$

“If *Pre* holds before *c*, **and** *c* **terminates**, then  
*Post* holds after *c*.”

$$[Pre] c [Post]$$

“If *Pre* holds before *c* **then** *c* **will terminate** and  
*Post* will hold after *c*.”

# Sample program verification

- Let's step through a simple program and see how it creates an MMIO event in the trace

```

function attempt_read_byte() returns (uint32_t err, uint32_t v) {
  uint32_t busy = MMIOREAD(SPI_READ_ADDR);
  if (busy >> 31) {
    err = 1;
  } else {
    err = 0;
    v = busy & 0xff;
  }
}

```

**Lemma** attempt\_read\_byte\_correct:  $\forall t\ m,$   
 $\text{vcgen}(\text{body attempt\_read\_byte})\ t\ m\ \text{empty}\ (\lambda t'\ m'\ l' \Rightarrow$   
 $\exists b, t' = (\text{"MMIOREAD"}, \text{SPI\_READ\_ADDR}, b)::t \wedge$   
 $(0 \leq b < 256 \wedge l'[\text{err}] = 0 \wedge l'[\text{v}] = b) \vee$   
 $(b \gg 31 = 1 \wedge l'[\text{err}] = 1).$

```

∀ t m, vcgen (
  uint32_t busy = MMIOREAD(SPI_READ_ADDR);
  if (busy >> 31) {
    err = 1;
  } else {
    err = 0;
    v = busy & 0xff;
  }
) t m empty (λ t' m' l' =>
  ∃b, t' = ("MMIOREAD", SPI_READ_ADDR, b)::t ∧
    (0 <= b < 256 ∧ l'[err] = 0 ∧ l'[v] = b) ∨
    (b >> 31 = 1 ∧ l'[err] = 1)

```

```

∀ t m, vcgen (
  uint32_t busy = MMIOREAD(SPI_READ_ADDR);
)
t m empty (λ t' m' l' =>
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    if (busy >> 31) {
      err = 1;
    } else {
      err = 0;
      v = busy & 0xff;
    }
  ) t' m' l' (λ t'' m'' l'' =>
    ∃b, t'' = ("MMIOREAD", SPI_READ_ADDR, b)::t ∧
      (0 <= b < 256 ∧ l''[err] = 0 ∧ l''[v] = b) ∨
      (b >> 31 = 1 ∧ l''[err] = 1)
  )
)

```

```

∀ t m, vcextern MMIOREAD t [SPI_READ_ADDR] (λ b =>
  (λ t' m' l' =>
    vcgen (
      if (busy >> 31) {
        err = 1;
      } else {
        err = 0;
        v = busy & 0xff;
      }
    ) t' m' l' (λ t'' m'' l'' =>
      ∃b, t'' = ("MMIOREAD", SPI_READ_ADDR, b)::t ∧
        (0 <= b < 256 ∧ l''[err] = 0 ∧ l''[v] = b) ∨
        (b >> 31 = 1 ∧ l''[err] = 1).
    ) ((("MMIOREAD", SPI_READ_ADDR, b)::t) m empty[busy:=b])
  )

```

```

∀ t m, vcextern MMIOREAD t [SPI_READ_ADDR] (λ b =>
  vcgen
  (
    if (busy >> 31) {
      err = 1;
    } else {
      err = 0;
      v = busy & 0xff;
    }
  )
  ((“MMIOREAD”, SPI_READ_ADDR, b)::t)
  m
  empty[busy:=b]
  (λ t'' m'' l'' =>
    ∃b, t'' = (“MMIOREAD”, SPI_READ_ADDR, b)::t ∧
      (0 <= b < 256 ∧ l''[err] = 0 ∧ l''[v] = b) ∨
      (b >> 31 = 1 ∧ l''[err] = 1)))

```

```

 $\forall$  t m, SPI_READ_ADDR  $\in$  MMIO_RANGE  $\wedge$   $\forall$  r, ( $\lambda$  b  $\Rightarrow$ 
  vcgen
  (
    if (busy >> 31) {
      err = 1;
    } else {
      err = 0;
      v = busy & 0xff;
    }
  )
  ((“MMIOREAD”, SPI_READ_ADDR, b)::t)
  m
  empty[busy:=b]
  ( $\lambda$  t'' m'' l''  $\Rightarrow$ 
     $\exists$ b, t'' = (“MMIOREAD”, SPI_READ_ADDR, b)::t  $\wedge$ 
      (0 <= b < 256  $\wedge$  l''[err] = 0  $\wedge$  l''[v] = b)  $\vee$ 
      (b >> 31 = 1  $\wedge$  l''[err] = 1))
  ) r

```

```

∀ t m, SPI_READ_ADDR ∈ MMIO_RANGE ∧ ∀ r,
  vcgen
  (
    if (busy >> 31) {
      err = 1;
    } else {
      err = 0;
      v = busy & 0xff;
    }
  )
  ((“MMIOREAD”, SPI_READ_ADDR, r)::t)
  m
  empty[busy:=r]
  (λ t'' m'' l'' =>
    ∃b, t'' = (“MMIOREAD”, SPI_READ_ADDR, b)::t ∧
      (0 <= b < 256 ∧ l''[err] = 0 ∧ l''[v] = b) ∨
      (b >> 31 = 1 ∧ l''[err] = 1))

```

```

V t m r,
  vcgen
  (
    if (busy >> 31) {
      err = 1;
    } else {
      err = 0;
      v = busy & 0xff;
    }
  )
  ((“MMIOREAD”, SPI_READ_ADDR, r)::t)
  m
  empty[busy:=r]
  (λ t'' m'' l'' =>
    ∃b, t'' = (“MMIOREAD”, SPI_READ_ADDR, b)::t ∧
      (0 <= b < 256 ∧ l''[err] = 0 ∧ l''[v] = b) ∨
      (b >> 31 = 1 ∧ l''[err] = 1))

```

```

∀ t m r,
  ∃ a, vcexpr m empty[busy:=r] (busy >> 31) a
    ∧ (a <> 0 ->
      vcgen (
        err = 1
      ) ((“MMIOREAD”, SPI_READ_ADDR, r)::t) m empty[busy:=r]
      (λ t'' m'' l'' =>
        ∃b, t'' = (“MMIOREAD”, SPI_READ_ADDR, b)::t ∧
          (0 <= b < 256 ∧ l''[err] = 0 ∧ l''[v] = b) ∨
          (b >> 31 = 1 ∧ l''[err] = 1)))
    ∧ (a = 0 ->
      vcgen (
        err = 0;
        v = busy & 0xff;
      ) ((“MMIOREAD”, SPI_READ_ADDR, r)::t) m empty[busy:=r]
      (λ t'' m'' l'' =>
        ∃b, t'' = (“MMIOREAD”, SPI_READ_ADDR, b)::t ∧
          (0 <= b < 256 ∧ l''[err] = 0 ∧ l''[v] = b) ∨
          (b >> 31 = 1 ∧ l''[err] = 1)))

```

```

V t m r,
  E a, a = r >> 31
    A (a <> 0 ->
      vcgen (
        err = 1
      ) ((“MMIOREAD”, SPI_READ_ADDR, r)::t) m empty[busy:=r]
      (λ t'' m'' l'' =>
        E b, t'' = (“MMIOREAD”, SPI_READ_ADDR, b)::t A
          (0 <= b < 256 A l''[err] = 0 A l''[v] = b) V
          (b >> 31 = 1 A l''[err] = 1)))
    A (a = 0 ->
      vcgen (
        err = 0;
        v = busy & 0xff;
      ) ((“MMIOREAD”, SPI_READ_ADDR, r)::t) m empty[busy:=r]
      (λ t'' m'' l'' =>
        E b, t'' = (“MMIOREAD”, SPI_READ_ADDR, b)::t A
          (0 <= b < 256 A l''[err] = 0 A l''[v] = b) V
          (b >> 31 = 1 A l''[err] = 1)))

```

```

∀ t m r,
  (r >> 31 <> 0 ->
    vcgen (
      err = 1
    ) ((“MMIOREAD”, SPI_READ_ADDR, r)::t) m empty[busy:=r]
    (λ t'' m'' l'' =>
      ∃b, t'' = (“MMIOREAD”, SPI_READ_ADDR, b)::t ∧
        (0 <= b < 256 ∧ l''[err] = 0 ∧ l''[v] = b) ∨
        (b >> 31 = 1 ∧ l''[err] = 1)))
  ∧ (r >> 31 = 0 ->
    vcgen (
      err = 0;
      v = busy & 0xff;
    ) ((“MMIOREAD”, SPI_READ_ADDR, r)::t) m empty[busy:=r]
    (λ t'' m'' l'' =>
      ∃b, t'' = (“MMIOREAD”, SPI_READ_ADDR, b)::t ∧
        (0 <= b < 256 ∧ l''[err] = 0 ∧ l''[v] = b) ∨
        (b >> 31 = 1 ∧ l''[err] = 1)))

```

```

∀ t m r,
  (r >> 31 <> 0 ->
   vcgen (
     err = 1
   ) ((“MMIOREAD”, SPI_READ_ADDR, r)::t) m empty[busy:=r]
  (λ t'' m'' l'' =>
    ∃b, t'' = (“MMIOREAD”, SPI_READ_ADDR, b)::t ∧
      (0 <= b < 256 ∧ l''[err] = 0 ∧ l''[v] = b) ∨
      (b >> 31 = 1 ∧ l''[err] = 1)))

```

```

∀ t m r,
  (r >> 31 <> 0 ->
    (λ t'' m'' l'' =>
      ∃ b, t'' = ("MMIOREAD", SPI_READ_ADDR, b)::t ∧
        (0 <= b < 256 ∧ l''[err] = 0 ∧ l''[v] = b) ∨
        (b >> 31 = 1 ∧ l''[err] = 1)))
    ((("MMIOREAD", SPI_READ_ADDR, r)::t) m empty[busy:=r][err:=1]

```

```

∀ t m r, r >> 31 <> 0 ->
  ∃ b, ((“MMIOREAD”, SPI_READ_ADDR, r)::t) = (“MMIOREAD”, SPI_READ_ADDR, b)::t ∧
    ((0 ≤ b < 256 ∧ empty[busy:=r][err:=1][err] = 0 ∧
      empty[busy:=r][err:=1][v] = b)
      ∨
      (b >> 31 = 1 ∧ empty[busy:=r][err:=1][err] = 1)))

```

```
∀ t m r, r >> 31 <> 0 ->  
  True ∧  
  ((0 ≤ r < 256 ∧ empty[busy:=r][err:=1][err] = 0 ∧  
    empty[busy:=r][err:=1][v] = r)  
  ∨  
  (r >> 31 = 1 ∧ empty[busy:=r][err:=1][err] = 1)))
```

$\forall t m r, r \gg 31 \langle \rangle 0 \rightarrow$   
 $(r \gg 31 = 1 \wedge \text{empty}[\text{busy}:=r][\text{err}:=1][\text{err}] = 1)$

$\forall t m r, r \gg 31 \langle \rangle 0 \rightarrow$

$r \gg 31 = 1 \wedge \text{empty}[\text{busy}:=r][\text{err}:=1][\text{err}] = 1$

# Viewing postconditions as continuations

$$\text{SEQ} \frac{\vdash \{P\} c_1 \{R\} \quad \vdash \{R\} c_2 \{Q\}}{\vdash \{P\} c_1; c_2 \{Q\}}$$

$$\mathcal{T}[\textit{let } x = e_1 \textit{ in } e_2] = \lambda k. \mathcal{T}[e_1] (\lambda x. \mathcal{T}[e_2] k)$$

k: continuation taking the result value of the expression

P: postcondition taking resulting trace t, memory m, and local vars l

$\text{vcgen } (c_1; c_2) \text{ t m l P} :=$

$\text{vcgen } c_1 \text{ t m l } (\lambda \text{ t' m' l' } \Rightarrow \text{vcgen } c_2 \text{ t' m' l' P})$

# Bedrock2 semantics for compiler correctness proof

- vcgen is a function
- But for compiler correctness, need an Inductive representing executions to drive proof by induction over executions of programs

$$\frac{\text{EVAL-UNOP} \quad (y, v_y) \in \ell \quad \text{evalunop}(op, v_y, v) \quad Q(m, \ell[x := v], \tau)}{(x = op(y))/m/\ell/\tau \Downarrow Q}$$

$$\frac{\text{EVAL-INPUT} \quad \forall n, Q(m, \ell[x := n], \tau :: \text{IN } n)}{(x = \text{input}())/m/\ell/\tau \Downarrow Q}$$

$$\frac{\text{EVAL-SEQ} \quad \begin{array}{l} c_1/m/\ell/\tau \Downarrow Q_1 \\ (\forall m' \ell' \tau', Q_1(m', \ell', \tau') \implies c_2/m'/\ell'/\tau' \Downarrow Q) \end{array}}{(c_1; c_2)/m/\ell/\tau \Downarrow Q}$$

$$\frac{\text{EVAL-STORE} \quad \begin{array}{l} (x, a) \in \ell \quad (a + n) \in \text{dom } m \\ (y, v) \in \ell \quad Q(m[(a + n) := v], \ell, \tau) \end{array}}{(x[n] = y)/m/\ell/\tau \Downarrow Q}$$

# Large-step semantics

$$\Downarrow \subseteq \mathbf{Config} \times \mathbf{FinalConfig}$$

where

$$\mathbf{Config} = \mathbf{Exp} \times \mathbf{Store}$$

$$\text{and } \mathbf{Final Config} = \mathbf{Int} \times \mathbf{Store} \subseteq \mathbf{Config}.$$

We write  $\langle e, \sigma \rangle \Downarrow \langle n, \sigma' \rangle$  to mean that  
 $(\langle e, \sigma \rangle, \langle n, \sigma' \rangle) \in \Downarrow$

# Compiler correctness

Simple case: Source and target language is the same

$$\forall t m l, c/m/l/t \Downarrow Q \longrightarrow \text{compile}(c)/m/l/t \Downarrow Q$$

“ $c/m/l/t \Downarrow Q$ ” is an Inductive Prop, and we can write proofs by induction on it

# Omnisemantics: Smoother Handling of Nondeterminism

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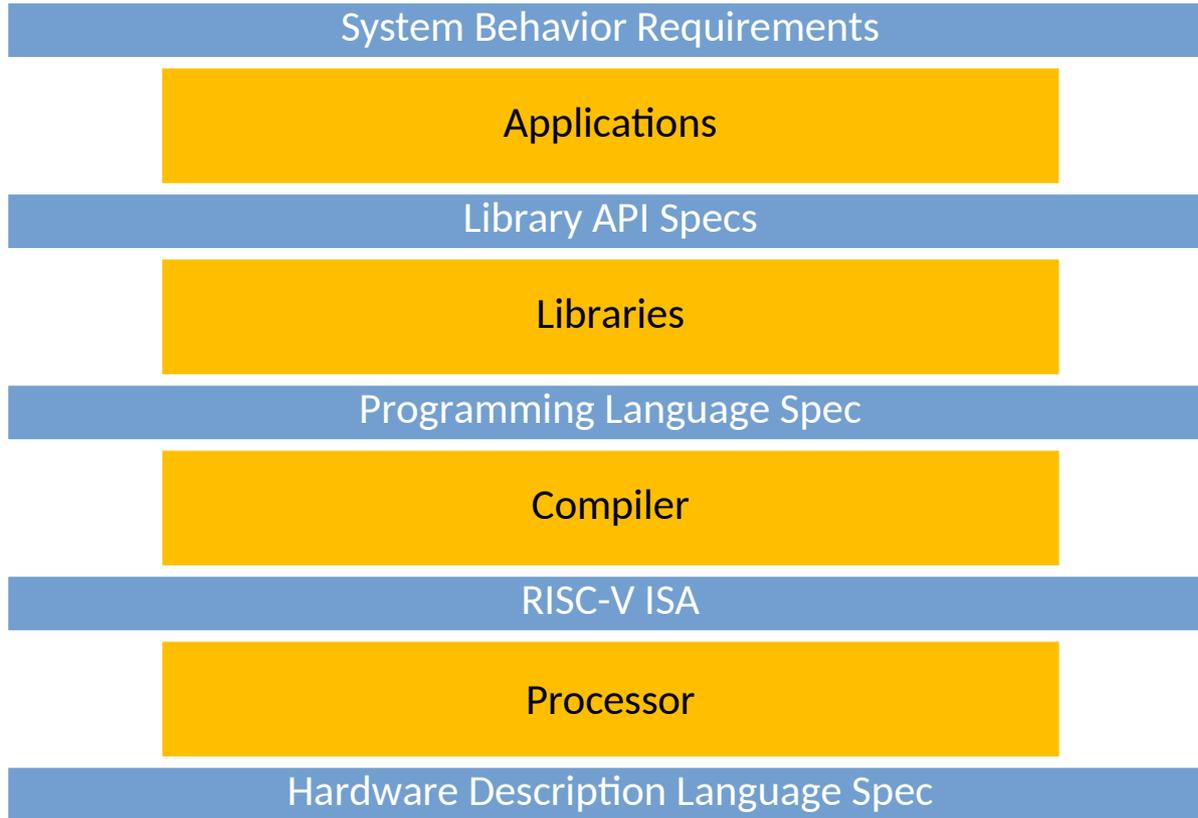
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This paper gives an in-depth presentation of the omni-big-step and omni-small-step styles of semantic judgments. These styles describe operational semantics by relating starting states to sets of outcomes rather than to individual outcomes. A single derivation of these semantics for a particular starting state and program describes all possible nondeterministic executions (hence the name *omni*), whereas in traditional small-step and big-step semantics, each derivation only talks about one single execution. This restructuring allows for straightforward modeling of languages featuring both nondeterminism and undefined behavior. Specifically, omnisemantics inherently assert *safety*, i.e. they guarantee that none of the execution branches gets stuck, while traditional semantics need either a separate judgment or additional error markers to specify safety in the presence of nondeterminism.

Omnisemantics can be understood as an inductively defined weakest-precondition semantics (or more generally, predicate-transformer semantics) that does not involve invariants for loops and recursion, but instead uses unrolling rules like in traditional small-step and big-step semantics. Omnisemantics have already been used in the past, but we believe that it has been under-appreciated and that it deserves a well-motivated, extensive and pedagogical presentation of its benefits. We also explore several novel aspects associated with these semantics, in particular their use in type-soundness proofs for lambda calculi, partial-correctness reasoning, and forward proofs of compiler correctness for terminating but potentially nondeterministic programs being compiled to nondeterministic target languages. All results in this paper are formalized in Coq.

# Apply semantics, PL, verification throughout the whole stack



# Questions?

## References:

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