HARVARD
John A. Paulson
School of Engineering
and Applied Sciences

CS153: Compilers Lecture 17: Control Flow Graph and Data Flow Analysis

Stephen Chong

https://www.seas.harvard.edu/courses/cs153

Announcements

- Project 5 out
  - Due Tuesday Nov 13 (14 days)
- Project 6 out
  - Due Tuesday Nov 20 (21 days)
- Project 7 will be released today
  - Due Thursday Nov 29 (30 days)

Today

- Control Flow Graphs
  - Basic Blocks
- Dataflow Analysis
  - Available Expressions

Optimizations So Far

- We've look only at local optimizations
  - Limited to "pure" expressions
  - Avoid variable capture by having unique variable names

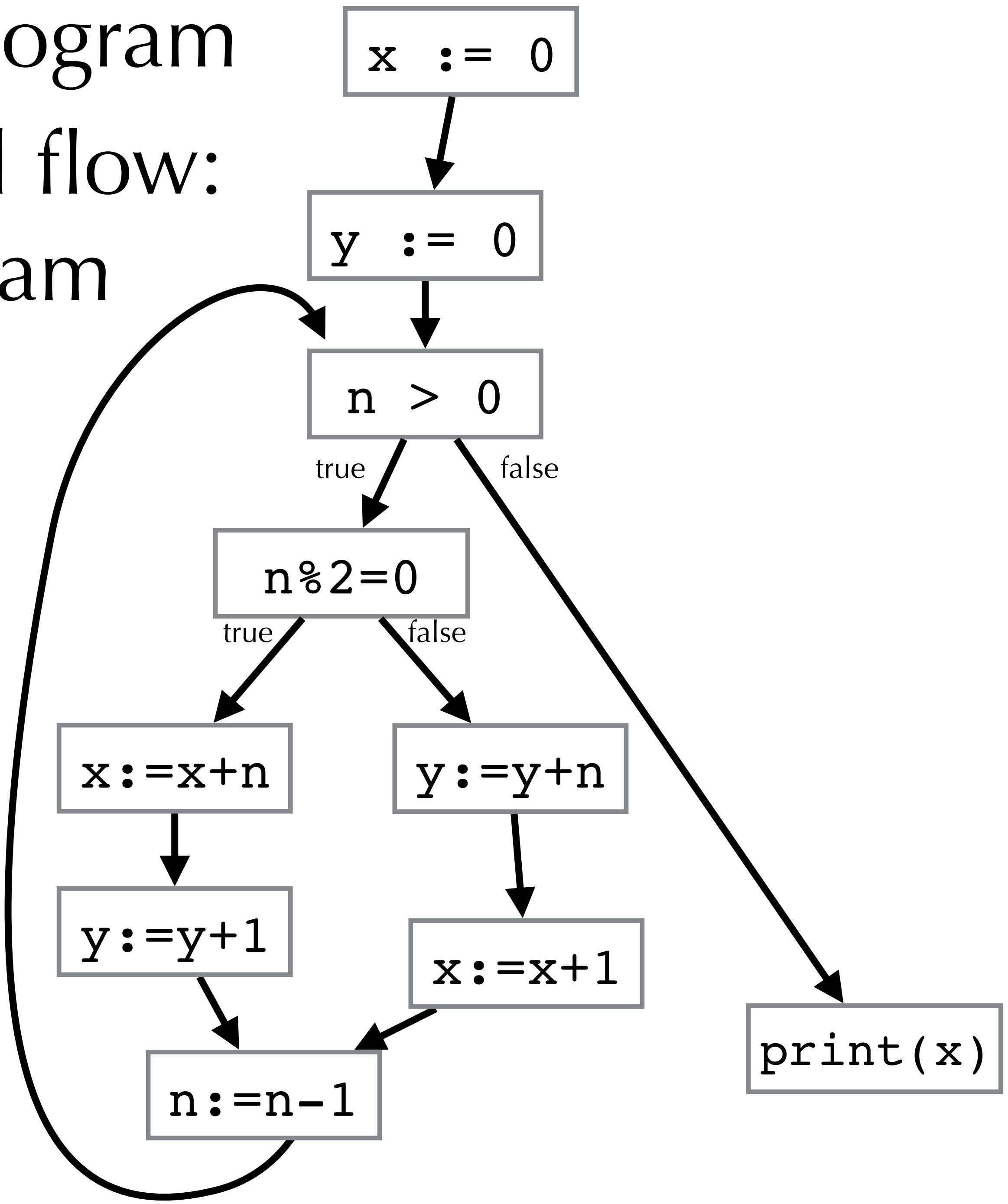
Next Few Lectures

- Imperative Representations
  - Like MIPS assembly at the instruction level.
    - except we assume an infinite # of temps
    - and abstract away details of the calling convention
  - But with a bit more structure.
- Organized into a Control-Flow graph
  - nodes: labeled basic blocks of instructions
    - single-entry, single-exit
    - i.e., no jumps, branching, or labels inside block
  - edges: jumps/branches to basic blocks
- Dataflow analysis
  - computing information to answer questions about data flowing through the graph.

Control-Flow Graphs

- Graphical representation of a program
- Edges in graph represent control flow: how execution traverses a program
- Nodes represent statements

```
x := 0;
y := 0;
while (n > 0) {
  if (n % 2 = 0) {
    x := x + n;
    y := y + 1;
  }
  else {
    y := y + n;
    x := x + 1;
 }
  n := n - 1;
}
print(x);
```



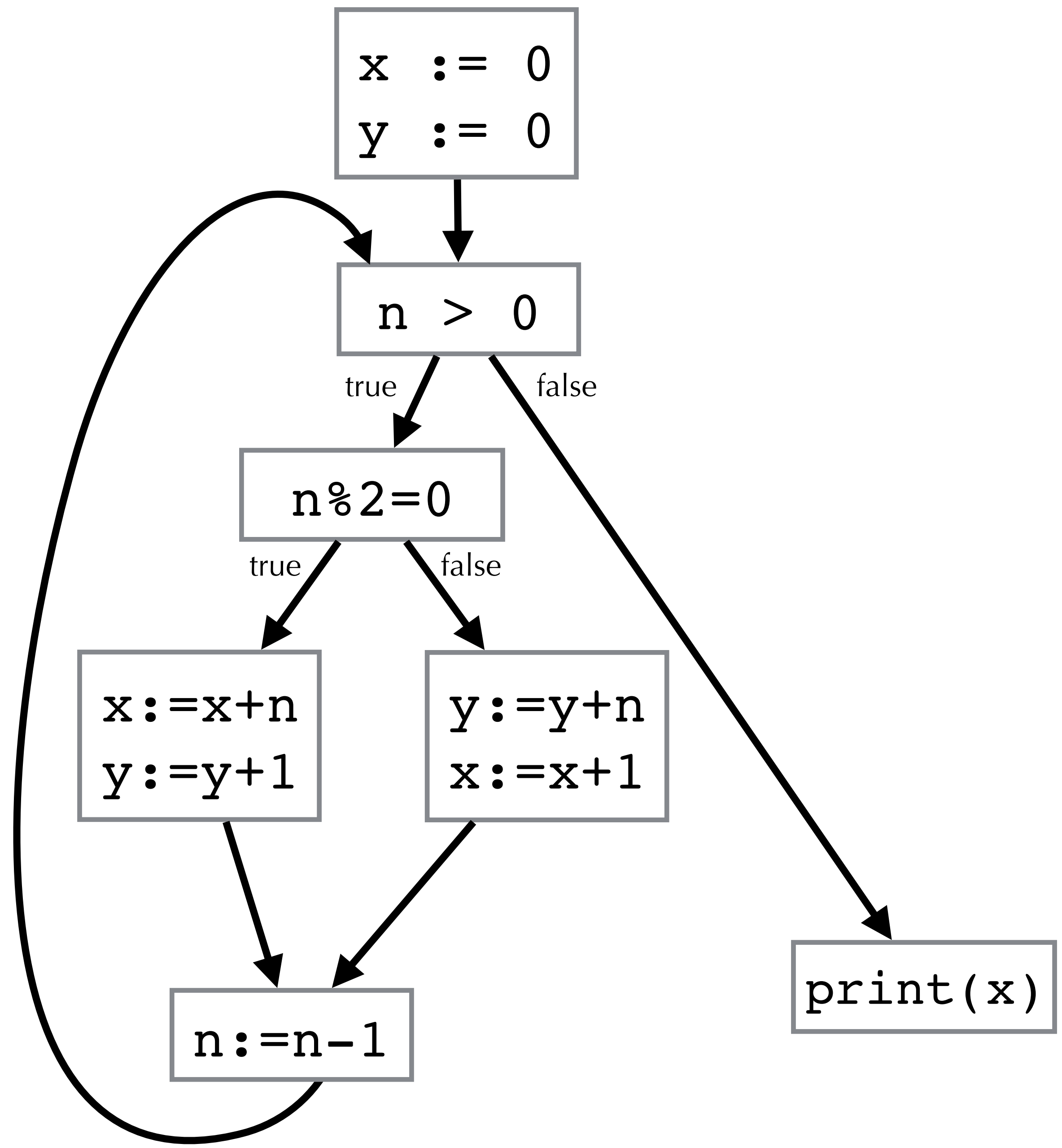
Basic Blocks

- We will require that nodes of a control flow graph are **basic blocks**
  - Sequences of statements such that:
    - Can be entered only at beginning of block
    - Can be exited only at end of block
      - Exit by branching, by unconditional jump to another block, or by returning from function
- Basic blocks simplify representation and analysis

Basic Blocks

- Basic block: single entry, single exit

```
x := 0;
y := 0;
while (n > 0) {
  if (n % 2 = 0) {
    x := x + n;
    y := y + 1;
  }
  else {
    y := y + n;
    x := x + 1;
 }
  n := n - 1;
}
print(x);
```



CFG Abstract Syntax

```
type operand =
    | Int of int | Var of var | Label of label

type block =
| Return of operand
| Jump of label
| Branch of operand * test * operand * label * label
| Move of var * operand * block
| Load of var * int * operand * block
| Store of var * int * operand * block
| Assign of var * primop * (operand list) * block
| Call of var * operand * (operand list) * block

type proc = { vars : var list,
              prologue: label, epilogue: label,
              blocks : (label * block) list }
```

Differences with Monadic Form

- Essentially MIPS assembly with infinite number of registers
- No lambdas, so easy to translate to MIPS modulo register allocation and assignment.
  - Monadic form requires extra pass to eliminate lambdas and make closures explicit (closure conversion, lambda lifting)
- Unlike Monadic Form, variables are **mutable**
- `Return` constructor is function return, not monadic return

Let's Revisit Optimizations

- Folding
  - `t:=3+4` becomes `t:=7`
- Constant propagation
  - `t:=7; B; u:=t+3; B'`
    becomes `t := 7; B; u:=7+3; B'`
  - Problem! `B` might assign a fresh value to `t`
- Copy propagation
  - `t:=u; B; v:=t+3; B'`
    becomes `t:=u; B; v:=u+3; B'`
  - Problem! `B` might assign a fresh value to `t` or `u`

Let's Revisit Optimizations

- Dead code elimination
  - `x:=e; B; jump L` becomes `B; jump L`
    - Problem! Block `L` might use `x`
  - `x:=e1;`B_1`; x:=e2;`B_2 becomes B_1`;x:=e2;`B_2 (`x` not used in B_1)
- Common sub-expression elimination
  - `x:=y+z;`B_1`;w := y+z;`B_2 becomes `x:=y+z;`B_1`;w:=x;`B_2
    - problem: B_1 might change `x`, `y`, or `z`

Optimization in Imperative Settings

- Optimization on a functional representation:
  - Only had to worry about variable capture.
  - Could avoid this by renaming variables so that they were unique.
  - then: `let x=p(v1,…,vn) in e` == `e[x↦p(v1,…,vn)]`
- Optimization in an imperative representation:
  - Have to worry about intervening updates
    - for defined variable, similar to variable capture.
    - but must also worry about free variables.
    - `x:=p(v1,…,vn);B` == `B[x↦p(v1,…,vn)]` only when `B` doesn't modify `x` or modify any of the v_i!
  - On the other hand, graph representation makes it possible to be more precise about the scope of a variable.

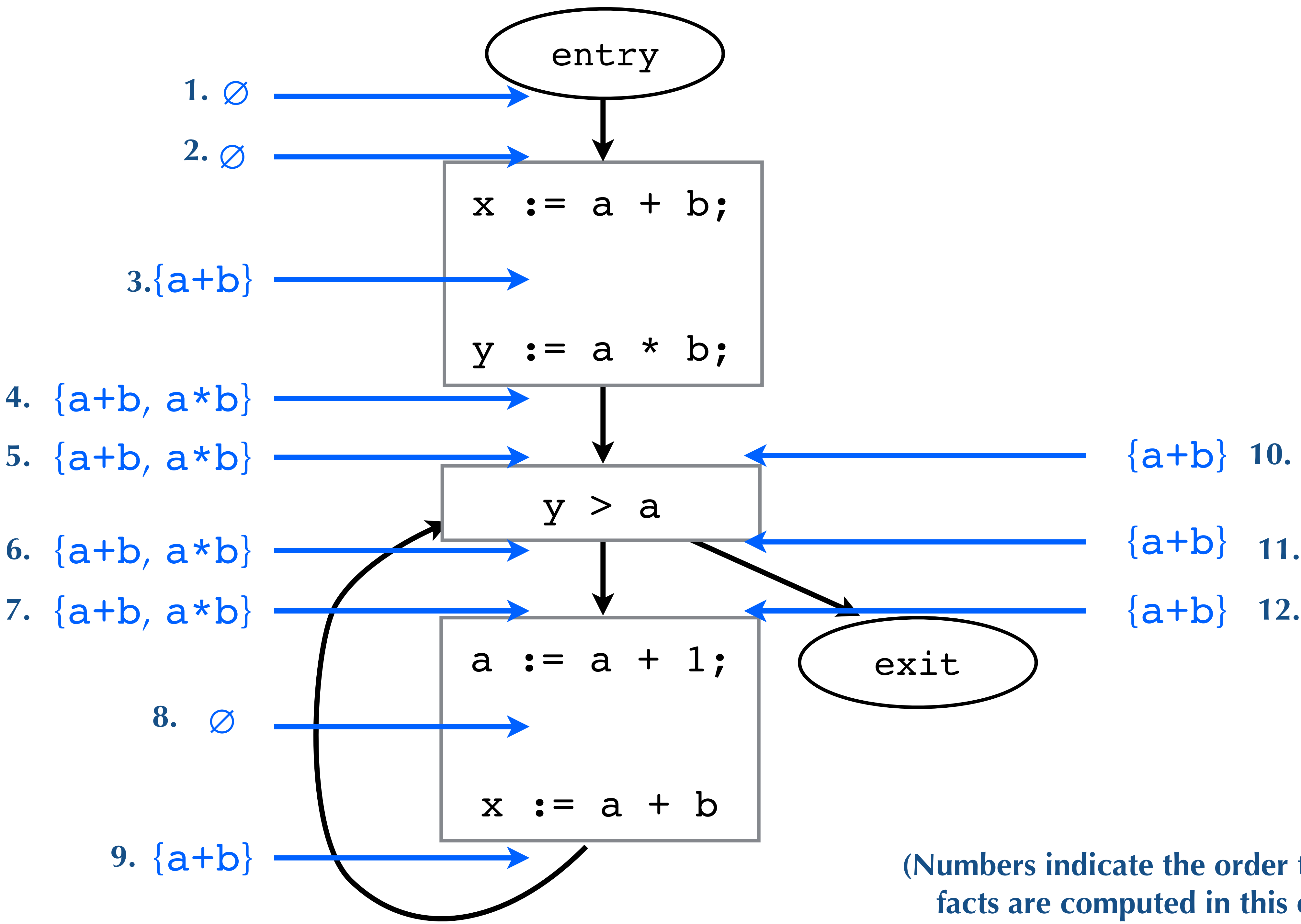
Dataflow Analysis

- To handle intervening updates we will compute **analysis facts** for each **program point**
  - There is a "program point" immediately before and after each instruction
- Analysis facts are facts about variables, expressions, etc.
  - Which facts we are interested in will depend on the particular optimization or analysis we are concerned with
- Given that some facts D hold at a program point before instruction S, after S executes some facts D' will hold
  - How S transforms D into D' is called the transfer function for S
- This kind of analysis is called dataflow analysis
  - Because given a control-flow graph, we are computing facts about data/variables and propagating these facts over the control flow graph

Available Expressions

- An expression `e` is **available** at program point *p* if on all paths from the entry to *p*, expression `e` is computed at least once, and there are no intervening assignment to `x` or to the free variables of `e`
- If `e` is available at *p*, we do not need to re-compute `e`
  - (i.e., for common sub-expression elimination)
- How do we compute the available expressions at each program point?

Available Expressions Example



(Numbers indicate the order that the facts are computed in this example.)

More Formally

- Suppose D is a set of expressions that are available at program point p
- Suppose p is immediately before "`x := e1; B`"
- Then the statement "`x:=e1`"
  - generates the available expression e_1, and
  - kills any available expression e_2 in D such that `x` is in variables(e_2)
- So the available expressions for `B` are:
  $(D \cup \{e_1\}) - \{\, e_2 \mid x \in \text{variables}(e_2) \,\}$

Gen and Kill Sets

- Can describe this analysis by the set of available expressions that each statement generates and kills!

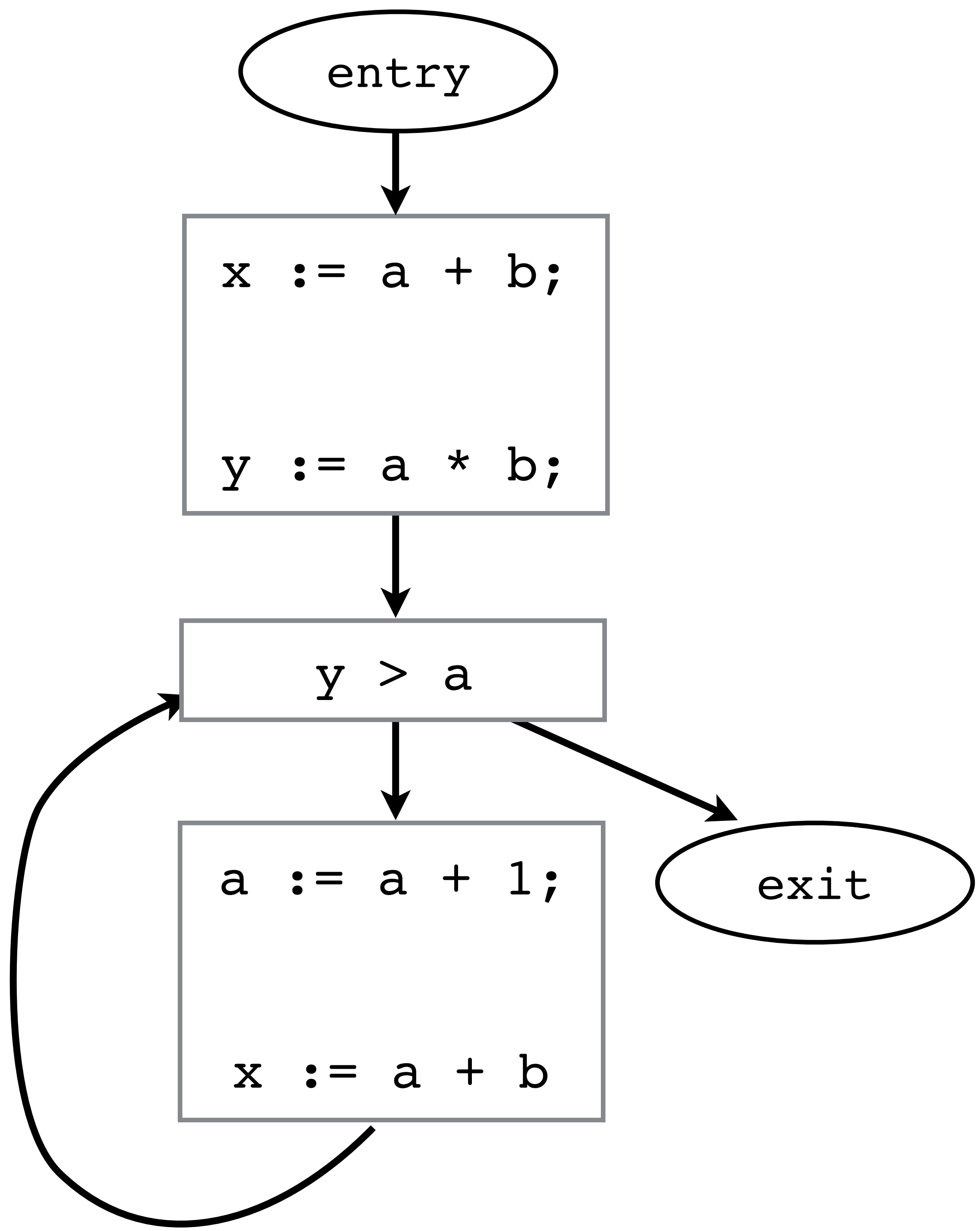
Stmt	Gen	Kill
`x:=v`	{ v }	{e | x in e}
`x:=v1 op v2`	{v1 op v2}	{e | x in e}
`x:=*(v+i)`	{}	{e | x in e}
`*(v+i):=x`	{}	{}
`jump L`	{}	{}
`return v`	{}	{}
`if v1 op v2 goto L1 else goto L2`	{}	{}
`x:=v(v1,...vn)`	{}	{e | x in e}

- Transfer function for stmt S: $\lambda D.\ (D \cup Gen_S) - Kill_S$

Available Expressions Example

- What is the effect of each statement on the facts?

Stmt	Gen	Kill
x := a + b	a+b	
y := a * b	a*b	
y > a		
a := a + 1	a+1	a+1 a+b a*b



Aliasing

- We don't track expressions involving memory (loads & stores).
  - We can tell whether variables names are equal.
  - We cannot (in general) tell whether two variables will have the same value.
  - If we track `*x` as an available expression, and then see `*y := e'`, don't know whether to kill `*x`
    - Don't know whether `x`'s value will be the same as `y`'s value

Function Calls

- Because a function call may access memory, and may have side effects, we can't consider them to be available expressions

Flowing Through the Graph

- How to propagate available expression facts over control flow graph?
- Given available expressions D_{in}[`L`] that flow into block labeled `L`, compute D_{out}[`L`] that flow out
  - Composition of transfer functions of statements in `L`'s block
- For each block `L`, we can define:
  - *succ*[`L`] = the blocks `L` might jump to
  - *pred*[`L`] = the blocks that might jump to `L`
- We can then flow D_{out}[`L`] to all of the blocks in *succ*[`L`]
  - They'll compute new D_{out}'s and flow them to their successors and so on
- How should we combine facts from predecessors?
  - e.g., if *pred*[`L`] = {L_1, L_2, L_3}, how do we combine $D_{out}[L_1]$, $D_{out}[L_2]$, $D_{out}[L_3]$ to get D_{in}[`L`] ?
  - Union or intersection?

Algorithm Sketch

- initialize D_{in}[`L`] to the empty set.
- initialize D_{out}[`L`] to the available expressions that flow out of block `L`, assuming D_{in}[`L`] are the set flowing in.
- loop until no change {
-   for each `L`:
-     In := ∩{D_{out}[`L'`] | `L'` in *pred*[`L`] }
-     if $In \neq D_{in}$[`L`] then {
-       D_{in}[`L`] := In
-       D_{out}[`L`] := flow D_{in}[`L`] through `L`'s block.
-     }
- }

Termination and Speed

- We know the available expressions dataflow analysis will terminate!
  - Each time through the loop each D_{in}[`L`] and D_{out}[`L`] either stay the same or increase
  - If all D_{in}[`L`] and D_{out}[`L`] stay the same, we stop
  - There's a finite number of assignments in the program and finite blocks, so a finite number of times we can increase D_{in}[`L`] and D_{out}[`L`]
- In general, if set of facts form a lattice, transfer functions monotonic, then termination guaranteed
- There are a number of tricks used to speed up the analysis:
  - Can keep a work queue that holds only those blocks that have changed
  - Pre-compute transfer function for a block (i.e., composition of transfer functions of statements in block)