HARVARD
School of Engineering
and Applied Sciences

Static Single Assignment Form
(and dominators, post-dominators, dominance frontiers…)

CS252r Spring 2011

(Almost all slides shamelessly stolen from Jeff Foster)

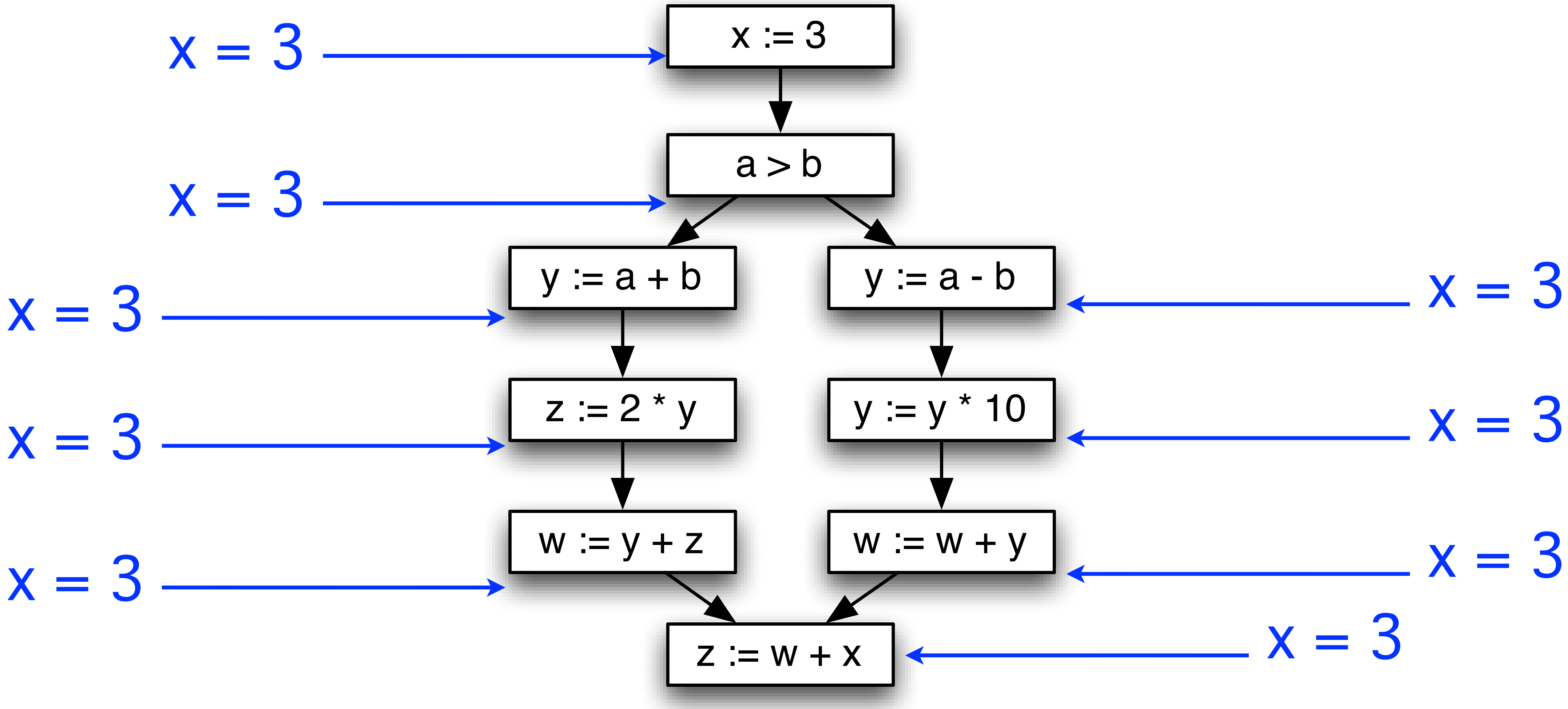
Motivation

- Data flow analysis needs to represent facts at every program point

- What if
  - There are a lot of facts and
  - There are a lot of program points?
  - ⇒ potentially takes a lot of space/time

- Most likely, we're keeping track of irrelevant facts

Example

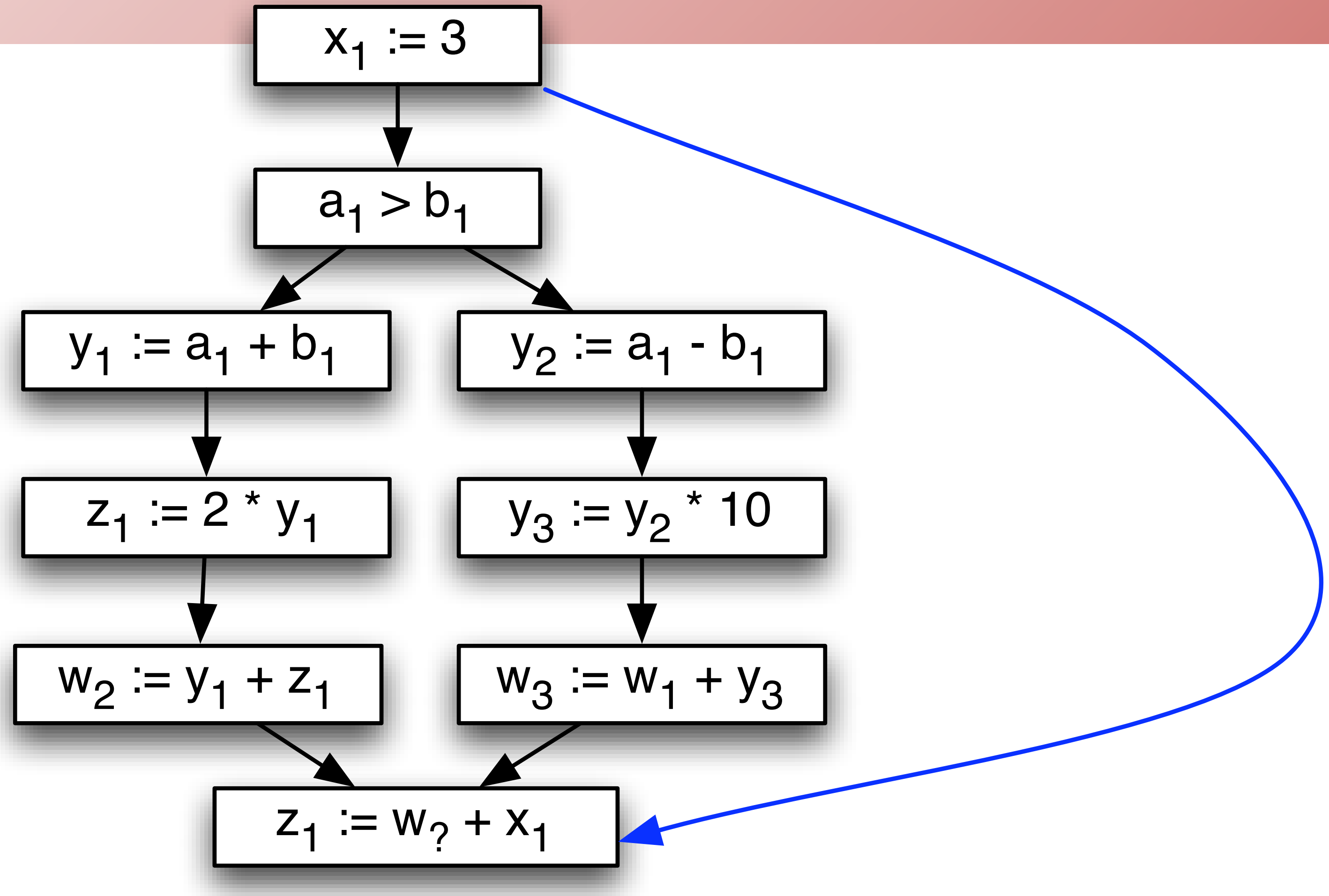


Sparse Representation

- Instead, we'd like to use a sparse representation
  - Only propagate facts about x where they're needed

- Enter **static single assignment** form
  - Each variable is defined (assigned to) exactly once
  - But may be used multiple times

Example: SSA


$x_1 := 3$

$a_1 > b_1$

$y_1 := a_1 + b_1$

$y_2 := a_1 - b_1$

$z_1 := 2 * y_1$

$y_3 := y_2 * 10$

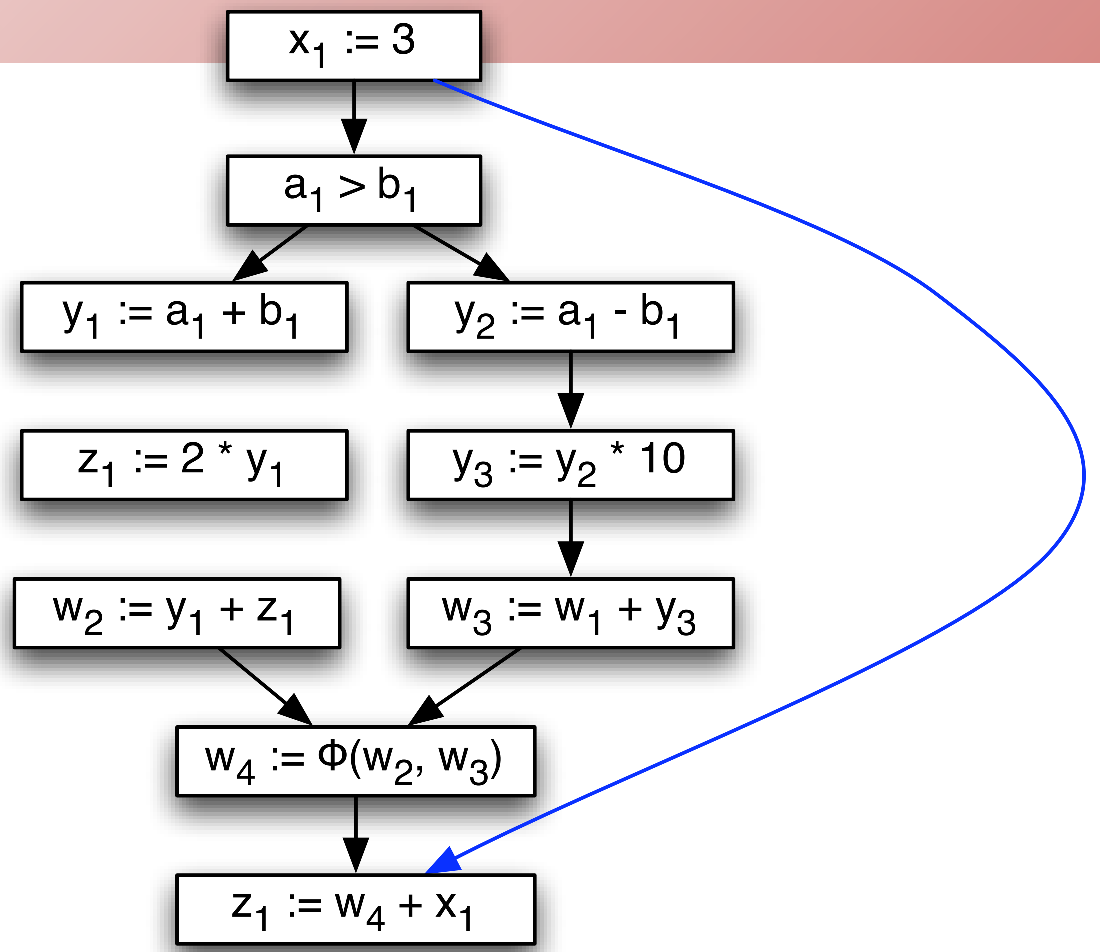
$w_2 := y_1 + z_1$

$w_3 := w_1 + y_3$

$z_1 := w_? + x_1$

- Add **SSA edges** from definitions to uses
  - No intervening statements define variable
  - Safe to propagate facts about x only along SSA edges

What About Joins?



- Add Φ functions/nodes to model joins
  - One argument for each incoming branch
  - Operationally: selects one of the arguments based on how control flow reach this node
  - Dataflow analysis: Intuitively, takes meet of arguments
  - At code generation time, need to eliminate Φ nodes

Constant Propagation Revisited

- Initialize facts at each program point
  - C(n) := $\top$
- Add all SSA edges to the worklist
- While the worklist isn't empty,
  - Remove an edge (x, y) from the worklist
  - C(y) := C(y) $\sqcap$ C(x)
  - Add to worklist SSA edges from y if C(y) changed

Def-Use Chains vs. SSA

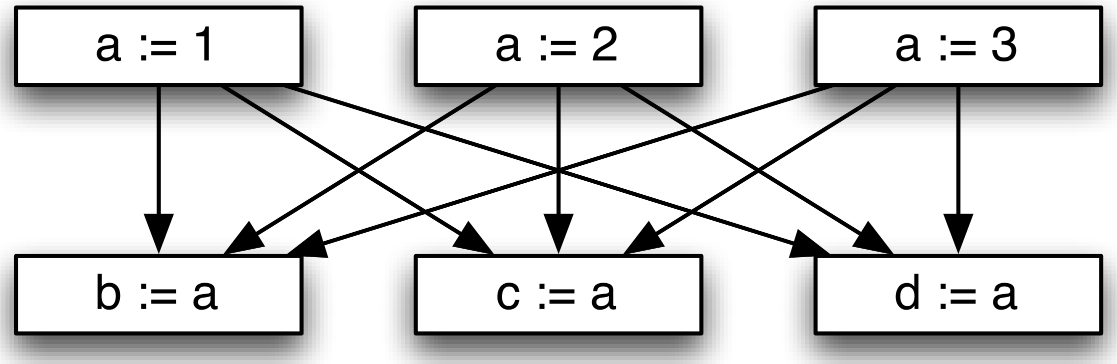
- Alternative: Don't do renaming; instead, compute simple def-use chains (reaching definitions)
  - Propagate facts along def-use chains

- Drawback: Potentially quadratic size

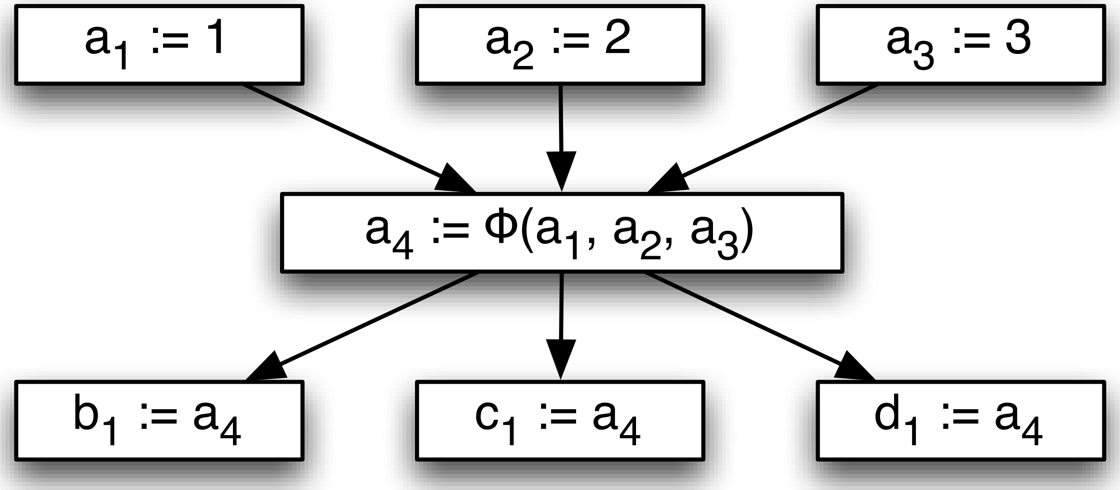
Def-Use Chains vs. SSA (cont'd)

```
case (...) of
  0:  a := 1;
  1:  a := 2;
  2:  a := 3;
end
case (...) of
  0:  b := a;
  1:  c := a;
  2:  d := a;
end
```

Def-Use Chains



SSA Form



Quadratic vs. (in practice) linear behavior

Conditional Constant Propagation

- So far, we assume that all branches can be taken
  - But what if some branches are never taken in practice?
    - Debugging code that can be enabled/disabled at run time
    - Macro expanded code with constants
    - Optimizations

- Idea: use constant propagation to decide which branches might be taken
  - Fits in neatly with SSA form

Nodes versus Edges

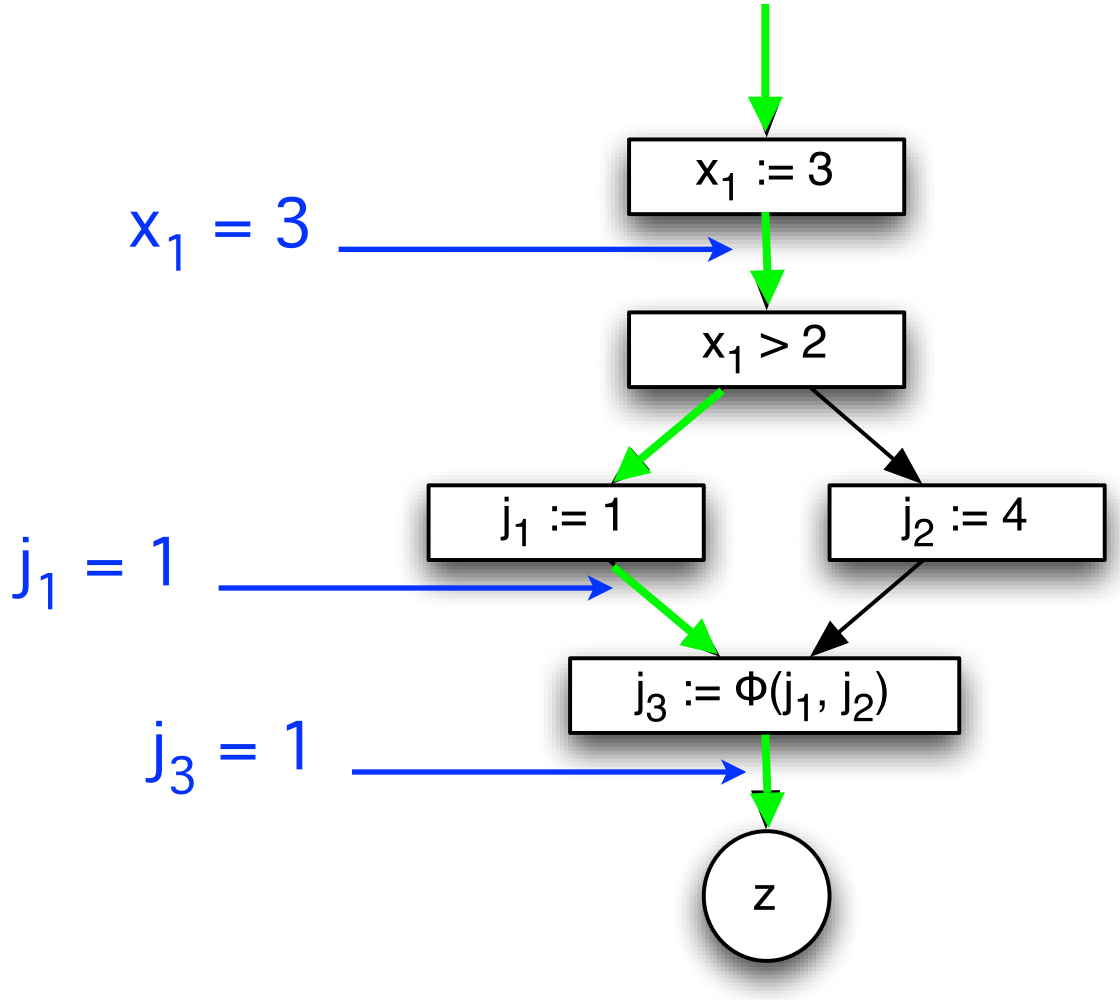
- So far, we've been hazy about whether data flow facts are associated with nodes or edges
  - Advantage of nodes: may be fewer of them
  - Advantage of edges: can trace differences on multiple paths to same node

- For this problem, we'll associate facts with edges

Conditional Execution

- Keep track of whether edges may be executed
  - Some may not be because they're on not-taken branch
  - Initially, assume no edges taken
  - At joins, don't propagate information from not-taken in-edges
- Side comment: Notice that we always, always start with the optimistic assumption
  - We need proof that a pessimistic fact holds
  - We're computing a **greatest fixpoint**

Example



Computing SSA Form

- Step 1: Place Φ nodes
  - Naive, impractical step 1: put a Φ function for every variable at the beginning of every block
- Step 2: Rename variables so only one definition per name

Computing SSA Form

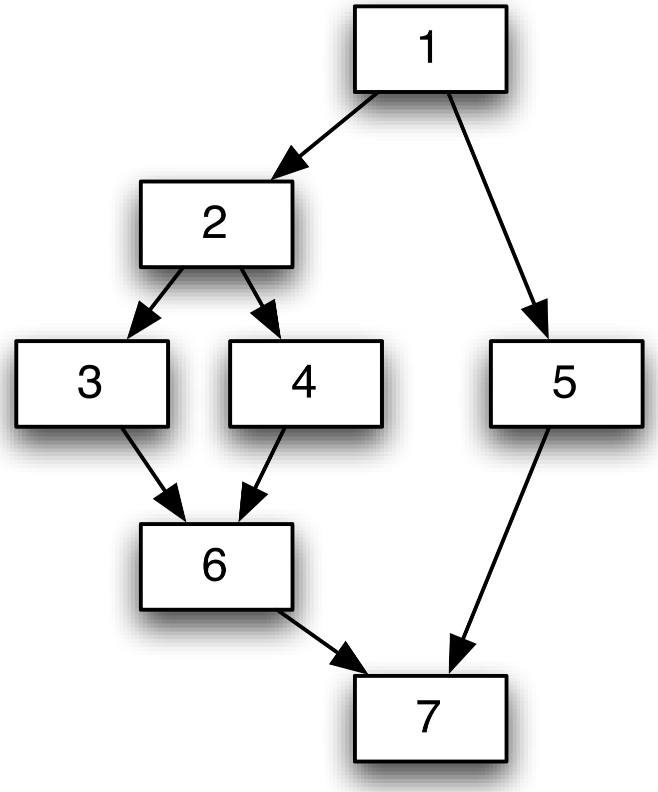
- Step 1a: Compute the dominance frontier
- Step 1b: Use dominance frontier to place Φ nodes
  - If node X contains assignment to a, put Φ function for a in dominance frontier of X
    - Adding Φ fn may require introducing additional Φ fn
- Step 2: Rename variables so only one definition per name

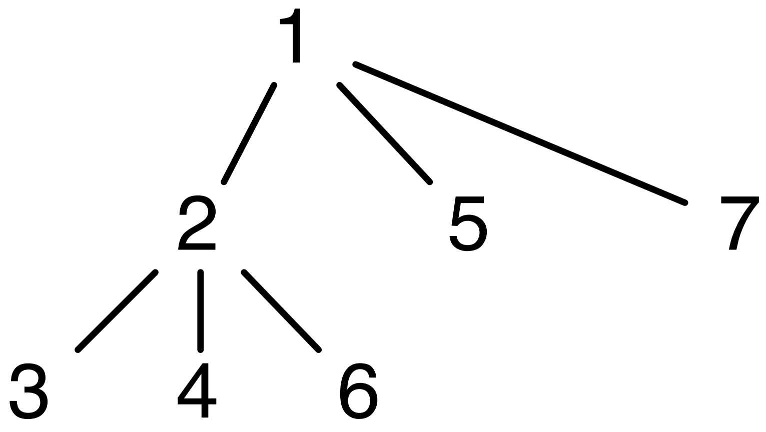
Dominators

- Let X and Y be nodes in the CFG
  - Assume single entry point Entry
- X **dominates** Y (written X≥Y) if
  - X appears on every path from Entry to Y
- Write X>Y (X **strictly dominates** Y) when X dominates Y but X≠Y
  - Note ≥ is reflexive

Dominator Tree

- The dominator relationship forms a tree
  - Edge from parent to child = parent dominates child
  - Note: edges are not same as CFG edges!



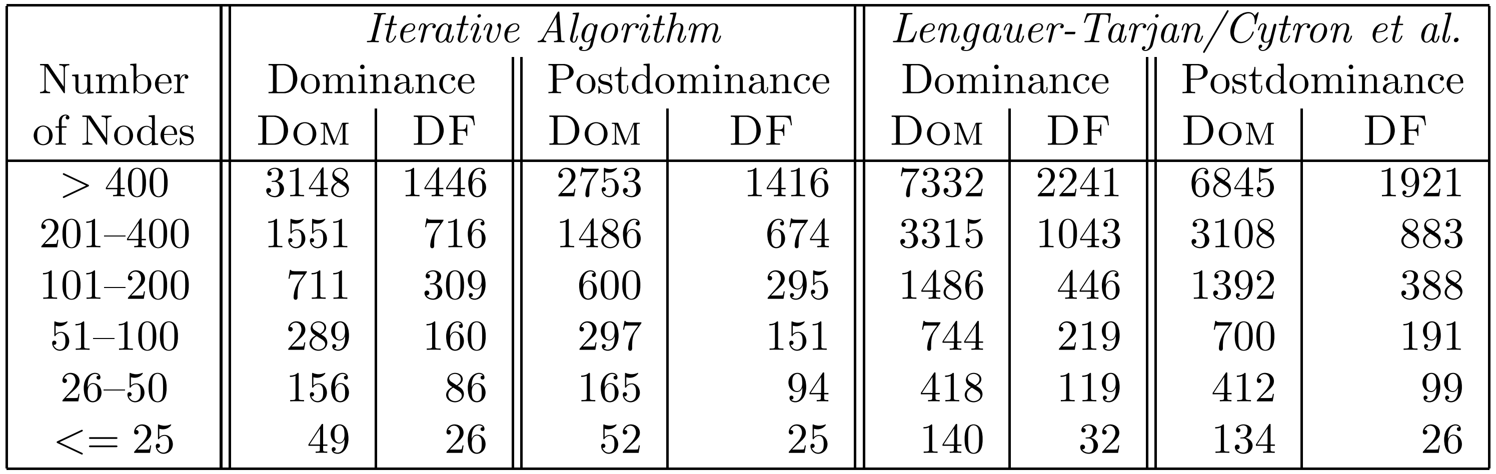


Computing Dominator Tree

- An algorithm due to Lengauer and Tarjan
  - Runs in time $O(E\alpha(E, N))$
    - E = # of edges, N = # of nodes
    - where $\alpha(\cdot)$ is the inverse Ackerman's function
    - Very slow growing; effectively constant in practice
  - Algorithm quite difficult to understand
    - But lots of pseudo-code available

Computing Dominator Tree

- “A Simple, Fast Dominance Algorithm” by Cooper, Harvey, Kennedy, 2001
  - Shows O(N^2) algorithm runs faster in practice than Lengauer and Tarjan
  - Intuitive algorithm, phrased as dataflow equations, solved with standard (reverse-postorder) iterative dataflow
  - Requires carefully engineered data structures



	Iterative Algorithm				*Lengauer-Tarjan/Cytron et al.*			
	Dominance		Postdominance		Dominance		Postdominance	
Number of Nodes	DOM	DF	DOM	DF	DOM	DF	DOM	DF
> 400	3148	1446	2753	1416	7332	2241	6845	1921
201–400	1551	716	1486	674	3315	1043	3108	883
101–200	711	309	600	295	1486	446	1392	388
51–100	289	160	297	151	744	219	700	191
26–50	156	86	165	94	418	119	412	99
<= 25	49	26	52	25	140	32	134	26

Average times by graph size, measured in $\frac{1}{100}$*'s of a second*

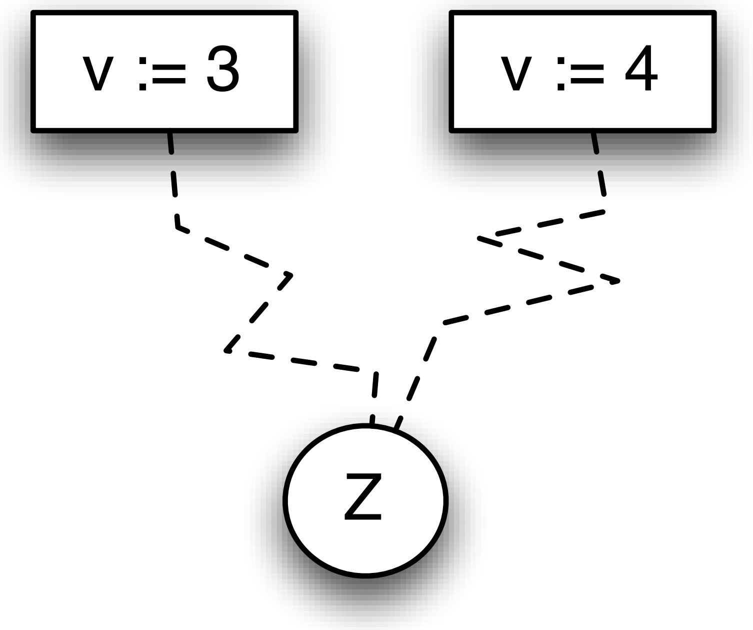
Table 1: Runtimes for 10,000 Runs of Our Fortran Test Suite, aggregated by Graph Size

Why Are Dominators Useful?

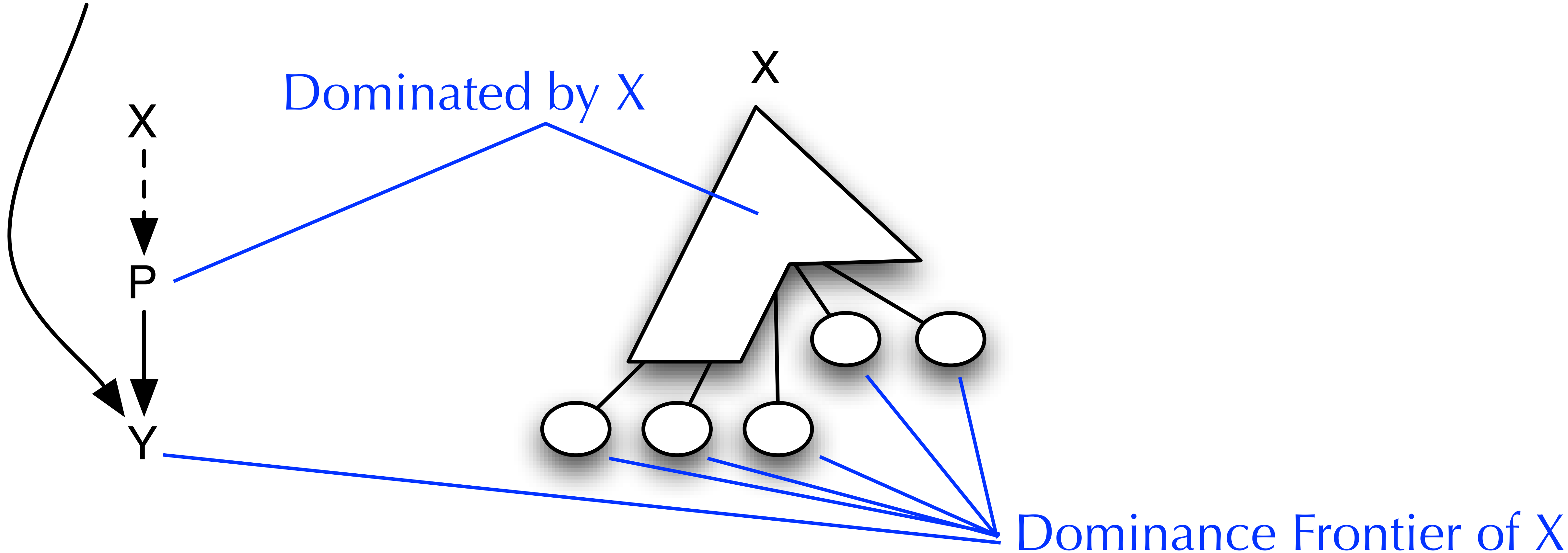
- Computing static single assignment form
- Computing control dependencies
- Identify (natural) loops in CFG
  - All nodes X dominated by entry node H, where X can reach H, and there is exactly one back edge (head dominates tail) in loop

Where do Φ Functions Go?

- We need a Φ function at node Z if
  - Two non-null CFG paths that both define v
  - Such that both paths start at two distinct nodes and end at Z



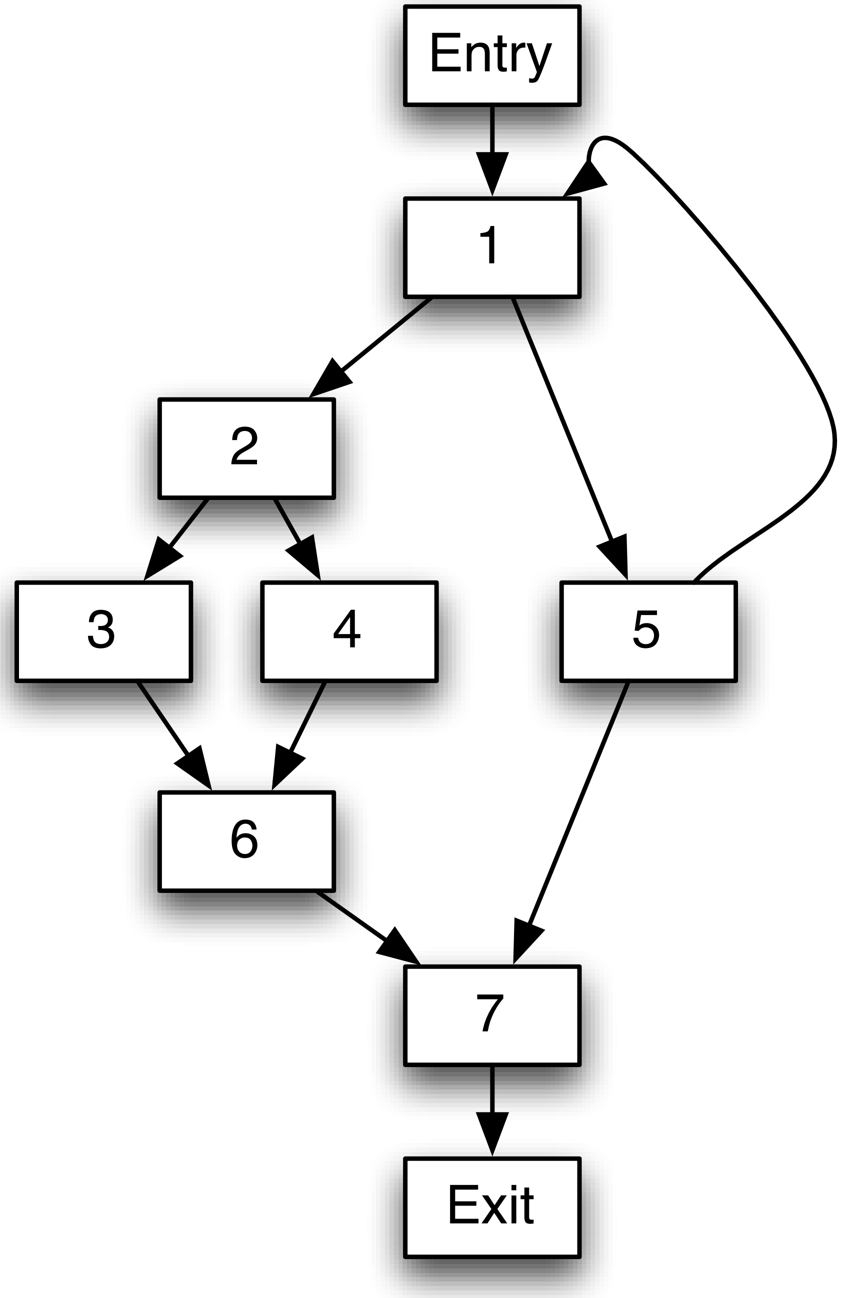
Dominance Frontiers: Illustration



Dominance Frontiers

- Y is in the dominance frontier of X iff
  - There exists a path from X to Exit through Y such that Y is the first node not strictly dominated by X
- Equivalently:
  - Y is the first node where a path from X to Exit and a path from Entry to Exit (not going through X) meet
- Equivalently:
  - X dominates a predecessor of Y
  - X does not strictly dominate Y

Example



$DF(1) = \{1\}$

$DF(2) = \{7\}$

$DF(3) = \{6\}$

$DF(4) = \{6\}$

$DF(5) = \{1, 7\}$

$DF(6) = \{7\}$

$DF(7) = \emptyset$

Computing SSA Form

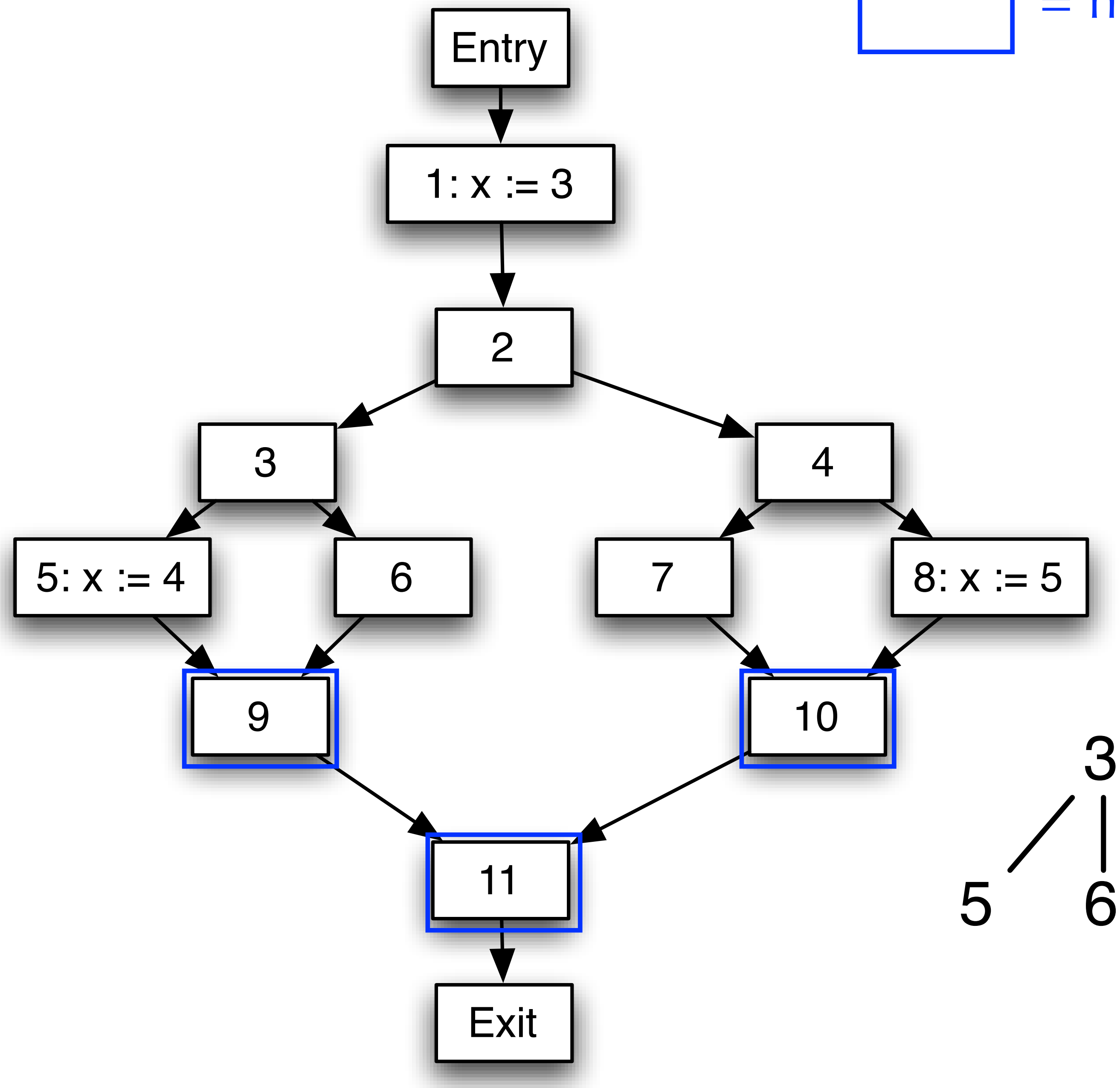
- Step 1a: Compute the dominance frontier
- Step 1b: Use dominance frontier to place Φ nodes
- Step 2: Rename variables so only one definition per name

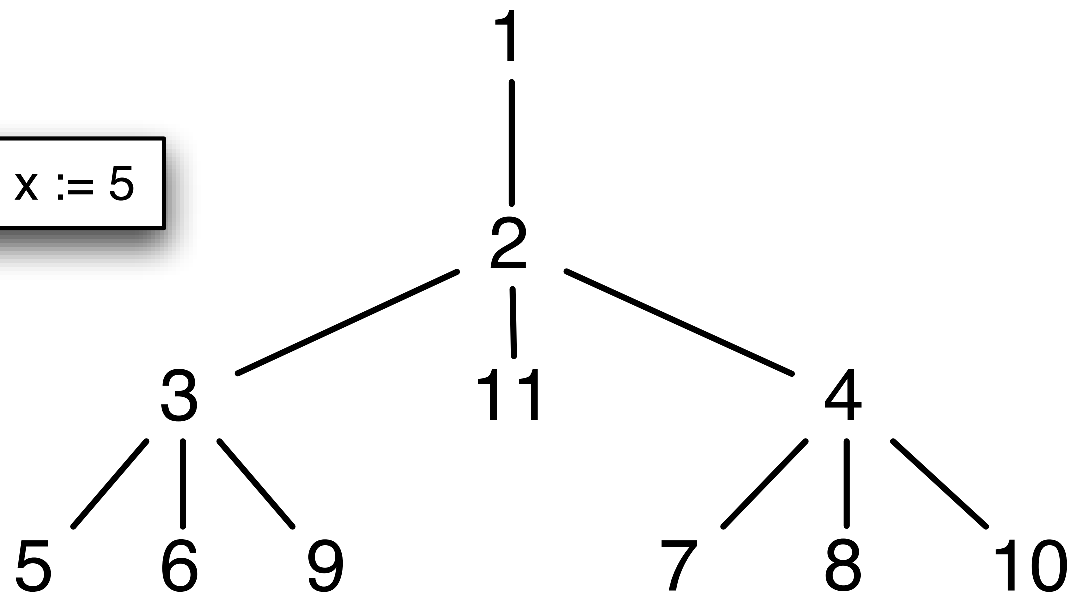
Step 1b: Placing Φ Functions for v

- Let S be the set of nodes that define v
- Need to place Φ function in every node in DF(S)
  - Recall, those are all the places where the definition of v in S and some other definition of v may meet
- But a Φ function adds another definition of v!
  - v := Φ(v, ..., v)
- So, iterate
  - $DF_1 = DF(S)$
  - $DF_{i+1} = DF(S \cup DF_i)$

Example

□ = need Φ function



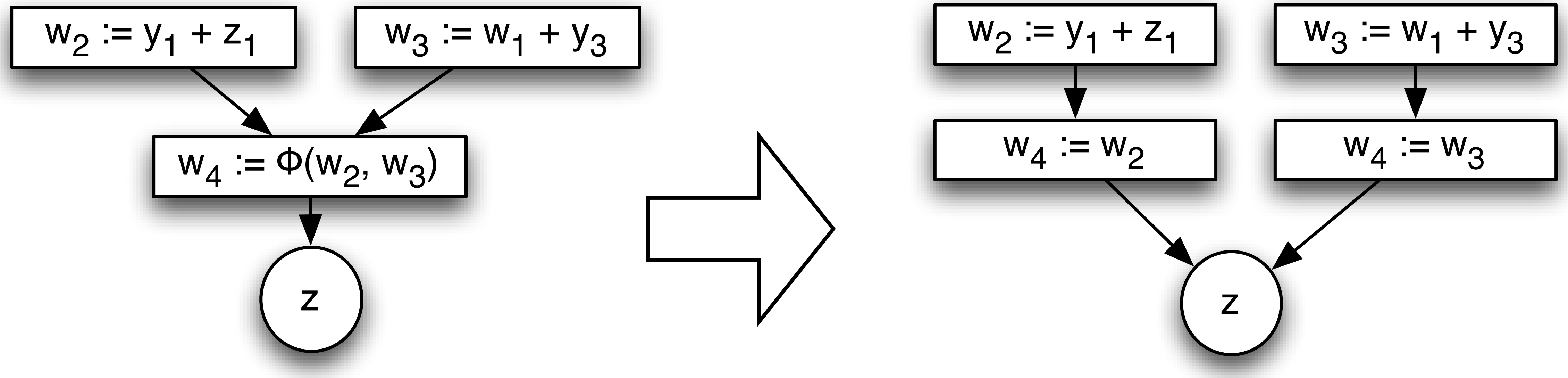


Step 2: Renaming Variables

- Top-down (DFS) traversal of dominator tree
  - At definition of v, push new # for v onto the stack
  - When leaving node with definition of v, pop stack
  - Intuitively: Works because there's a Φ function, hence a new definition of v, just beyond region dominated by definition
- Can be done in O(E+|DF|) time
  - Linear in size of CFG with Φ functions

Eliminating Φ Functions

- Basic idea: Φ represents facts that value of join may come from different paths
  - So just set along each possible path



Eliminating Φ Functions in Practice

- Copies performed at Φ fns may not be useful
  - Joined value may not be used later in the program
    - (So why leave it in?)

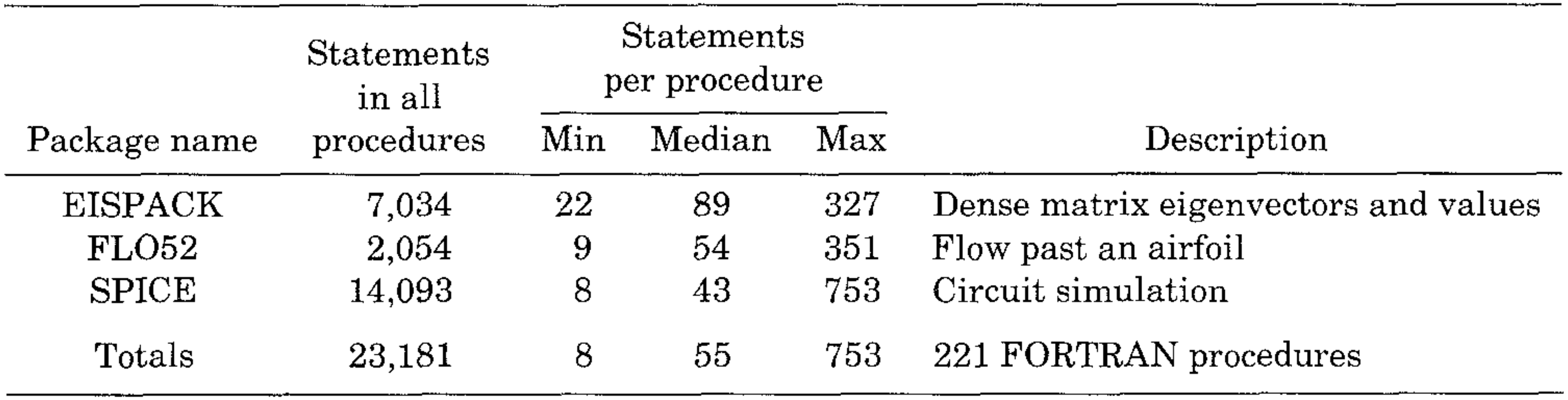
- Use dead code elimination to kill useless Φs

- Subsequent register allocation will map the (now very large) number of variables onto the actual set of machine register

Efficiency in Practice

- Claimed:
  - SSA grows linearly with size of program
  - No correlation between ratio and program size

Table I. Summary Statistics of Our Experiment



Package name	Statements in all procedures	Statements per procedure			Description
		Min	Median	Max	
EISPACK	7,034	22	89	327	Dense matrix eigenvectors and values
FLO52	2,054	9	54	351	Flow past an airfoil
SPICE	14,093	8	43	753	Circuit simulation
Totals	23,181	8	55	753	221 FORTRAN procedures

Cytron, Ferrante, Rosen, Wegman, and Zadeck, Efficiently Computing Static Single Assignment Form and the Control Dependence Graph, TOPLAS 13(4), Oct 1991.

Efficiency in Practice (cont'd)

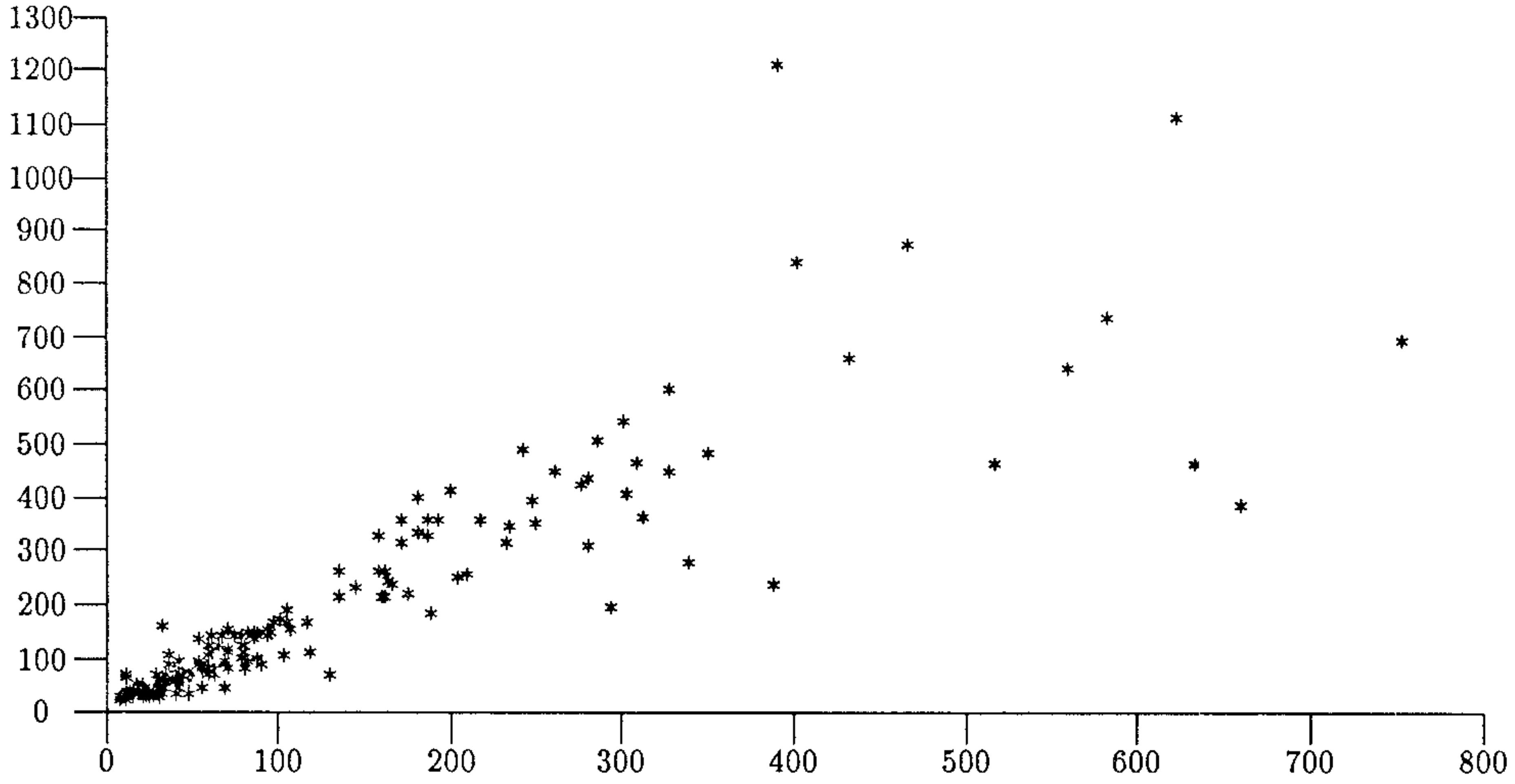


Fig. 21. Number of ϕ-functions versus number of program statements.

- Convincing?

Arrays

- Need to handle array accesses
  - A[i] := A[j] + B[k]
- Problem: How do we know whether A[i], A[j], and B[k] are all distinct?
  - Could have A=B, e.g., foo(int A[], int B[]){} ... foo(a,a)
  - Could have i=j
- History: significant research on determining array dependencies, for parallelizing compilers

Arrays (cont’d)

- One possibility: make arrays **immutable**
  - Then don’t need to worry about updates to them

```
* := A(i);
A(j) := V;
* := A(k) + 2;
```

```
* := A(i);
A := Update(A, j,V);
T := A(k);
* := T + 2;
```

- Update(A, j, V) makes a copy of A
  - Then try to collapse unnecessary copies

- Convincing?

Structures

- Can treat structures as sets of variables or as an array
  - with field name like an index into array

```
* := A.f;              * := X;      // X = A.f
A.g := V;              Y := V;      // Y = A.g
* := A.f + A.g         * := X + Y
```

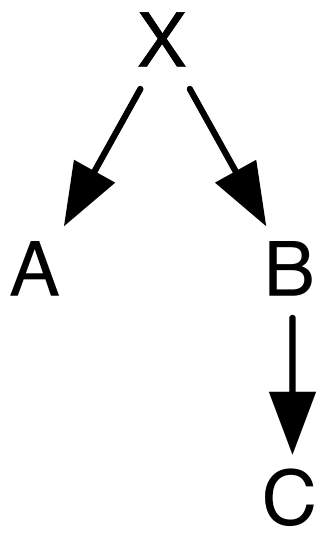
- Problems?

Pointers

- For each statement S, let
  - MustMod(S) = variables always modified by S
  - MayMod(S) = variables sometimes modified by S
    - So if v∉MayMod(S), then S must not modify v
  - MayUse(S) = variables sometimes used by S
- Then assume that statement S
  - writes to MayMod(S)
  - reads MayUse(S) ∪ (MayMod(S) - MustMod(S))
- Convincing? We'll talk more about pointers later in the course

Control Dependence

- Y is **control dependent** on X if whether Y is executed depends on a test at X



- A, B, and C are control dependent on X

Postdominators and Control Dependence

- Y **postdominates** X if every path from X to Exit contains Y
  - I.e., if X is executed, then Y is always executed
- Then, Y is control dependent on X if
  - There is a path $X \rightarrow Z_1 \rightarrow \cdots \rightarrow Z_n \rightarrow Y$ such that Y postdominates all Z_i and
  - Y does not postdominate X
  - I.e.,there is some path from X on which Y is always executed, and there is some path on which Y is not executed

Dominance Frontiers, Take 2

- Postdominators are just dominators on the CFG with the edges reversed

- To see what Y is control dependent on, we want to find the Xs such that in the reverse CFG
  - There is a path $X \leftarrow Z_1 \leftarrow \cdots \leftarrow Z_n \leftarrow Y$ where
    - for all i, $Y \geq Z_i$ and
    - $Y \not> X$
- I.e., we want to find $DF(Y)$ in the reverse CFG!