Harvard School of Engineering and Applied Sciences — CS 152: Programming Languages

Induction; Small-step operational semantics; Large-step operational semantics Section and Practice Problems

Week 3

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1 Induction

Let's inductively define a set of integers **Quux** with the following inference rules.

$$\textsc{Rule1}\ \frac{}{8 \in \textbf{Quux}} \qquad \textsc{Rule2}\ \frac{}{5 \in \textbf{Quux}} \qquad \textsc{Rule3}\ \frac{a \in \textbf{Quux} \quad b \in \textbf{Quux}}{c \in \textbf{Quux}}\ c = a + b + 1$$

(a) Of the rules above (i.e., RULE1, RULE2, and RULE3), which are axioms and which are inductive rules?

(b) Give a derivation showing that 11 is in the set **Quux**.

(c) Give a derivation showing that 20 is in the set **Quux**.

(d) Write down the inductive reasoning principle for **Quux**. That is, if you wanted to prove that for some property P, for all $a \in$ **Quux** we have $P(a)$, what would you need to show? (See Lecture 3 §2.2 and §2.3.)

(e) Prove that for all $a \in$ **Quux**, there exists $i \in \mathbb{Z}$ such that $a = 3 \times i - 1$.

Make sure that you follow the Recipe for Inductive Proofs! See Lecture 3 §2.5. What set are you inducting on? What is the property you are trying to prove? Go through each case.

(f) Is 2 in the set **Quux**? If so, give a derivation proving it.

2 Small-step operational semantics

Consider the small-step operational semantics for the language of arithmetic expressions (Lecture 2). Let σ_0 be a store that maps all program variables to zero.

(a) Show a derivation that $\langle 3 + (5 \times \mathsf{bar}), \sigma_0\rangle \longrightarrow \langle 3 + (5 \times 0), \sigma_0\rangle$.

(b) What is the sequence of configurations that $\langle \mathsf{foo} := 5;\ (\mathsf{foo} + 2) \times 7, \sigma_0\rangle$ steps to? (You don't need to show the derivations for each step, just show what configuration $\langle \mathsf{foo} := 5;\ (\mathsf{foo} + 2) \times 7, \sigma_0\rangle$ steps to in one step, then two steps, then three steps, and so on, until you reach a final configuration.)

(c) Find an integer n and store σ' such that $\langle((6+(\mathsf{foo} := (\mathsf{bar} := 3; 5); 1+\mathsf{bar}))+\mathsf{bar})\times\mathsf{foo}, \sigma_0\rangle \longrightarrow^* \langle n, \sigma'\rangle$.

(d) Is the relation $\longrightarrow$ reflexive? Is it symmetric? Is it anti-symmetric? Is it transitive?

(For each of these questions, if the answer is "no", what is a suitable counterexample? If any of the answers are "yes", think about how you would prove it.)

3 Large-step operational semantics

Consider the large-step operational semantics for the language of arithmetic expressions (Lecture 4). Let σ_0 be a store that maps all program variables to zero.

(a) Show a derivation that $\langle 3 + (5 \times \mathsf{bar}), \sigma_0\rangle \Downarrow \langle 3, \sigma_0\rangle$.

(b) Find an integer n and store σ' such that $\langle \mathsf{foo} := 5;\ (\mathsf{foo} + 2) \times 7, \sigma_0 \rangle \Downarrow \langle n, \sigma' \rangle$.

If you have time and a big piece of paper, give the derivation of $\langle \mathsf{foo} := 5;\ (\mathsf{foo} + 2) \times 7, \sigma_0 \rangle \Downarrow \langle n, \sigma' \rangle$.

(c) Is the relation $\Downarrow$ reflexive? Is it symmetric? Is it anti-symmetric? Is it transitive?

(For each of these questions, if the answer is "no", what is a suitable counterexample? If any of the answers are "yes", think about how you would prove it.)