John A. Paulson
School of Engineering and Applied Sciences

## CS153: Compilers <br> Lecture 8: Lexing

## Stephen Chong

https://www.seas.harvard.edu/courses/cs153
Contains content from lecture notes by Steve Zdancewic and Greg Morrisett

## Announcements

- Homework 3 (LLVMlite) out
-Due Tuesday Oct 15
- Start early!!!
- Challenging assignment; HW4 will be released Oct 8


## Basic Architecture: Review

Source Code


## Parsing

## Lexical Analysis

## Syntax <br> Analysis

```
if price>500
    then tax=.08
```

| if |
| :--- |
| price |
| $>$ |
| 500 |
| then |
| tax |
| $=$ |
| .08 |



## Today



- Lexical analysis: breaks input sequence of characters into individual words, aka "tokens"


## Lexical Tokens

- A language classifies lexical tokens into token types

| Type | Examples |  |
| :--- | :--- | :--- |
| ID | foo $\quad$ n14 | last |
| NUM | 73 | 0 | 00 

- So, a token type specifies a set of acceptable tokens.
- Reserved words are tokens that cannot be used as identifiers
-E.g., IF, VOID, RETURN


## Example 1

- Given a program

$$
\begin{aligned}
& \text { if (price>500) } \\
& \text { then tax=.08 }
\end{aligned}
$$

the lexical analysis returns the sequence of tokens

IF LPAREN ID(price) GT NUM(500) RPAREN THEN ID(tax) EQ REAL(0.08)

## Example 2

- Given a program

$$
\begin{aligned}
& \text { if }(\text { price }>500) \\
& \text { then } t a x=1 x a b
\end{aligned}
$$

the lexical analysis returns

## ERROR

because $1 \times \mathrm{xab}$ is neither a number nor an identifier.

| Type | Examples |  |  |
| :--- | :--- | :--- | :--- |
| ID | foo n14 | last |  |
| NUM | 73 | 0 | 00 |
| 515 | 082 |  |  |
| THEN | then |  |  |

- The lexical analysis can help in reporting where an error occurs in the code.
- By recognizing ' $\backslash \mathrm{n}$ ' as a token and incrementing the line number.


## Example 3

- Given a program

$$
\text { if (price>500) } \begin{aligned}
\text { thn tax }=.08
\end{aligned}
$$

the lexical analysis returns
IF LPAREN ID(price) GT NUM(500) RPAREN ID(thn) ID(tax) EQ REAL(0.08)

| Type | Examples |  |  |
| :--- | :--- | :--- | :--- |
| ID | foo n14 | last |  |
| NUM | 73 | 0 | 00 |
| 515 | 082 |  |  |
| THEN | then |  |  |

- Is this an error at the level of lexical analysis?
- No, it is an error at the level of syntax analysis (next lectures)!


## Towards Implementing A Lexical Analysis

- Recall: Lexical analysis breaks input into tokens.
- The lexical analysis needs to decide the token type for a given string (i.e., sequence of characters).



## Let's simplify...

- Recall: Lexical analysis breaks input into tokens.
-The lexical analysis needs to decide the token type for a given string (i.e., sequence of characters).



## A Set Membership Question

- Recall: a token type specifies a set of acceptable tokens (i.e., strings).
- The set of acceptable tokes for NUM is $\{0,1,2,3, \ldots\}$.
- But this set is infinite...



## A Set Membership Question

- How can we mechanically decide if a string belongs to a (possibly infinite) set $S$ of strings?
- An approach:
- Use a finite representation of S.
- Regular expressions
- Check whether the string is accepted by such a finite representation.
- Deterministic finite-state automata


## Regular Expressions

- Each regular expression represents a set of strings.
- Examples
-( $0 \mid 1$ )* 0
- Binary numbers that are multiples of 2
- $b^{*}\left(a b b^{*}\right)^{*}(a \mid \varepsilon)$
- Strings of a's and b's without consecutive a's
-(a|b)*aa(a|b)*
-Strings of a's and b's with consecutive a's


## Regular Expressions (RE)

- Grammar
- $\varnothing$ (matches no string)
- $\varepsilon$ (epsilon - matches empty string)
- Literals ('a', 'b', '2', ' ${ }^{\prime}$ ', etc.) drawn from alphabet
- Concatenation ( $\mathrm{R}_{1} \mathrm{R}_{2}$ )
- Alternation ( $\mathrm{R}_{1} \mid \mathrm{R}_{2}$ )
- Kleene star ( $\mathrm{R}^{*}$ )


## Set of Strings

- $\llbracket \varnothing \rrbracket=\{$ \}
- $\llbracket \varepsilon \rrbracket=\left\{{ }^{\prime \prime \prime}\right\}$
- $\llbracket{ }^{\prime} a^{\prime} \rrbracket=\left\{{ }^{\prime} a^{\prime \prime}\right\}$
- $\llbracket R_{1} R_{2} \rrbracket=\left\{s \mid s=\alpha \wedge \beta\right.$ and $\alpha \in \llbracket R_{1} \rrbracket$ and $\left.\beta \in \llbracket R_{2} \rrbracket\right\}$
$\bullet \llbracket R_{1} \mid R_{2} \rrbracket=\left\{s \mid s \in \llbracket R_{1} \rrbracket\right.$ or $\left.s \in \llbracket R_{2} \rrbracket\right\}$
$=\llbracket R_{1} \rrbracket \cup \llbracket R_{2} \rrbracket$
$\bullet \llbracket R^{*} \rrbracket=\llbracket \varepsilon \mid R R^{*} \rrbracket$

$$
\begin{gathered}
=\left\{s \mid s={ }^{\prime \prime \prime} \text { or } s=\alpha \wedge \beta \text { and } \alpha \in \llbracket R \rrbracket\right. \\
\text { and } \left.\beta \in \llbracket R^{*} \rrbracket\right\}
\end{gathered}
$$

## Syntactic Sugar

- [0-9] shorthand for $0|1| \ldots \mid 9$
$\bullet R$ ? shorthand for $(R \mid \varepsilon)$ (i.e., $R$ is optional)
- $R+$ shorthand for $\left(R R^{*}\right)$ (i.e., at least one $R$ )


## Regular Expressions to Specify Token Types!

| Reg Exp | Token Type |
| :--- | :---: |
| if | IF |
| $[a-z][a-z 0-9] *$ | ID |
| $[0-9]+$ | NUM |
| $([0-9]+" . "[0-9] *)$ <br> $([0-9] * " . "[0-9]+)$ | REAL |

-Question: What is the token type of input iffy?
-We want the token ID(iffy) rather than IF.

- In general, we want the longest match:
- longest initial substring of the input that can match a regular expression is taken as next token


## Recall: A Set Membership Question

- Lexical analysis breaks input into tokens.
- The lexical analysis needs to decide the token type for a given string (i.e., sequence of characters).



## A Matching Question

- Lexical analysis breaks input into tokens.
-The lexical analysis needs to decide the token type for a given string (i.e., sequence of characters).



## From RE to DFA

- A Deterministic Finite-state Automaton (DFA) can be used to decide whether an input matches a regular expression.
- Example: DFA for regular expression [0-9]+ :



## Other DFAs



## Combined Finite Automaton



- This DFA takes as an input a sequence of characters and returns a Token Type (if the input is accepted).
- So, this DFA can be used for Lexical Analysis.


## Using DFAs

- Usually record transition function as array indexed by state and characters (i.e., transition table)
- See Appel Chap 2.3 for an example.


## How is a RE converted to a DFA?

1. Convert RE to a Nondeterministic Finite-state Automaton (NFA).
2. Convert NFA to DFA.

## RE to NFA conversion

- Epsilon $\varepsilon$

- Literal 'a'

- Concatenation $\mathrm{R}_{1} \mathrm{R}_{2}$

- Alternation $\mathrm{R}_{1} \mid \mathrm{R}_{2}$



## RE to NFA conversion

- Kleene star R*



## NFA to DFA conversion (intuition)

-The NFA of a regular expression R can be easily composed from NFAs of subexpressions of R.

- But executing an NFA under input strings is harder and less efficient than executing a DFA due to the nondeterminism.
- So, we convert NFAs to DFAs.
- Basic idea: each state in DFA will represent a set of states of the NFA.


## Example: NFA to DFA

NFA:


DFA:


## Example: NFA to DFA

NFA:


DFA:


## Example: NFA to DFA

NFA:


DFA:


Check that this DFA is, in fact, deterministic!

## Lexical Analysis Summary

- Use a regular expression $R_{i}$ to specify the set strings for each Token Type.
-Example: [0-9]+ specifies the set of strings for NUM
- Construct the NFA formed by $\left(R_{1}\left|R_{2}\right| \ldots \mid R_{n}\right)$.
- Construct the DFA for this NFA.
- Produce the transition table for that DFA.
- Implement longest match.


## Using a Lexer Generator

- The designer of a lexical analysis follows the first step of the previous slide.
-The remaining steps are automatically performed by the lexer generator!


## A Lexer Generator in ML

- Provide regular expressions for token types in file mllexeg.mll
-Run lexer generator: ocamllex mllexeg.mll
- The lever generator produces the final transition table at file mllexeg.ml


## Structure of ocamllex File

```
{ header }
let ident = regexp ...
rule entrypoint1 [arg1 ... argn] =
    parse regexp { action }
        regexp { action }
and entrypoint2 [arg1 ... argn] =
    parse ...
and ...
{ trailer }
```

- Header and trailer are arbitrary OCaml code, copied to the output file
- Can define abbreviations for common regular expressions
- Rules are turned into (mutually recursive) functions with args1 ... argn lexbuf
- lexbuf is of type Lexing.lexbuf
- Result of function is the result of ml code action


## A hand-written Lexer

- See file lexer.ml

