HARVARD
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CS153: Compilers Lecture 19: Loop Optimization II

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https://www.seas.harvard.edu/courses/cs153

Pre-class Puzzle

- The following code multiplies two *N*x*N* matrices, `a` and `b`. What equivalent code would likely execute faster?

```
for (int j = 0; j < N; j++) {
    for (int i = 0; i < N; i++) {
        c[i][j] = 0;
        for (int k = 0; k < N; k++) {
            c[i][j] += a[i][k] * b[k][j];
        }
    }
}
```

Announcements

- Project 5 out
  - Due Tuesday Nov 13 (7 days)
- Project 6 out
  - Due Tuesday Nov 20 (14 days)
- Project 7 out
  - Due Thursday Nov 29 (23 days)

Today

- Loop optimization ctd
  - Loop-invariant removal
  - Induction variable reduction
  - Loop fusion
  - Loop fission
  - Loop unrolling
  - Loop interchange
  - Loop peeling
  - Loop tiling
  - Loop parallelization

Loop-Invariant Removal

Loop Invariants

- An assignment `x := v1 op v2` is **invariant** for a loop if for each operand v_1 and v_2 either
  - the operand is constant, or
  - all of the definitions that reach the assignment are outside the loop, or
  - only one definition reaches the assignment and it is a loop invariant

Example

```
L0:   t := 0
      a := x
L1:   i := i + 1
      b := 7
      t := a + b
      *i := t
      if i<N goto L1 else L2

L2:   x := t
```

Hoisting

- We would like to **hoist** invariant computations out of the loop
- But this is trickier than it sounds:
  - We need to potentially introduce an extra node in the CFG, right before the header to place the hoisted statements (the **pre-header**)
  - Even then, we can run into trouble…

Valid Hoisting Example

```
L0:   t := 0


L1:   i := i + 1
      t := a + b
      *i := t
      if i<N goto L1 else L2


L2:   x := t
```

Valid Hoisting Example

```
L0:  t := 0
     t := a + b


L1:  i := i + 1

     *i := t
     if i<N goto L1 else L2


L2:  x := t
```

Invalid Hoisting Example

```
L0:  t := 0


L1:  i := i + 1
     *i := t
     t := a + b
     if i<N goto L1 else L2


L2:  x := t
```

Although `t`'s definition is loop invariant, hoisting conflicts with this use of `t`

Conditions for Safe Hoisting

- An invariant assignment *d*: `x:= v`$_1$ `op v`$_2$ is safe to hoist if:
  - *d* dominates all loop exits at which **x** is live and
  - there is only one definition of **x** in the loop, and
  - **x** is not live at the entry point for the loop (the pre-header)

Induction Variable Reduction

Induction Variables

```
        s := 0
        i := 0
L1:     if i >= n goto L2
        j := i*4
        k := j+a
        x := *k
        s := s+x
        i := i+1
L2:     ...
```

- Can express `j` and `k` as linear functions of `i` where the coefficients are either constants or loop-invariant
  - `j` = `4*i + 0`
  - `k` = `4*i + a`

Induction Variables

```
       s := 0
       i := 0
L1:    if i >= n goto L2
       j := i*4
       k := j+a
       x := *k
       s := s+x
       i := i+1
L2:    ...
```

- Note that `i` only changes by the same amount each iteration of the loop
- We say that `i` is a **linear induction variable**
- It's easy to express the change in `j` and `k`
  - Since `j` = `4*i + 0` and `k` = `4*i + a`, if `i` changes by *c*, `j` and `k` change by `4*`*c*

Detecting Induction Variables

- **Definition:** `i` is a **basic induction variable** in a loop *L* if the only definitions of `i` within *L* are of the form `i:=i+`*`c`* or `i:=i-`*`c`* where *`c`* is loop invariant
- **Definition:** `k` is a **derived induction variable** in loop *L* if:
  - 1.There is only one definition of `k` within *L* of the form `k:=j*c` or `k:=j+`*`c`* where `j` is an induction variable and *`c`* is loop invariant; and
  - 2.If `j` is an induction variable in the family of `i` (i.e., linear in `i`) then:
    - the only definition of `j` that reaches `k` is the one in the loop; and
    - there is no definition of `i` on any path between the definition of `j` and the definition of `k`
- If `k` is a derived induction variable in the family of `j` and `j` = *`a`*`*i+`*`b`* and, say, `k:=j*`*`c`*, then *k* = *`a`*`*`*`c`*`*i+`*`b`*`*`*`c`*

Strength Reduction

- For each derived induction variable `j` where `j` = e_1`*i + `e_0 make a fresh temp `j'`
- At the loop pre-header, initialize `j'` to e_0
- After each `i:=i+`*`c`*, define `j':=j'+(`e_1`*`*`c`*`)`
  - note that e_1`*`*`c`* can be computed in the loop header (i.e., it's loop invariant)
- Replace the unique assignment of j in the loop with `j := j'`

Example

```
        s  := 0
        i  := 0
L1:     if i >= n goto L2
        j  := i*4
        k  := j+a
        x  := *k
        s  := s+x
        i  := i+1

L2:     ...
```

- `i` is basic induction variable
- `j` is derived induction variable in family of `i`
  - `j` = `4*i + 0`
- `k` is derived induction variable in family of `j`
  - `k` = `4*i + a`

Example

```
      s  := 0
      i  := 0
      j' := 0
      k' := a
L1:   if i >= n goto L2
      j  := i*4
      k  := j+a
      x  := *k
      s  := s+x
      i  := i+1
L2:   ...
```

- `i` is basic induction variable
- `j` is derived induction variable in family of `i`
  - `j` = `4*i + 0`
- `k` is derived induction variable in family of `j`
  - `k` = `4*i + a`

Example

```
      s  := 0
      i  := 0
      j' := 0
      k' := a
L1:   if i >= n goto L2
      j  := i*4
      k  := j+a
      x  := *k
      s  := s+x
      i  := i+1
      j' := j'+4
      k' := k'+4
L2:   ...
```

- `i` is basic induction variable
- `j` is derived induction variable in family of `i`
  - `j` = `4*i + 0`
- `k` is derived induction variable in family of `j`
  - `k` = `4*i + a`

Example

```
     s  := 0
     i  := 0
     j' := 0
     k' := a

L1:  if i >= n goto L2
     j  := j'
     k  := k'
     x  := *k
     s  := s+x
     i  := i+1
     j' := j'+4
     k' := k'+4

L2:  ...
```

- `i` is basic induction variable
- `j` is derived induction variable in family of `i`
  - `j` = `4*i + 0`
- `k` is derived induction variable in family of `j`
  - `k` = `4*i + a`

Example

```
      s  := 0
      i  := 0
      j' := 0
      k' := a

L1:   if i >= n goto L2
      x  := *k'
      s  := s+x
      i  := i+1
      j' := j'+4
      k' := k'+4

L2:   ...
```

- `i` is basic induction variable
- `j` is derived induction variable in family of `i`
  - `j` = `4*i + 0`
- `k` is derived induction variable in family of `j`
  - `k` = `4*i + a`

Useless Variables

```
      s  := 0
      i  := 0
      j' := 0
      k' := a

L1:   if i >= n goto L2
      x  := *k'
      s  := s+x
      i  := i+1
      j' := j'+4
      k' := k'+4

L2:   ...
```

- A variable is **useless** for *L* if it is dead at all exits from L and its only use is in a definition of itself
  - E.g., `j'` is useless
- Can delete useless variables

Useless Variables

```
      s  := 0
      i  := 0
      j' := 0
      k' := a

L1:   if i >= n goto L2
      x  := *k'
      s  := s+x
      i  := i+1
      k' := k'+4

L2:   ...
```

- A variable is **useless** for *L* if it is dead at all exits from L and its only use is in a definition of itself
  - E.g., `j'` is useless
- Can delete useless variables

Useless Variables

```
       s  := 0
       i  := 0
       k’:= a

L1:    if i >= n goto L2
       x  := *k’
       s  := s+x
       i  := i+1
       k’:= k’+4

L2:    ...
```

- A variable is **useless** for *L* if it is dead at all exits from L and its only use is in a definition of itself
  - E.g., `j’` is useless
- Can delete useless variables

Almost Useless Variables

```
      s  := 0
      i  := 0
      k' := a
L1:   if i >= n goto L2
      x  := *k'
      s  := s+x
      i  := i+1
      k' := k'+4

L2:   ...
```

- A variable is **almost useless** for *L* if it is used only in comparison against loop invariant values and in definitions of itself, and there is some other non-useless induction variable in same family
  - E.g., `i` is useless
- An almost-useless variable may be made useless by modifying comparison
  - See Appel for details

Loop Fusion and Loop Fission

- Fusion: combine two loops into one
- Fission: split one loop into two

Loop Fusion

- Before

```
int acc = 0;
for (int i = 0; i < n; ++i) {
  acc += a[i];
  a[i] = acc;
}
for (int i = 0; i < n; ++i) {
  b[i] += a[i];
}
```

- After

```
int acc = 0;
for (int i = 0; i < n; ++i) {
  acc += a[i];
  a[i] = acc;
  b[i] += acc;
}
```

- What are the potential benefits? Costs?
- Locality of reference

Loop Fission

- Before

```
for (int i = 0; i < n; ++i) {
  a[i] = e1;
  b[i] = e2;  // e1 and e2 independent
}
```

- After

```
for (int i = 0; i < n; ++i) {
  a[i] = e1;
}
for (int i = 0; i < n; ++i) {
  b[i] = e2;
}
```

- What are the potential benefits? Costs?
- Locality of reference

Loop Unrolling

- Make copies of loop body
- Say, each iteration of rewritten loop performs 3 iterations of old loop

Loop Unrolling

- Before

```
for (int i = 0; i < n; ++i) {
  a[i] = b[i] * 7 + c[i] / 13;
}
```

- After

```
for (int i = 0; i < n % 3; ++i) {
    a[i] = b[i] * 7 + c[i] / 13;
}
for (; i < n; i += 3) {
  a[i] = b[i] * 7 + c[i] / 13;
  a[i + 1] = b[i + 1] * 7 + c[i + 1] / 13;
  a[i + 2] = b[i + 2] * 7 + c[i + 2] / 13;
}
```

- What are the potential benefits? Costs?
- Reduce branching penalty, end-of-loop-test costs
- Size of program increased

Loop Unrolling

- If fixed number of iterations, maybe turn loop into sequence of statements!
- Before

```
for (int i = 0; i < 6; ++i) {
  if (i % 2 == 0) foo(i); else bar(i);
}
```

- After

```
foo(0);
bar(1);
foo(2);
bar(3);
foo(4);
bar(5);
```

Loop Interchange

- Change order of loop iteration variables

Loop Interchange

- Before

```
for (int j = 0; j < n; ++j) {
  for (int i = 0; i < n; ++i) {
    a[i][j] += 1;
  }
}
```

- After

```
for (int i = 0; i < n; ++i) {
  for (int j = 0; j < n; ++j) {
    a[i][j] += 1;
  }
}
```

- What are the potential benefits? Costs?
  - Locality of reference

Loop Peeling

- Split first (or last) few iterations from loop and perform them separately

Loop Peeling

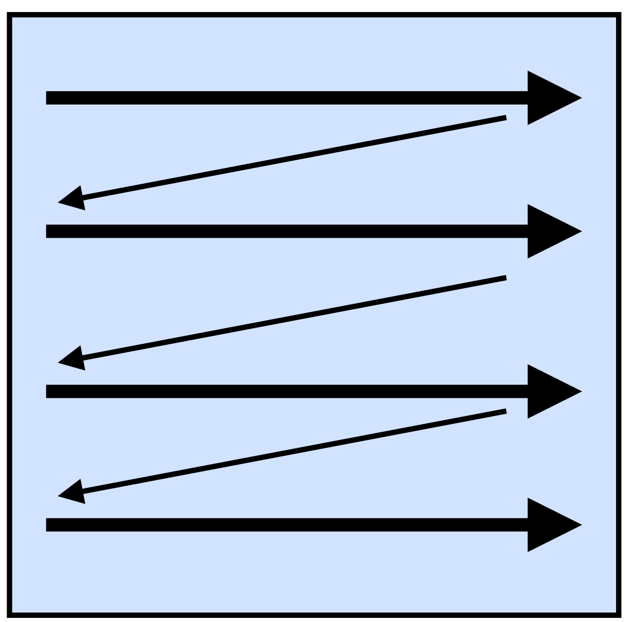
- Before
```
for (int i = 0; i < n; ++i) {
  b[i] = (i == 0) ? a[i] : a[i] + b[i-1];
}
```

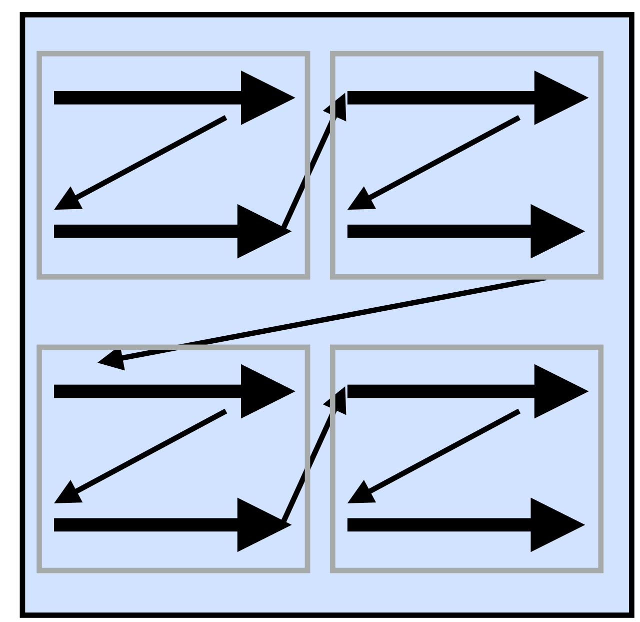
- After
```
b[0] = a[0];
for (int i = 1; i < n; ++i) {
  b[i] = a[i] + b[i-1];
}
```

- What are the potential benefits? Costs?

Loop Tiling

- For nested loops, change iteration order





Loop Tiling

- Before

```
for (i = 0; i < n; i++) {
  c[i] = 0;
  for (j = 0; j < n; j++) {
    c[i] = c[i] + a[i][j] * b[j];
  }
}
```

- After:

```
for (i = 0; i < n; i += 4) {
    c[i] = 0;
    c[i + 1] = 0;
    for (j = 0; j < n; j += 4) {
      for (x = i; x < min(i + 4, n); x++) {
        for (y = j; y < min(j + 4, n); y++) {
          c[x] = c[x] + a[x][y] * b[y];
        }
      }
    }
  }
```

- What are the potential benefits? Costs?

Loop Parallelization

- Before

```
for (int i = 0; i < n; ++i) {
  a[i] = b[i] + c[i]; // a, b, and c do not overlap
}
```

- After

```
for (int i = 0; i < n % 4; ++i) a[i] = b[i] + c[i];
for (; i < n; i = i + 4) {
  __some4SIMDadd(a+i,b+i,c+i);
}
```

- What are the potential benefits? Costs?