HARVARD
John A. Paulson
School of Engineering
and Applied Sciences

CS153: Compilers
Lecture 9: Recursive Parsing

Stephen Chong

https://www.seas.harvard.edu/courses/cs153

Contains content from lecture notes by Greg Morrisett

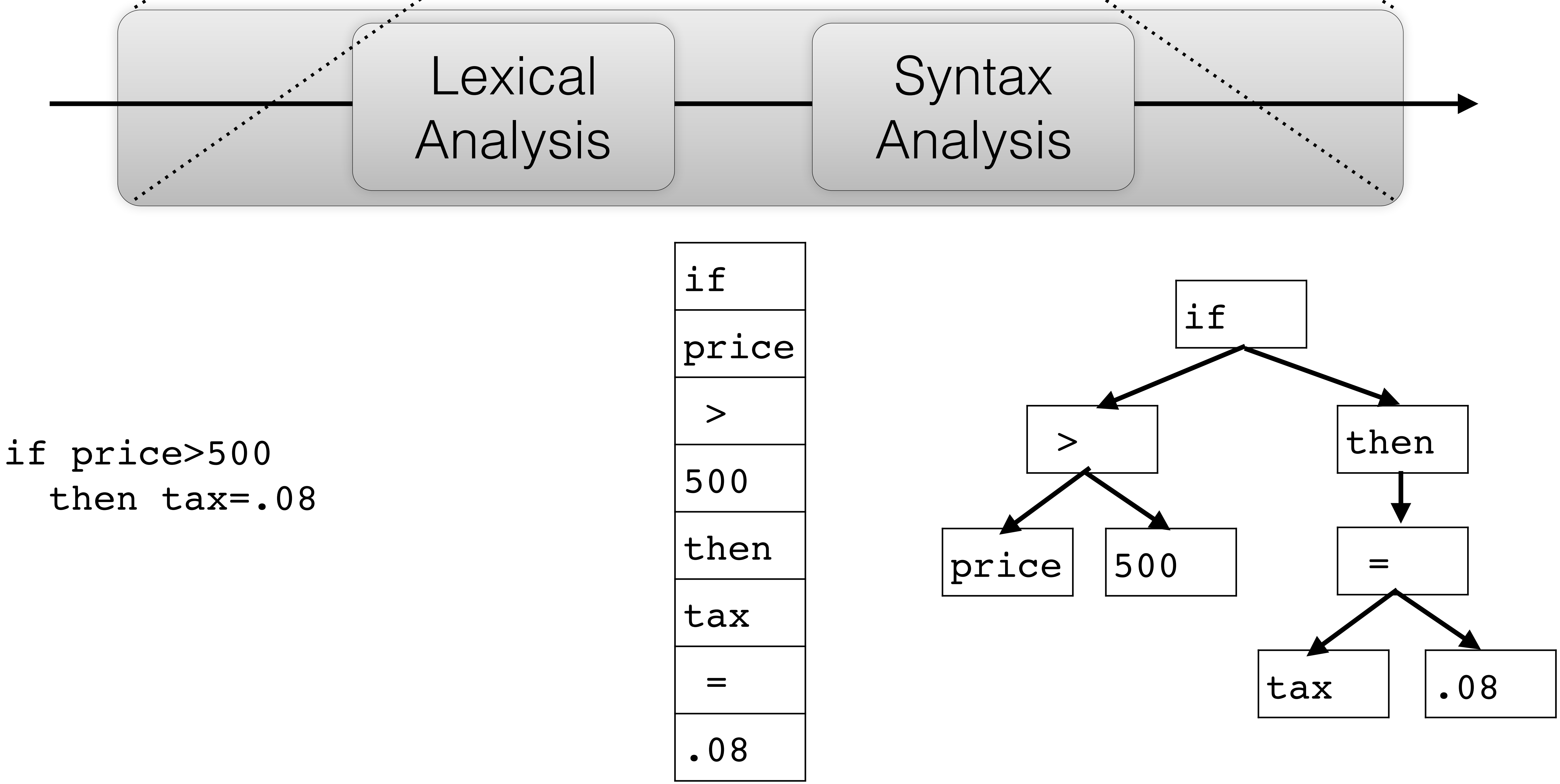
Announcements

- HW3 LLVMlite out
  - Due Oct 15
- New TF! Zach Yedidia
  - Some more office hours will be added soon

Today

- Parsing
  - Context-free grammars
  - Derivations
  - Parse trees
  - Ambiguous grammars
  - Recursive descent parsing
  - Parser combinators

Parsing



Lexical Analysis
Syntax Analysis
if price>500
then tax=.08
if
price
>
500
then
tax
=
.08
if
>
then
price
500
=
tax
.08

Parsing

- Two pieces conceptually:
  - Recognizing syntactically valid phrases.
  - Extracting semantic content from the syntax.
    - E.g., What is the subject of the sentence?
    - E.g., What is the verb phrase?
    - E.g., Is the syntax ambiguous? If so, which meaning do we take?
      - "Time flies like an arrow", "Fruit flies like a banana"
      - "2 * 3 + 4"
      - "x ^ f y"
- In practice, solve both problems at the same time.

Specifying the Language

- A language is a set of strings. We need to specify what this set is.
- Can we use regular expressions?
- In MLLex, we named regular expressions e.g.,
  - `digits = [0-9]+`
  - `sum = (digits “+”)* digits`
  - Defines sums of the form `4893 + 48 + 92`
- But what if we wanted to add parentheses to the language?
  - `digits = [0-9]+`
  - `sum = expr “+” expr`
  - `expr = digits | “(“ sum “)”`

Specifying the Language

- It's impossible for finite automaton to recognize language with balanced parentheses!
- MLLex just treats digits as an abbreviation of the regex [0-9]+
    - This doesn't add expressive power to the language
- Doesn't work for example above: try expanding the definition of `sum` in `expr`:
    - `expr = digits | "(" sum ")"`
    - `expr = digits | "(" expr "+" expr ")"`
    - But `expr` is an abbreviation, so we expand it and get
    - ```
      expr = digits |
          "(" (digits | "(" expr "+" expr ")")
                "+" (digits | "(" expr "+" expr ")") ")"
      ```
    - Uh oh...
      ```

Context-Free Grammars

- Additional expressive power of recursion is exactly what we need!
- Context Free Grammars (CFGs) are regular expressions with recursion
- CFGs provide declarative specification of syntactic structure
- CFG has set of **productions** of the form

  symbol → *symbol symbol* … *symbol*

  with zero or more symbols on the right
- Each symbol is either **terminal** (i.e., token from the alphabet) or **non-terminal** (i.e., appears on the LHS of some production)
  - No terminal symbols appear on the LHS of productions

CFG example

$S \to S; S$
$S \to$ `id :=` E
$S \to$ `print (` L `)`

$E \to$ `id`
$E \to$ `num`
$E \to E + E$
$E \to (S, E)$

$L \to E$
$L \to L, E$

- Terminals are: `id print num , + ( ) := ;`
- Non-terminals are: S, E, L
  - S is the start symbol
- E.g., one sentence in the language is

  `id := num; id := (id := num+num, id+id)`

  - Source text (before lexical analysis) might have been

    `a := 7; b := (c := 30+5, a+c)`

Derivations

- To show that a sentence is in the language of a grammar, we can perform a **derivation**
  - Start with start symbol, repeatedly replace a non-terminal by its right hand side
- E.g.,
  - S
  - S ; S
  - `id := ` E ; S
  - `id := ` E ; `id := ` E
  - `id := num; id := ` E
  - …
  - `id := num; id := (id := num+num, id+id)`

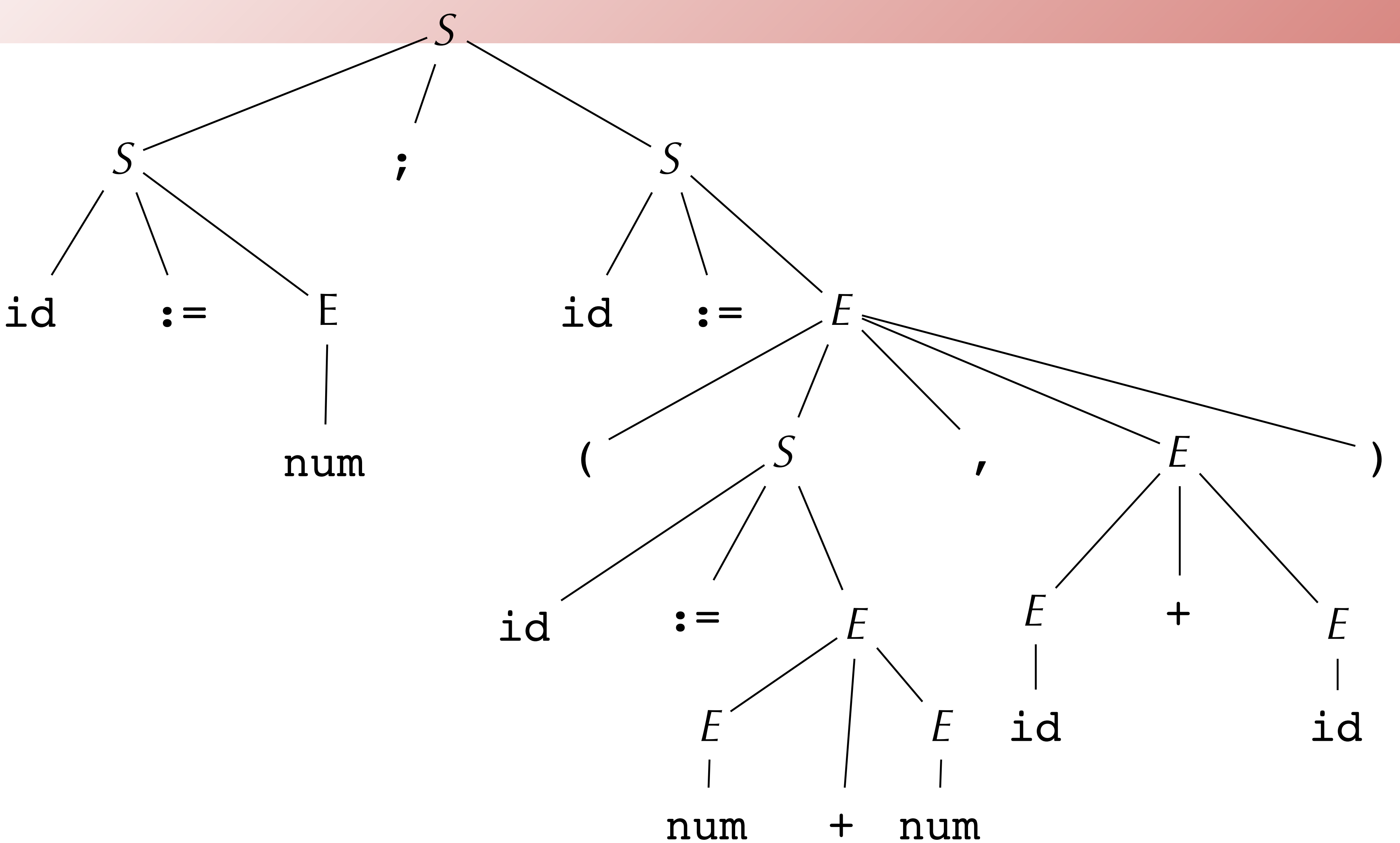
$S \rightarrow S$ `;` S
$S \rightarrow$ `id :=` E
$S \rightarrow$ `print (` L `)`
$E \rightarrow$ `id`
$E \rightarrow$ `num`
$E \rightarrow E + E$
$E \rightarrow$ `(` S`,` E `)`
$L \rightarrow E$
$L \rightarrow L$`,` E

CFGs and Regular Expressions

- CFGs are strictly more expressive than regular expressions

How can you translate a regular expression into a CFG?

Parse Tree

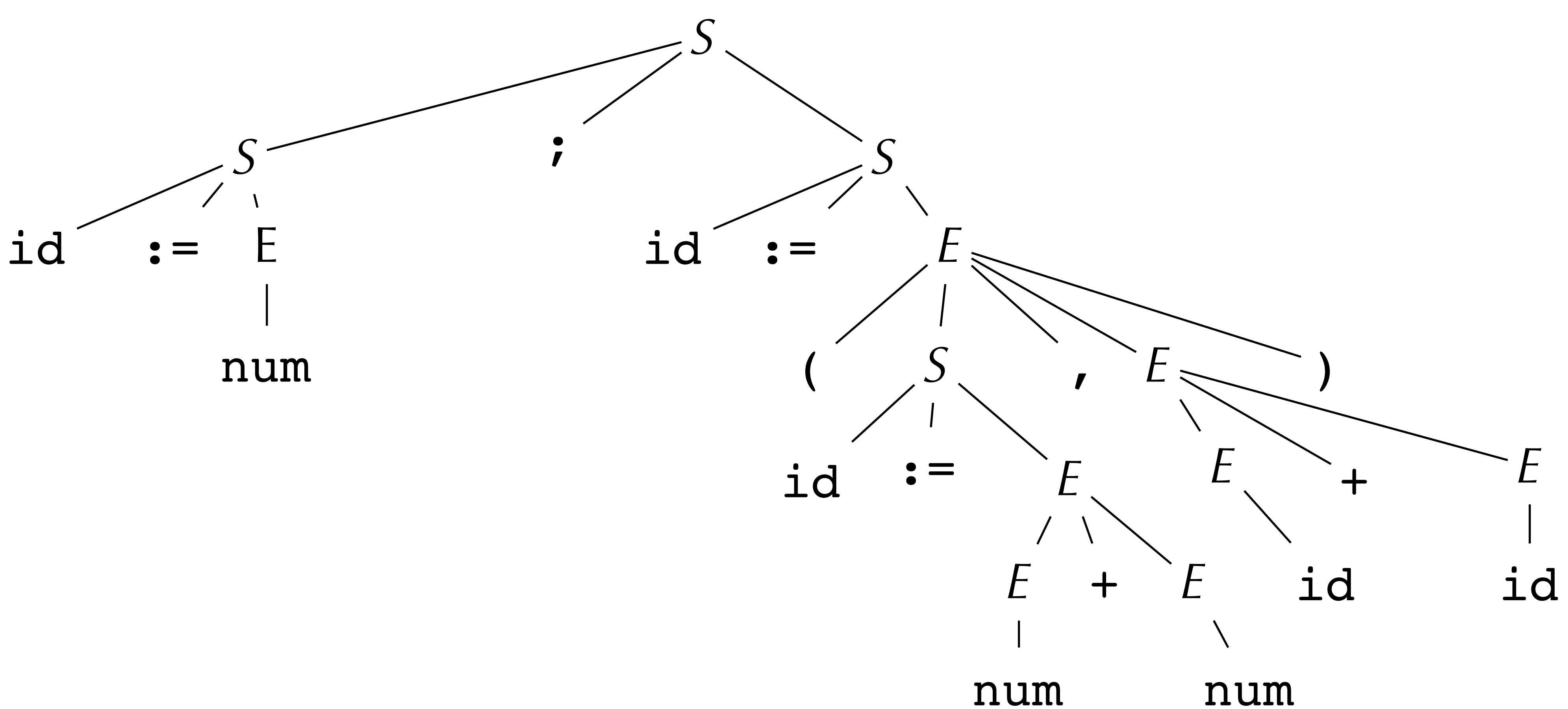


$S \rightarrow S$ **;** S
$S \rightarrow$ `id` `:=` E
$S \rightarrow$ `print` `(` L `)`
$E \rightarrow$ `id`
$E \rightarrow$ `num`
$E \rightarrow E + E$
$E \rightarrow$ `(`S`,` E`)`
$L \rightarrow E$
$L \rightarrow L$`,` E

- A **parse tree** connects each symbol to the symbol it was derived from
- A derivation is, in essence, a way of constructing a parse tree.
  - Two different derivations may have the same parse tree

How to Build a Parse Tree/ Find a Derivation

- Conceptually, two possible ways:
  - Start from start symbol, choose a non-terminal and expand until you reach the sentence
  - Start from the terminals and replace phrases with non-terminals



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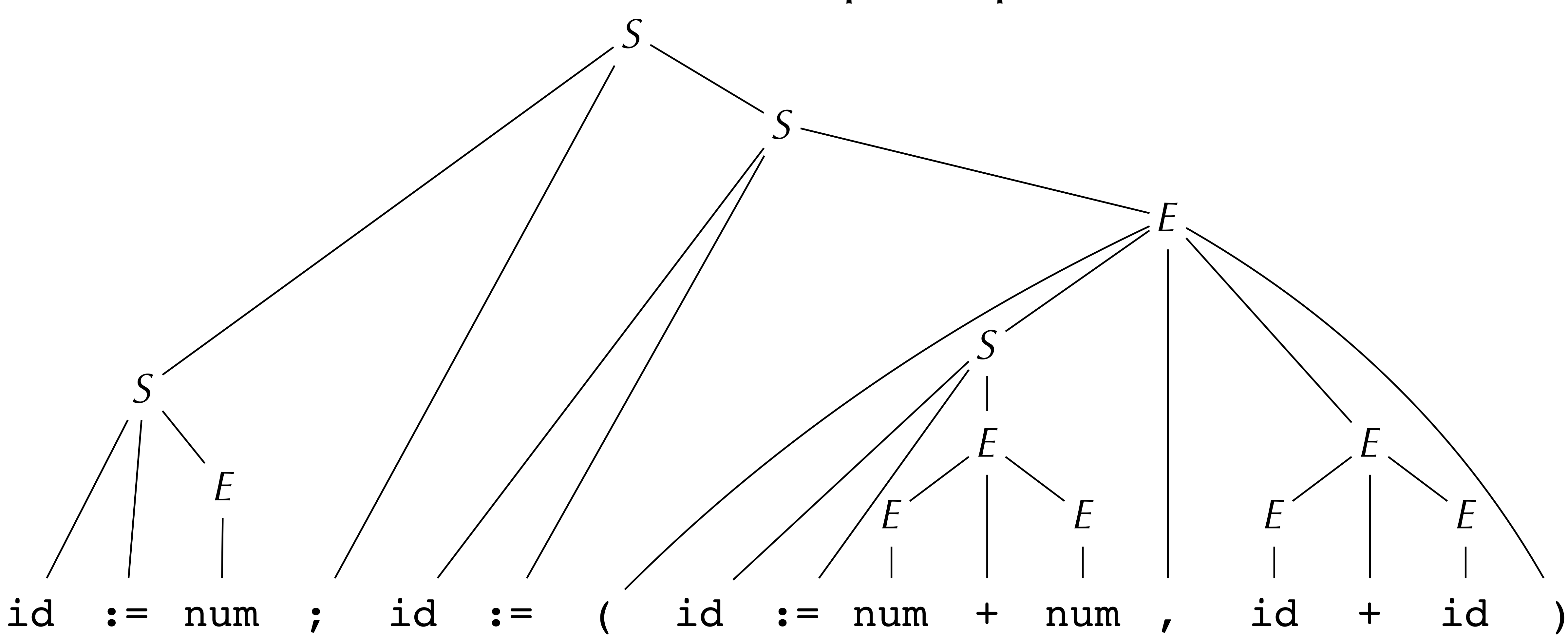
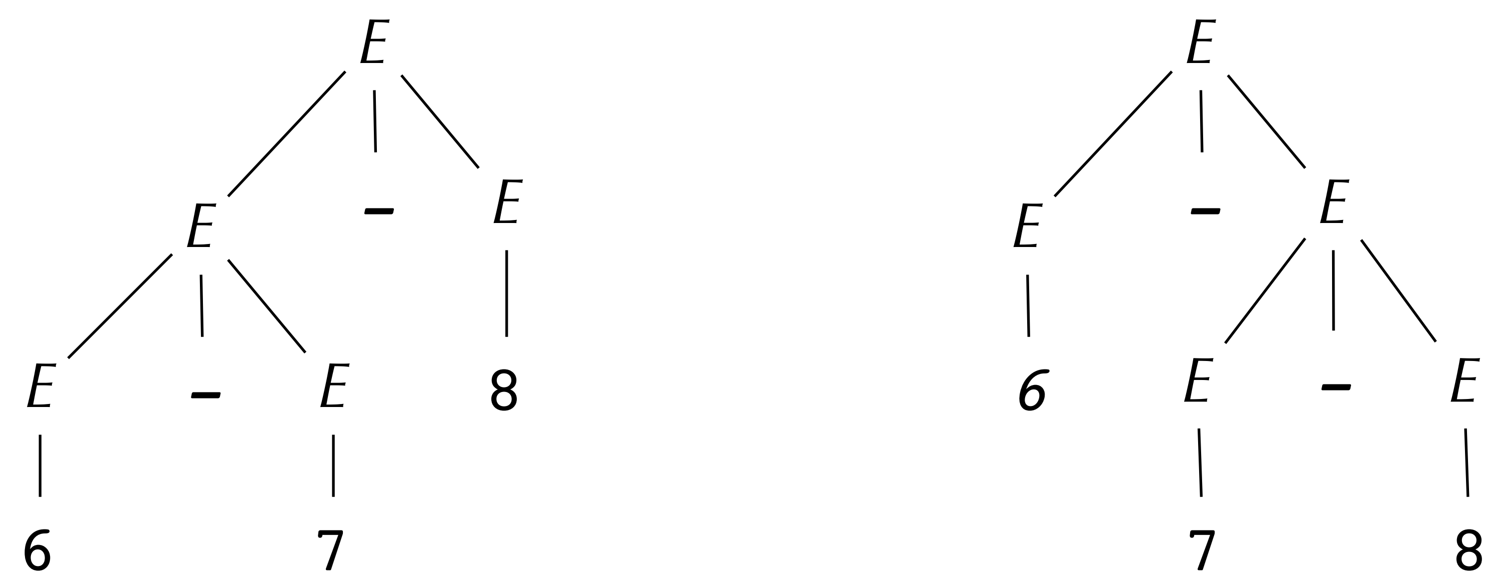


Ambiguous Grammar

- A grammar is **ambiguous** if it can derive a sentence with two different parse trees
- E.g.,

$$
\begin{aligned}
E &\rightarrow \texttt{id} \\
E &\rightarrow \texttt{num} \\
E &\rightarrow E \,\texttt{*}\, E \\
E &\rightarrow E \,\texttt{/}\, E \\
E &\rightarrow E \,\texttt{+}\, E \\
E &\rightarrow E \,\texttt{-}\, E \\
E &\rightarrow \texttt{(}\, E \,\texttt{)}
\end{aligned}
$$



- Ambiguity is usual bad: different parse trees often have different meaning!
- But we can usually eliminate ambiguity by transforming the grammar

Fixing Ambiguity Example

- We would like `*` to **bind higher** than `+` (aka, `*` to have **higher precedence** than +)
  - So `1+2*3` means 1+(2*3) instead of (1+2)*3
- We would like each operator to **associate to the left**
  - So `6-7-8` means (6-7)-8 instead of 6-(7-8)
- Symbol *E* for expression, *T* for term, *F* for factor

$E \rightarrow E + T$	$T \rightarrow T * F$	$F \rightarrow$ `id`
$E \rightarrow E - T$	$T \rightarrow T \,/\, F$	$F \rightarrow$ `num`
$E \rightarrow T$	$T \rightarrow F$	$F \rightarrow$ `(`E`)`

How to Parse

Top down

- Manual, (recursive descent)
  - Easy to write
  - Good error messages
  - Tedious, hard to maintain
- Parsing Combinators
  - Encode grammars as higher-order functions
  - Essentially, functions generate a recursive descent parser
- Antlr http://www.antlr.org/

- Yacc

Bottom up

- ...

Recursive Descent

- See file `recdesc-a.ml`
- Try the following:
  - `exp_parse “32”`
  - `exp_parse “let foo = 7 in 42”`
  - `exp_parse “let foo = 7 let bar”`

Recursive Descent

- See file `recdesc-b.ml`
- More direct implementation of grammar

$E \rightarrow E + T$	$T \rightarrow T * F$	$F \rightarrow$ `id`
$E \rightarrow E - T$	$T \rightarrow T / F$	$F \rightarrow$ `num`
$E \rightarrow T$	$T \rightarrow F$	$F \rightarrow$ `(`E`)`

- Each non-terminal is a function

Left Recursion

- Recursive descent parsing doesn't handle left recursion well!
- We can refactor grammar to avoid left recursion
- E.g., transform left recursive grammar

$$
\begin{array}{lll}
E \rightarrow E + T & T \rightarrow T * F & F \rightarrow \texttt{id} \\
E \rightarrow E - T & T \rightarrow T \, / \, F & F \rightarrow \texttt{num} \\
E \rightarrow T & T \rightarrow F & F \rightarrow \texttt{(} E \texttt{)}
\end{array}
$$

to

$$
\begin{array}{lll}
E \rightarrow T \, E' & T \rightarrow F \, T' & F \rightarrow \texttt{id} \\
E' \rightarrow + \, T \, E' & T' \rightarrow * \, F \, T' & F \rightarrow \texttt{num} \\
E' \rightarrow - \, T \, E' & T' \rightarrow / \, F \, T' & F \rightarrow \texttt{(} E \texttt{)} \\
E' \rightarrow & T' \rightarrow &
\end{array}
$$

Left Recursion

- See file `recdesc-c.ml`
- Try the following:
  - `exp_parse "6 - 7 - 8";;` *Observe the left associativity*

Parser Combinators

- Parser combinators are an elegant functional-programming technique for parsing
  - Higher-order functions that accept parsers as input and returns a new parser as output
- That's what our code already is!