HARVARD
John A. Paulson
School of Engineering
and Applied Sciences

# Dataflow analysis

*CS252r Fall 2015*

*(Based on lecture notes by Jeff Foster)*

# New England Security Day

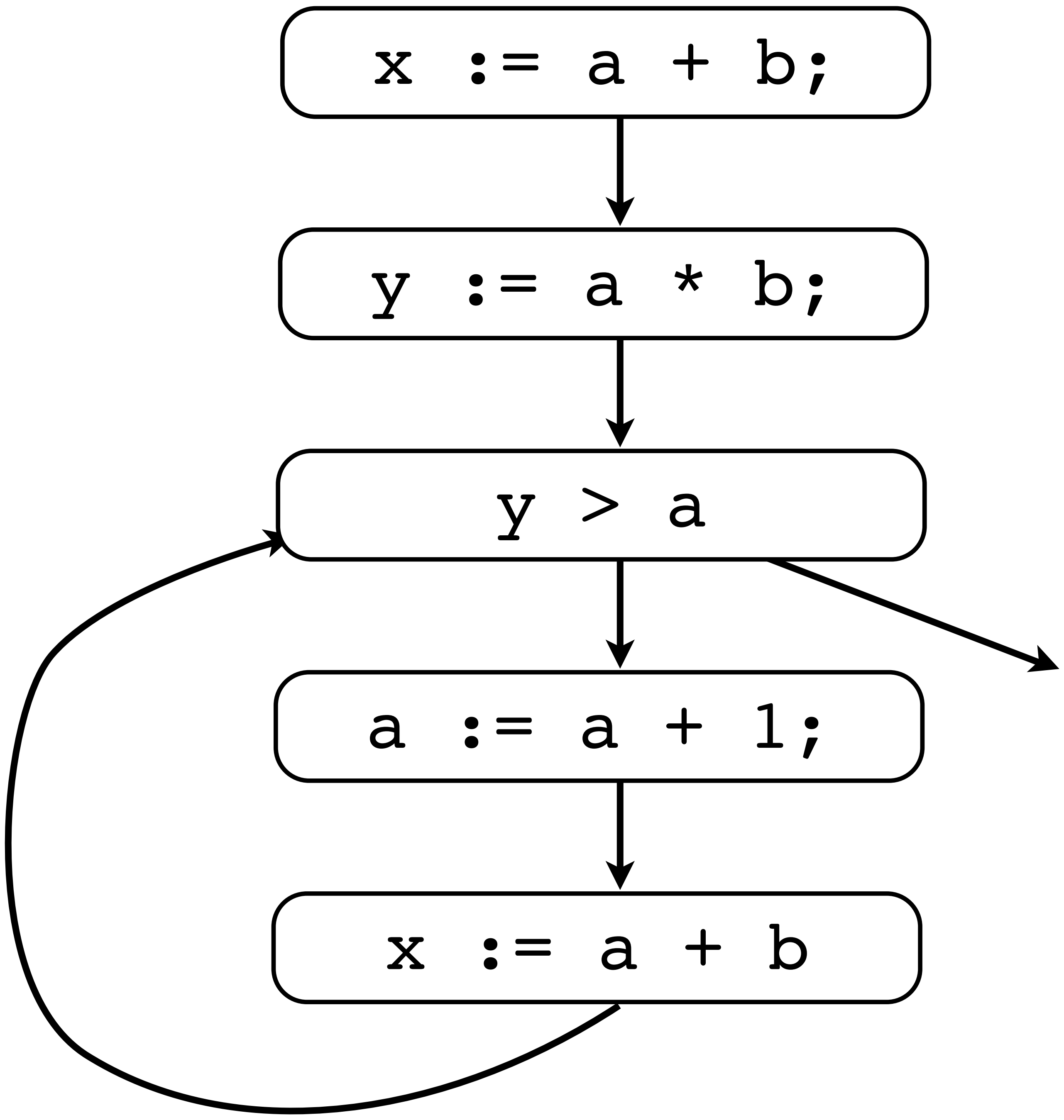
- Class outing! Thursday September 17
- If you plan on going, need to register TODAY
  - https://docs.google.com/forms/d/1PuRiIXr4qcQTcVY2MlTE6Ywzsi5J9kePSexGo73bJQ0/viewform?c=0&w=1
- More info at http://nesd.cs.umass.edu.

# Control flow graph

- A **control flow graph** is a representation of a program that makes certain analyses (including dataflow analyses) easier
- A directed graph where
  - Each node represents a statement
  - Edges represent control flow
- Statements may be
  - Assignments: `x := y` or `x := y op z` or `x := op y`
  - Branches: `goto L` or `if b then goto L`
  - etc.

# Control-flow graph example

```
x := a + b;
y := a * b;
while (y > a) {
   a := a + 1;
   x := a + b
}
```

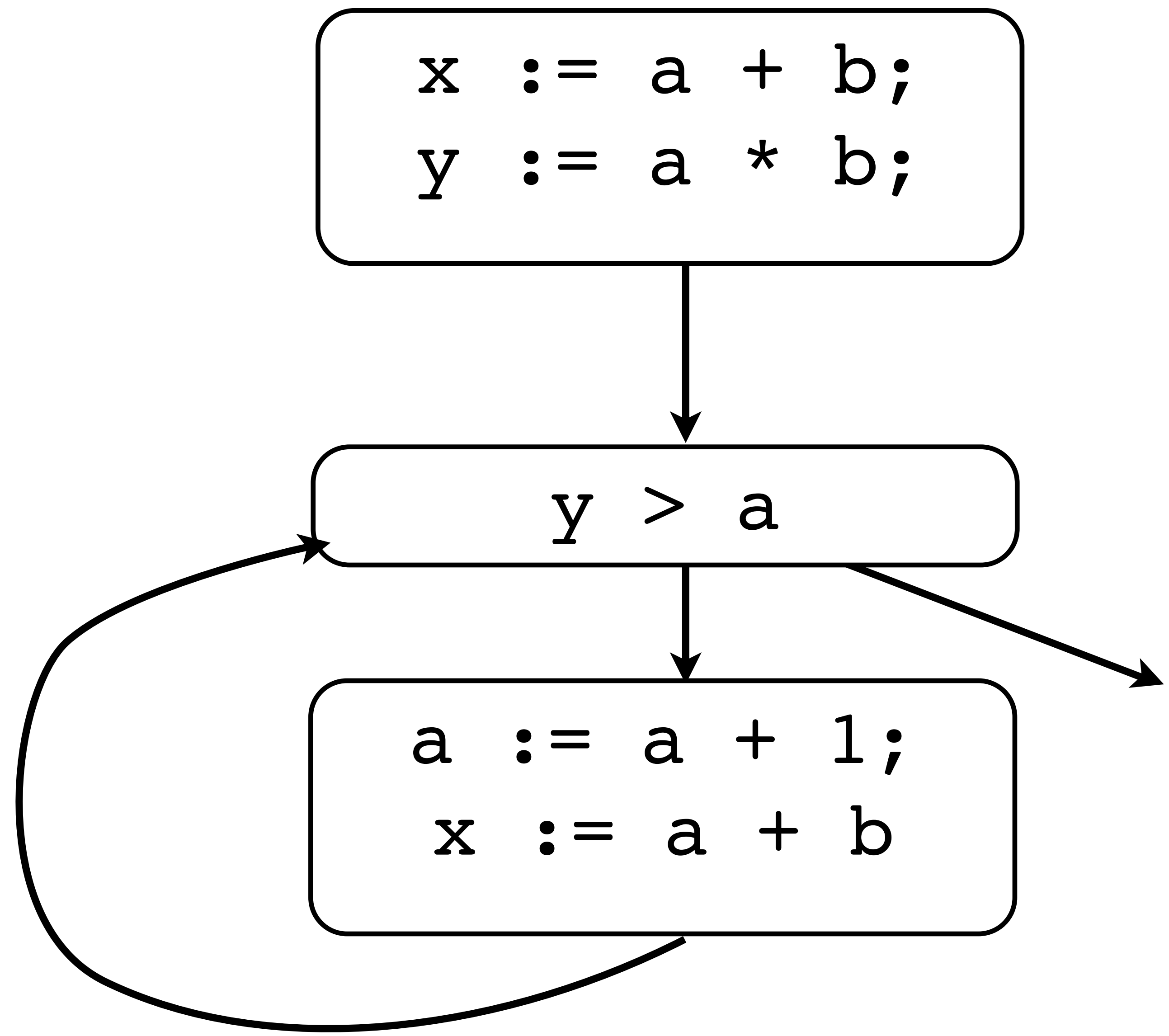


# Variations on CFGs

- Usually don't include declarations (e.g., `int x;`) in the CFG
  - But there's usually something in the implementation
- May want a **unique entry** and **unique exit** node
  - Won't matter for the examples we give
- May group statements into **basic blocks**
  - Basic block: a sequence of instructions with no branches into or out of the block.
    - i.e., execution starts only at the beginning of the block, and executes all of the block. Final statement in block may be a branch.

# Control-flow graph with basic blocks

```
x := a + b;
y := a * b;
while (y > a) {
   a := a + 1;
   x := a + b
}
```

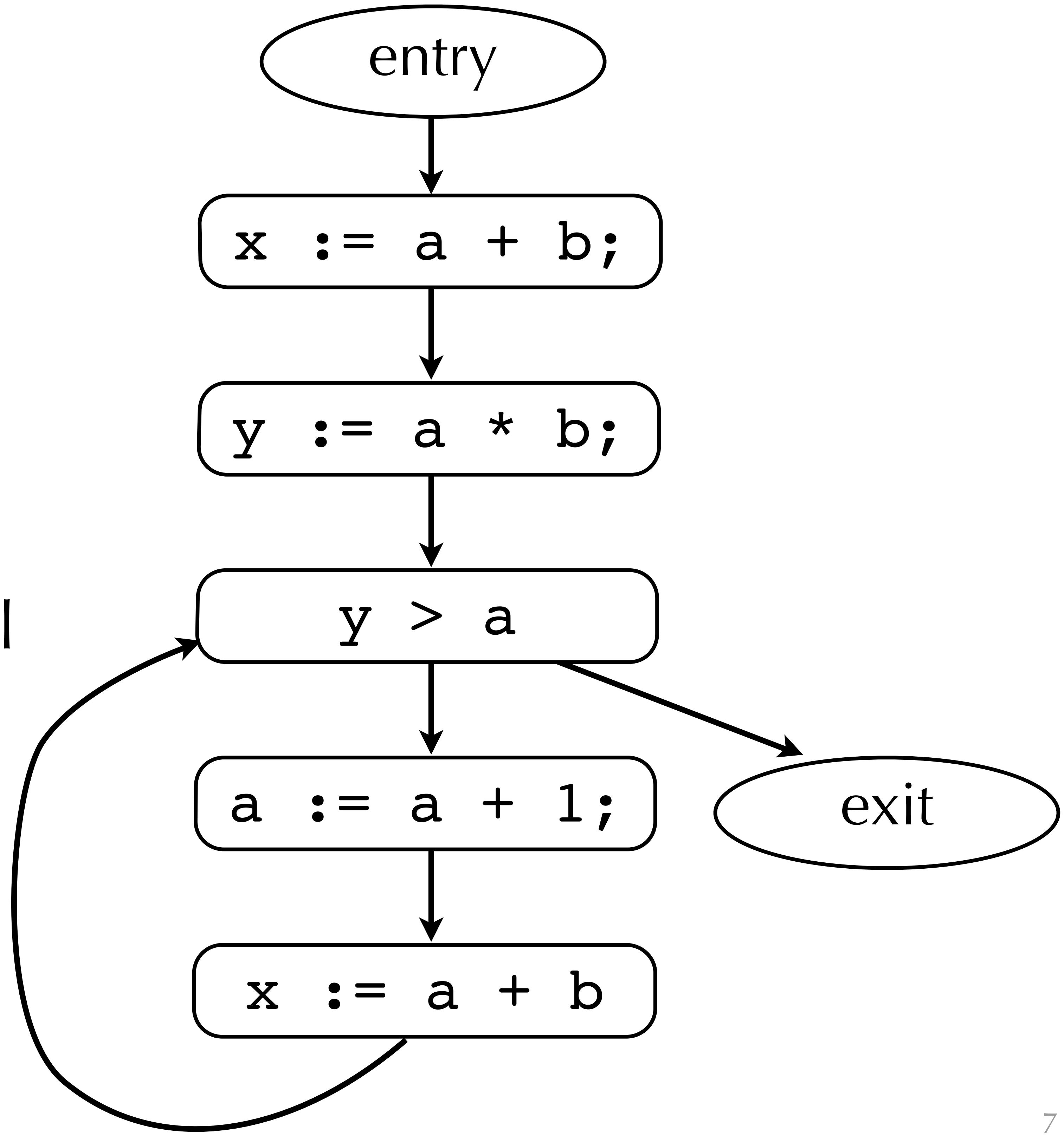



- Can lead to more efficient implementations
- More complicated to explain, so for the meantime we'll use single statement blocks

# Graph example with entry and exit

```
x := a + b;
y := a * b;
while (y > a) {
   a := a + 1;
   x := a + b
}
```

- All nodes without a normal predecessor should be pointed to by entry
- All nodes without a successor should point to exit



# CFG vs AST

- CFGs are much simpler than ASTs
- Fewer forms, less redundancy, only simple expressions
- But AST is a more faithful representation
  - CFGs introduce temporaries
  - Lose block structure of program
- ASTs are
  - Easier to report errors and other messages
  - Easier to explain to programmer
  - Easier to unparse to produce readable code

# Dataflow analysis

- A framework for proving facts about programs
- Reasons about lots of little facts
- Little or no interaction between facts
  - Works best on properties about how program computes
- Based on all paths through program
  - Including infeasible paths

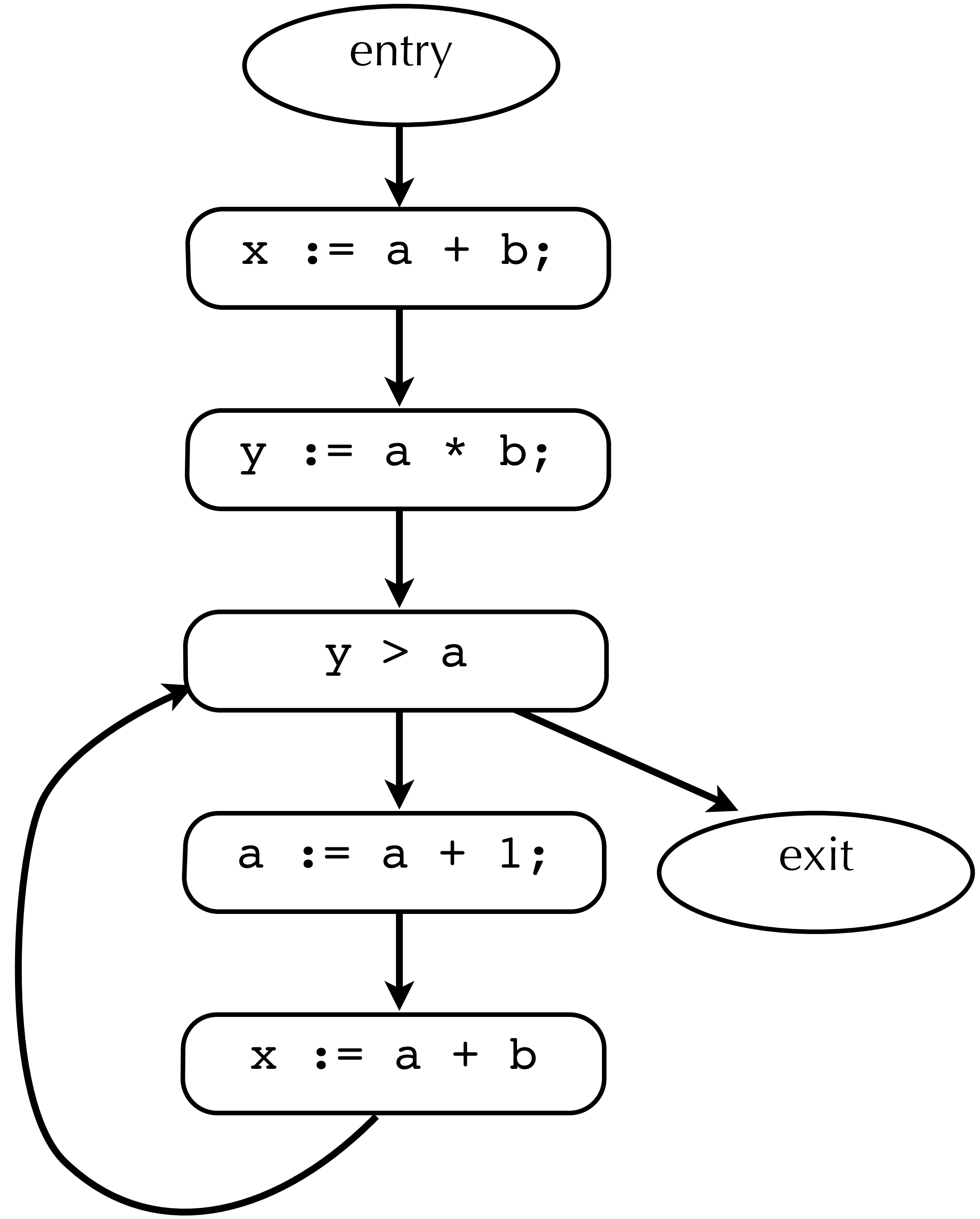
- Let's consider some dataflow analyses

# Available expressions

- An expression *e* is **available** at program point *p* if
  - *e* is computed on every path to *p*,and
  - the value of *e* has not changed since the last time *e* was computed on the paths to *p*
- Available expressions can be used to optimize code
  - If an expression is available, don’t need to recompute it (provided it is stored in a register somewhere)

# Data flow facts

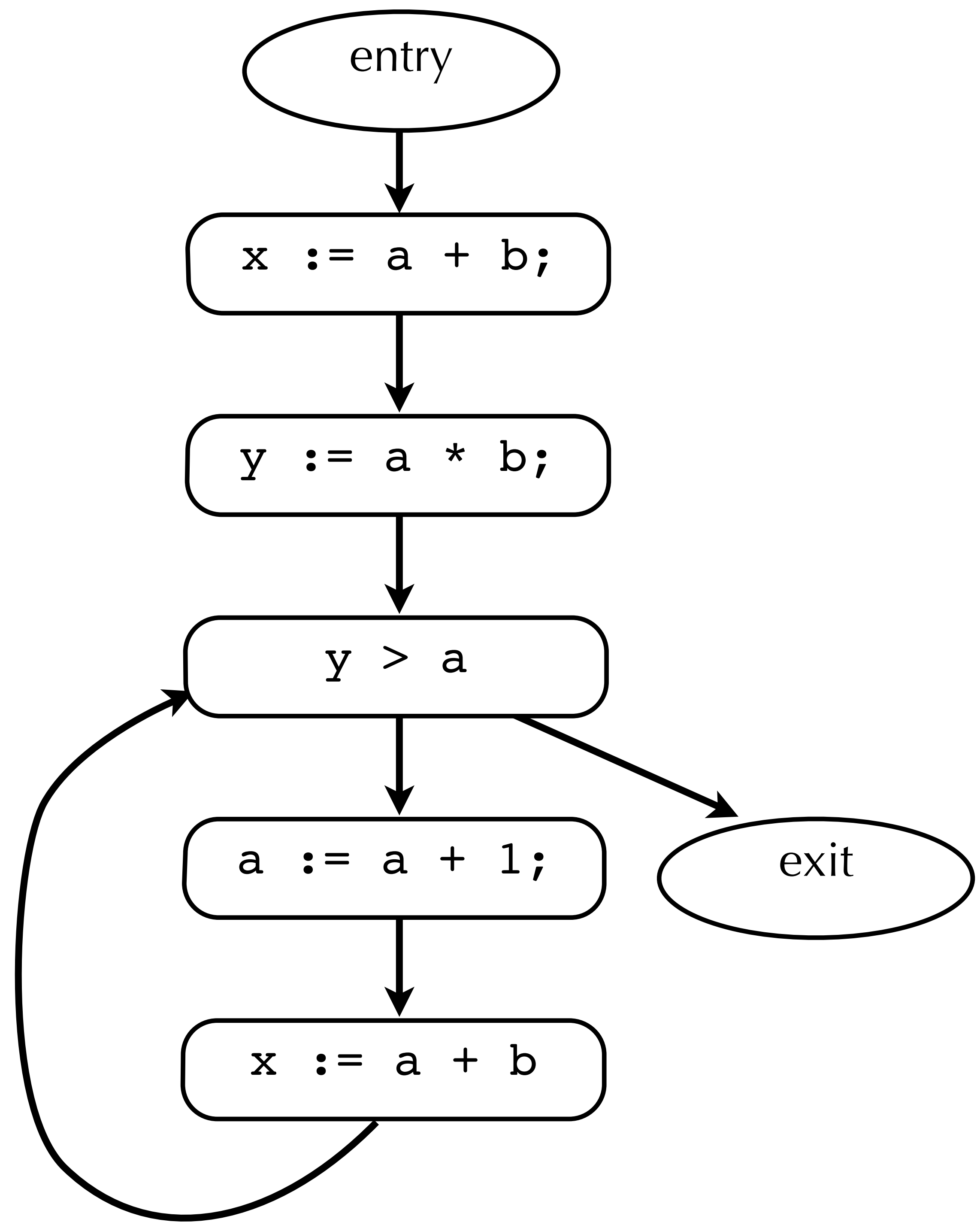
- Is expression e available?
- Facts
  - "a + b is available"
  - "a * b is available"
  - "a + 1 is available"
- For each program point, we will compute which facts hold.



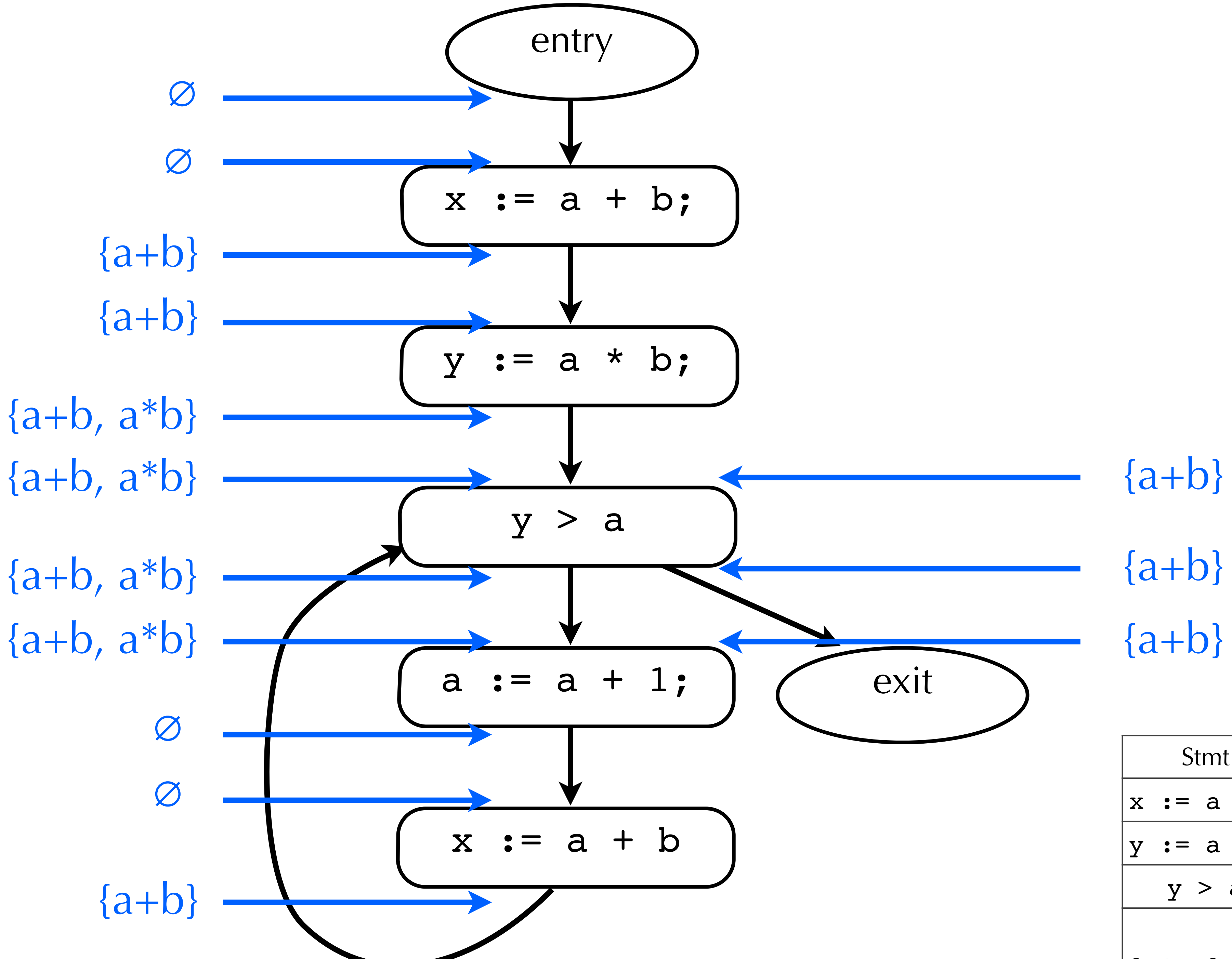
# Gen and Kill

- What is the effect of each statement on the facts?

| Stmt | Gen | Kill |
|---|---|---|
| `x := a + b` | `a + b` | |
| `y := a * b` | `a * b` | |
| `y > a` | | |
| `a := a + 1` | | `a + 1`<br>`a + b`<br>`a * b` |



# Computing available expressions




| Stmt | Gen | Kill |
|---|---|---|
| x := a + b | a + b | |
| y := a * b | a * b | |
| y > a | | |
| a := a + 1 | | a + 1<br>a + b<br>a * b |

# Terminology

- A **join point** is a program point where two or more branches meet
- Available expressions is a **forward must analysis**
  - **Forward** = Data flow from in to out
  - **Must** = At join points, only keep facts that hold on *all* paths that are joined

# Data flow equations

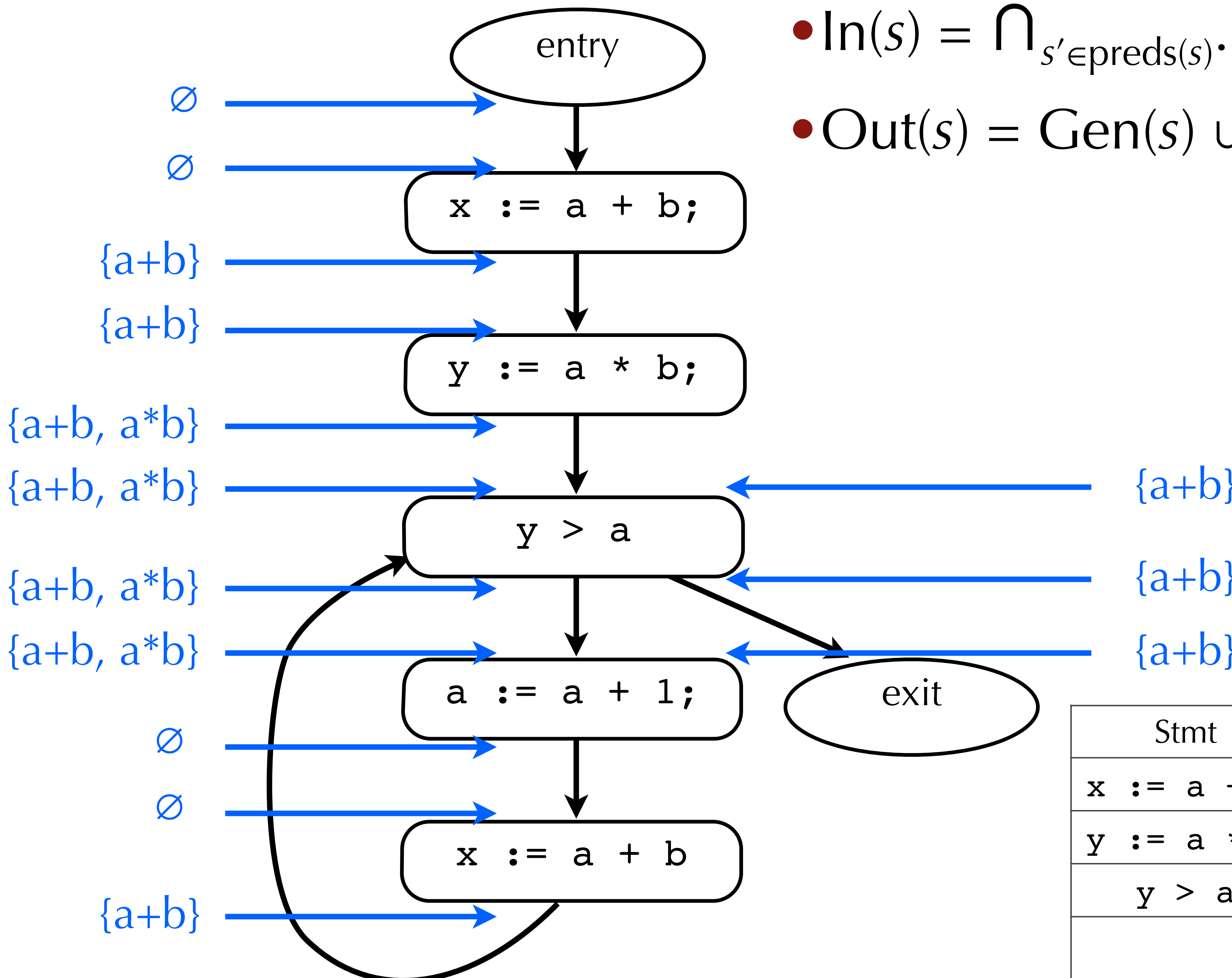
- Let $s$ be a statement
  - succs($s$) = { immediate successor stmts of $s$ }
  - preds($s$) = { immediate predecessor stmts of $s$ }
  - In($s$) = facts that holds just before executing $s$
  - Out($s$) = facts that hold just after executing $s$

- $\text{In}(s) = \bigcap_{s' \in \text{preds}(s)}.\ \text{Out}(s')$
- $\text{Out}(s) = \text{Gen}(s) \cup (\text{In}(s) - \text{Kill}(s))$

*What shows that this is a must analysis?*
*What shows that this is a forward analysis?*

# Computing available expressions

- $In(s) = \bigcap_{s' \in preds(s)} Out(s')$
- $Out(s) = Gen(s) \cup (In(s) - Kill(s))$




| Stmt | Gen | Kill |
|---|---|---|
| x := a + b | a + b | |
| y := a * b | a * b | |
| y > a | | |
| a := a + 1 | | a + 1<br>a + b<br>a * b |

# Liveness analysis

- A variable *v* is **live** at program point *p* if
  - *v* will be used on some execution path originating from *p* before *v* is overwritten
- Optimization
  - If a variable is not live, no need to keep it in a register
  - If variable is dead at assignment, can eliminate assignment

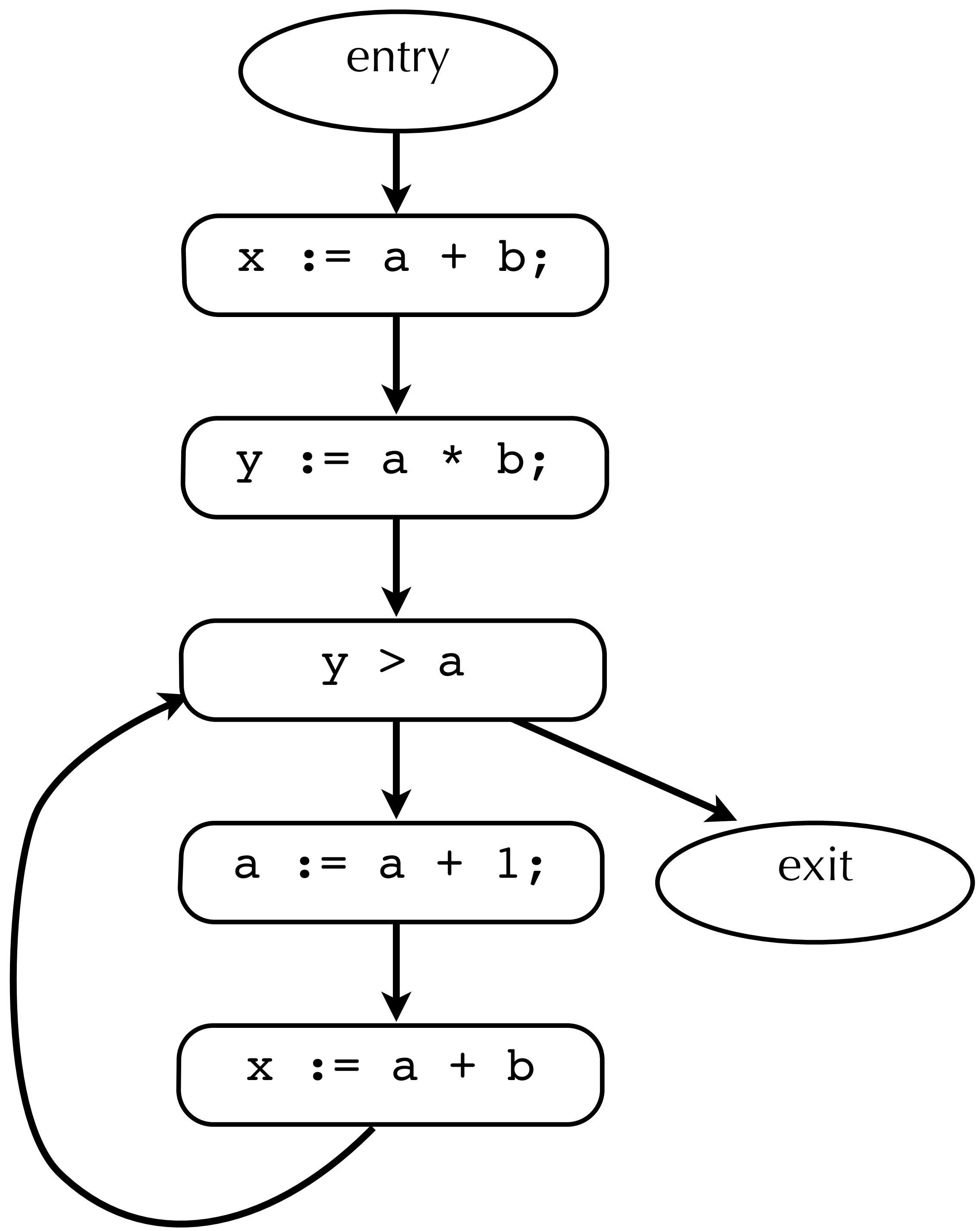
# Data flow equations

- Available expressions is a forward must analysis
  - Propagate facts in same direction as control flow
  - Expression is available only if available on all paths
- Liveness is a **backwards may analysis**
  - To know if a variable is live, we need to look at the future uses of it. We propagate facts backwards, from Out to In
  - Variable is live if it is used on some path
- $\text{Out}(s) = \bigcup_{s' \in \text{succs}(s)} \text{In}(s')$
- $\text{In}(s) = \text{Gen}(s) \cup (\text{Out}(S) - \text{Kill}(s))$

# Gen and Kill

- What is the effect of each statement on the facts?

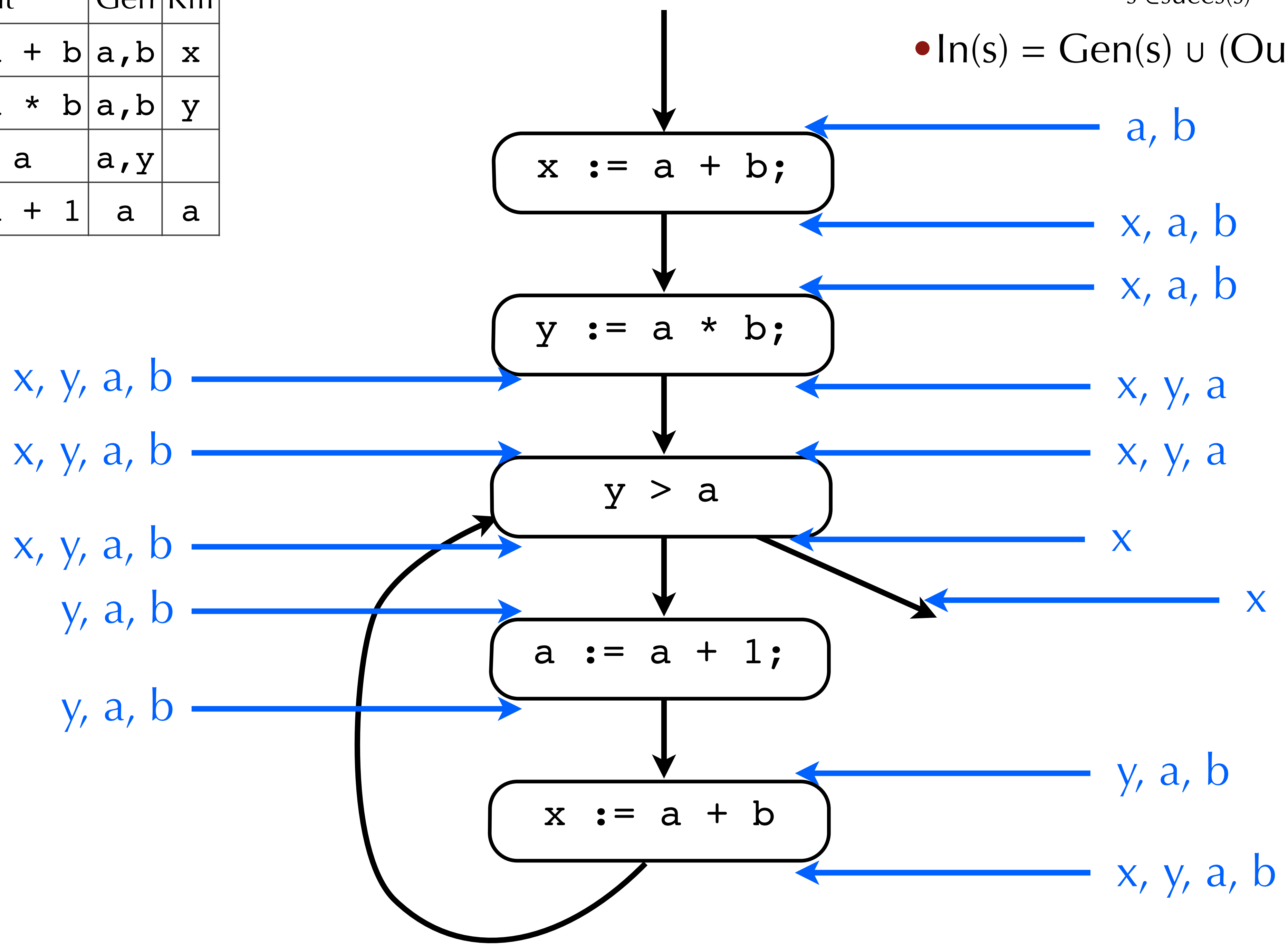
| Stmt | Gen | Kill |
|---|---|---|
| x := a + b | a,b | x |
| y := a * b | a,b | y |
| y > a | a,y | |
| a := a + 1 | a | a |



# Computing live variables

| Stmt | Gen | Kill |
|---|---|---|
| x := a + b | a,b | x |
| y := a * b | a,b | y |
| y > a | a,y | |
| a := a + 1 | a | a |

- $Out(s) = \bigcup_{s' \in succs(s)} In(s')$
- $In(s) = Gen(s) \cup (Out(S) - Kill(s))$



# Very busy expressions

- An expression *e* is very busy at point *p* if
  - On every path from *p*, expression *e* is evaluated before the value of *e* is changed
- Optimization
  - Can hoist very busy expression computation
- What kind of problem?
  - Forward or backward?
  - May or must?

# Reaching definitions

- A definition of a variable *v* is an assignment to *v*
- A definition of variable *v* reaches point *p* if
  - There is a path from the definition of *v* to *p*
  - There is no intervening assignment to *v* on that path
  - Also called **def-use** information
- What kind of problem?
  - Forward or backward?
  - May or must?

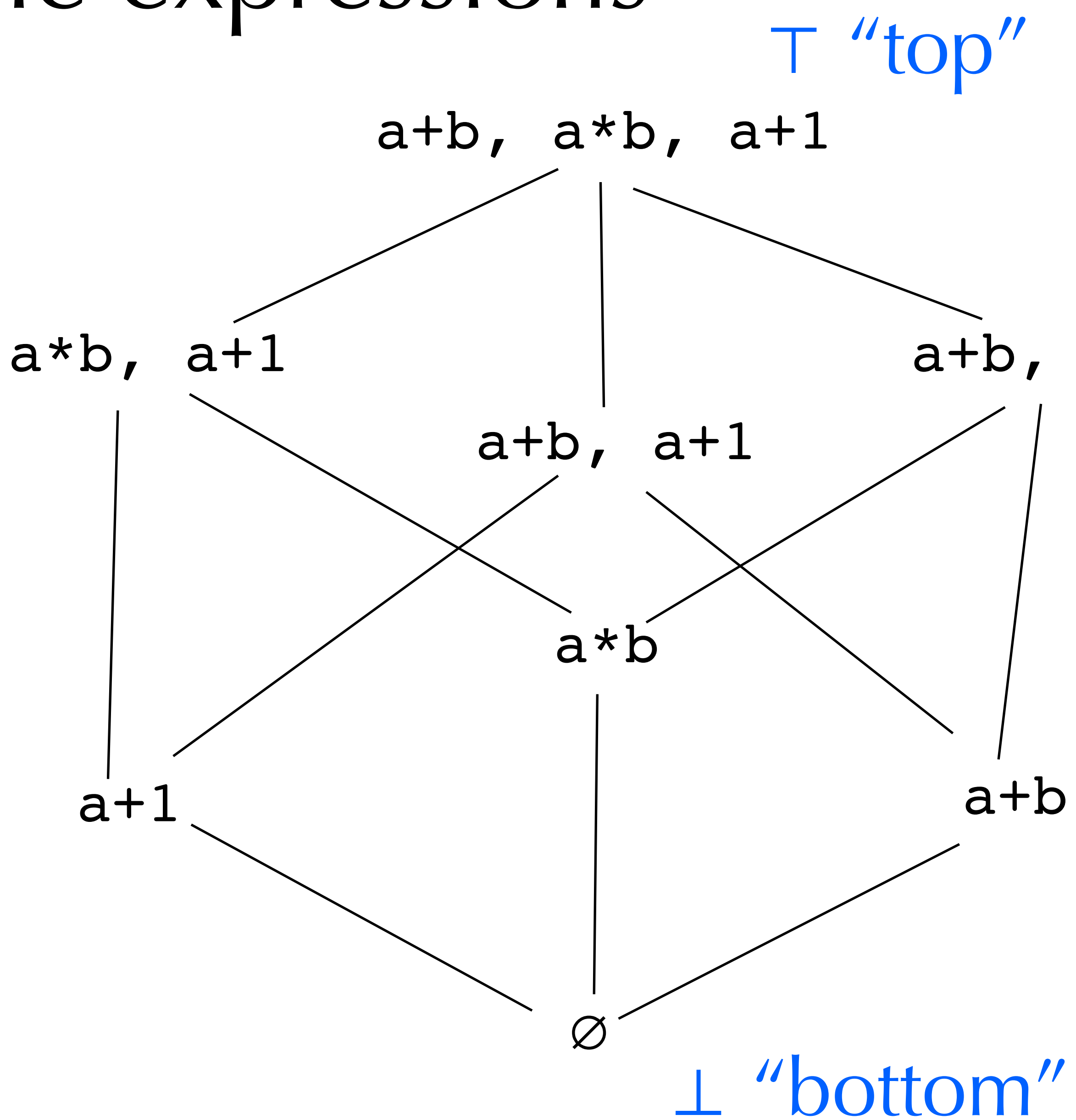
# Space of data flow analyses

- Most dataflow analyses can be categorized in this way
  - A few don't fit, need bidrectional flow
- Lots of literature on data flow analyses

| | **May** | **Must** |
|---|---|---|
| **Forward** | Reaching definitions | Available expressions |
| **Backward** | Live variables | Very busy expressions |

# Data flow facts and lattices

- Typically, data flow facts form **lattices**
- E.g., available expressions



# Partial orders and lattices

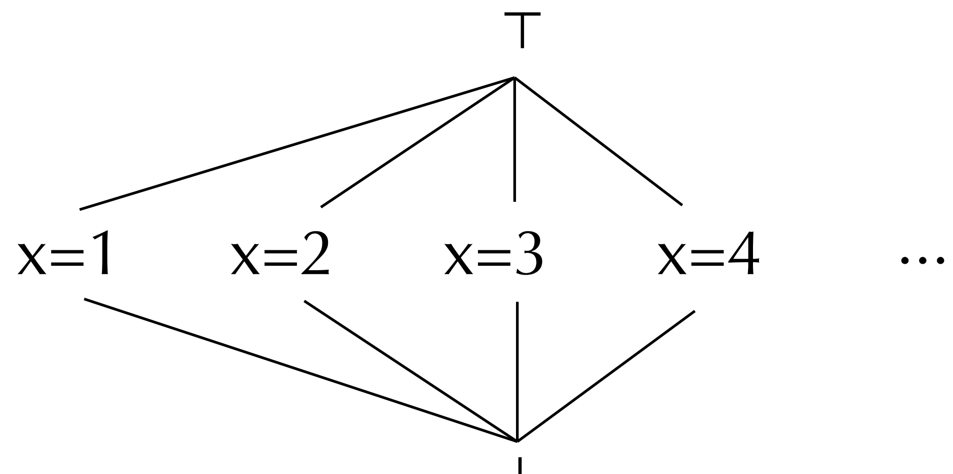
- A **partial order** is a pair $(P,\leq)$ such that
  - $\leq$ is a relation over $P$ $(\leq\ \subseteq P\times P)$
  - $\leq$ is reflexive, anti-symmetric, and transitive
- A partial order is a **lattice** if every two elements of $P$ have a unique least upper bound and greatest lower bound.
  - $\sqcap$ is the meet operator: $x \sqcap y$ is the greatest lower bound of $x$ and $y$
    - $x \sqcap y \leq x$ and $x \sqcap y \leq y$
    - if $z \leq x$ and $z \leq y$ then $z \leq x \sqcap y$
  - $\sqcup$ is the join operator: $x \sqcup y$ is the least upper bound of $x$ and $y$
    - $x \leq x \sqcup y$ and $y \leq x \sqcup y$
    - if $x \leq z$ and $y \leq z$ then $x \sqcup y \leq z$
- A **join semi-lattice** (meet semi-lattice) has only the join (meet) operator defined

# Complete lattices

- A partially ordered set is a **complete lattice** if meet and join are defined for all subsets (i.e., not just for all pairs)
  - (What's an example of a lattice that is not a complete lattice?)
- A complete lattice always has a bottom element and a top element
- A finite lattice always has a bottom element and a top element

# Useful lattices

- $(2^S, \subseteq)$ forms a lattice for any set $S$
  - $2^S$ is the **powerset** of S, the set of all subsets of S.
- If $(S, \leq)$ is a lattice, so is $(S, \geq)$
  - i.e., can “flip” the lattice
- Lattice for constant propagation



# Forward must data flow algorithm

```
Out(s) = ⊤ for all statements s
W := { all statements }                      (worklist)
repeat {
    Take s from W
    In(s) := ∩_{s′ ∊ pred(s)} Out(s′)
    temp := Gen(s) ∪ (In(s) - Kill(s))
    if (temp != Out(s)) {
        Out(s) := temp
        W := W ∪ succ(s)
    }
} until W = ∅
```

# Monotonicity

- A function $f$ on a partial order is **monotonic** if
  - if $x \leq y$ then $f(x) \leq f(y)$

- Functions for computing In(s) and Out(s) are monotonic
  - $\text{In}(s) := \bigcap_{s' \in \text{pred}(s)} \text{Out}(s')$
  - $\text{temp} := \text{Gen}(s) \cup (\text{In}(s) - \text{Kill}(s))$ A function $f_s$ of In(s)

- Putting them together: $\text{temp} := f_s(\bigcap_{s' \in \text{pred}(s)} \text{Out}(s'))$

# Termination

- We know the algorithm terminates
- In each iteration, either W gets smaller, or Out(s) decreases for some s
  - Since function is monotonic
- Lattice has only finite height, so for each s, Out(s) can decrease only finitely often

```
Out(s) = ⊤ for all statements s
W := { all statements }
repeat {
    Take s from W
    In(s) := ∩s′ ∊ pred(s) Out(s′)
    temp := Gen(s) ∪ (In(s) - Kill(s))
    if (temp != Out(s)) {
        Out(s) := temp
        W := W ∪ succ(s)
    }
} until W = ∅
```

# Termination

- A descending chain in a lattice is a sequence $x_0 > x_1 > \ldots$
- The height of a lattice is the length of the longest descending chain in the lattice
- Then, dataflow must terminate in O(*nk*) time
  - *n* = # of statements in program
  - *k* = height of lattice
  - assumes meet operation and transfer function takes O(1) time

# Fixpoints

- Dataflow tradition: Start with Top, use meet
  - To do this, we need a meet semilattice with top
    - complete meet semilattice = meets defined for any set
    - finite height ensures termination
  - Computes greatest fixpoint

- Denotational semantics tradition: Start with Bottom, use join
  - Computes least fixpoint

# Forward must data flow algorithm

```
Out(s) = ⊤ for all statements s
W := { all statements }                    (worklist)
repeat {
    Take s from W
    In(s) := ∩_{s′ ∈ pred(s)} Out(s′)
    temp := Gen(s) ∪ (In(s) - Kill(s))
    if (temp != Out(s)) {
        Out(s) := temp
        W := W ∪ succ(s)
    }
} until W = ∅
```

# Forward data flow again

Out(s) = ⊤ for all statements s
W := { all statements }
repeat {
    Take s from W
    temp := $f_s(\sqcap_{s' \in pred(s)} Out(s'))$
    if (temp != Out(s)) {
        Out(s) := temp
        W := W ∪ succ(s)
    }
} until W = ∅

*Transfer function for statement s*

# Which lattice to use?

- Available expressions
  - $P$ = sets of expressions
  - Meet operation $\sqcap$ is set intersection $\cap$
  - $\top$ is set of all expressions
- Reaching definitions
  - $P$ = sets of definitions (assignment statements)
  - Meet operation $\sqcap$ is set union $\cup$
  - $\top$ is empty set
- Monotonic transfer function *fs* is defined based on gen and kill sets.

# Distributive data flow problems

- If $f$ is monotonic, then we have

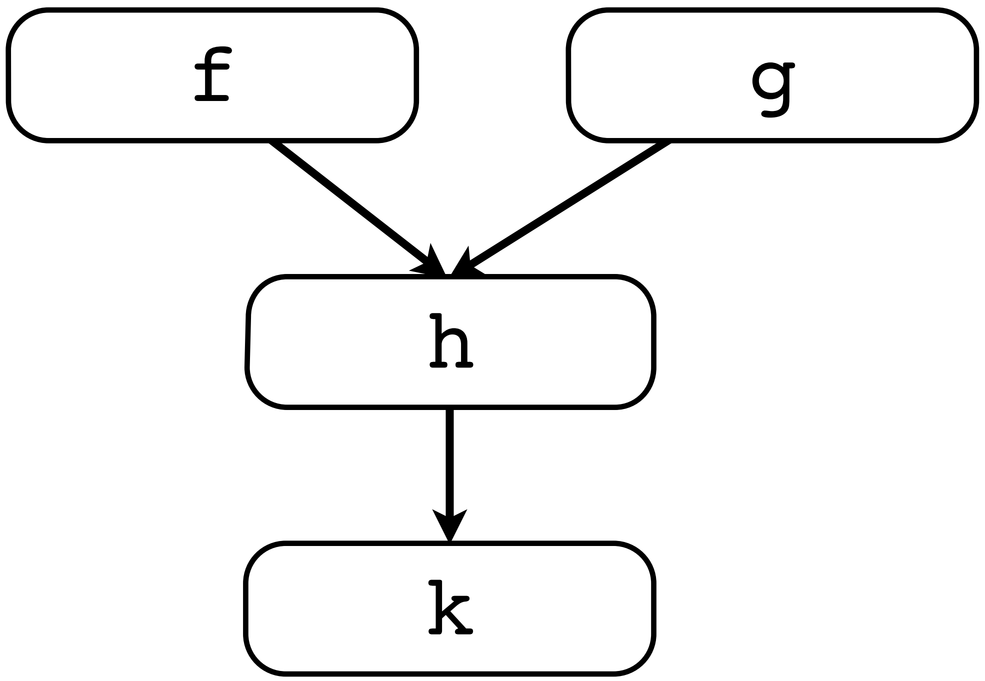
$$f(x \sqcap y) \leq f(x) \sqcap f(y)$$

- If $f$ is distributive then we have

$$f(x \sqcap y) = f(x) \sqcap f(y)$$

# Benefit of distributivity

- Joins lose no information




- $k(h(f(\top) \sqcap g(\top)))$
  $= k(h(f(\top)) \sqcap h(g(\top)))$
  $= k(h(f(\top))) \sqcap k(h(g(\top)))$

# Accuracy of data flow analysis

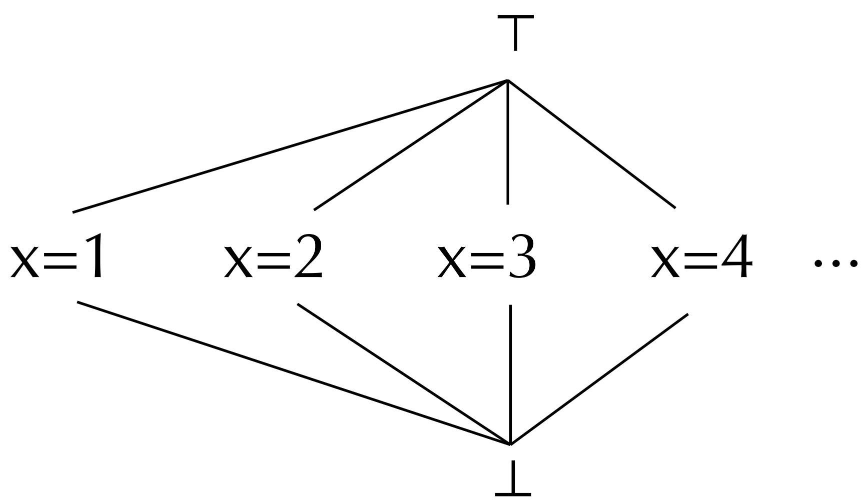
- Ideally we would like to compute the **meet over all paths** (MOP) solution:
  - Let $f_s$ be the transfer function for statement $s$
  - If $p$ is a path $s1,\ldots,sn$, let $f_p = f_{sn};\ldots;f_{s1}$
  - Let paths($s$) be the set of paths from the entry to $s$
  - $\text{MOP}(s) = \sqcap_{p \in \text{paths}(s)} f_p(\top)$
- **Theorem:** If the transfer functions are distributive, then solving using the data flow equations in the standard way produces the MOP solution

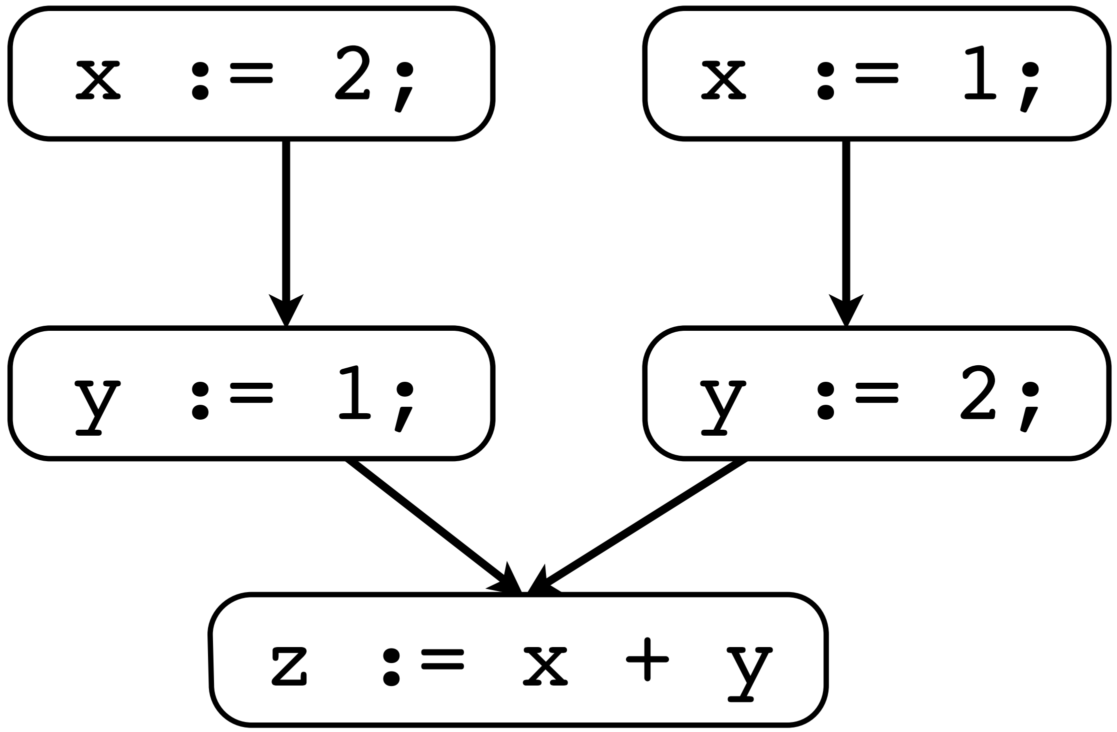
# What problems are distributive?

- Analyses of how the program computes
  - E.g.,
    - Live variables
    - Available expressions
    - Reaching definitions
    - Very busy expressions
- All Gen/Kill problems are distributive

# Non-distributive example

- Constant propagation




- In general, analysis of what the program computes is not distributive
- **Thm**: MOP for In(s) will always be ⊑ iterative dataflow solution
  - i.e., the iterative dataflow solution over-approximates the MOP

# Practical implementation

- Data flow facts are assertions that are true or false at a program point
- Can represent set of facts as bit vector
  - Fact *i* represented by bit *i*
  - Intersection=bitwise and, union=bitwise or, etc
- “Only” a constant factor speedup
  - But very useful in practice

# Basic blocks

- A **basic block** is a sequence of statements such that
  - No branches to any statement except the first
  - No statement in the block branches except the last
- In practical data flow implementations
  - Compute Gen/Kill for each basic block
    - Compose transfer functions
  - Store only In/Out for each basic block
  - Typical basic block is about 5 statements

# Order is important

- Assume forward data flow problem
  - Let $G=(V,E)$ be the CFG
  - Let $k$ be the height of the lattice
- If $G$ acyclic, visit in topological order
  - Visit head before tail of edge
- Running time O(|E|)
  - No matter what size the lattice

# Order is important

- If $G$ has cycles, visit in reverse postorder
  - Order from depth-first search
- Let $Q$ = max # back edges on cycle-free path
  - Nesting depth
  - Back edge is from node to ancestor on DFS tree
- Then if $\forall x.\ f(x) \leq x$ (sufficient, but not necessary)
  - Running time is $O((Q + 1)|E|)$

# Flow sensitivity

- Data flow analysis is **flow sensitive**
  - The order of statements is taken into account
    - I.e., we keep track of facts **per program point**

- Alternative: **Flow-insensitive** analysis
  - Analysis the same regardless of statement order
  - Standard example: types describe facts that are true at all program points
    - `/*x:int*/` `x:=…` `/*x:int*/`

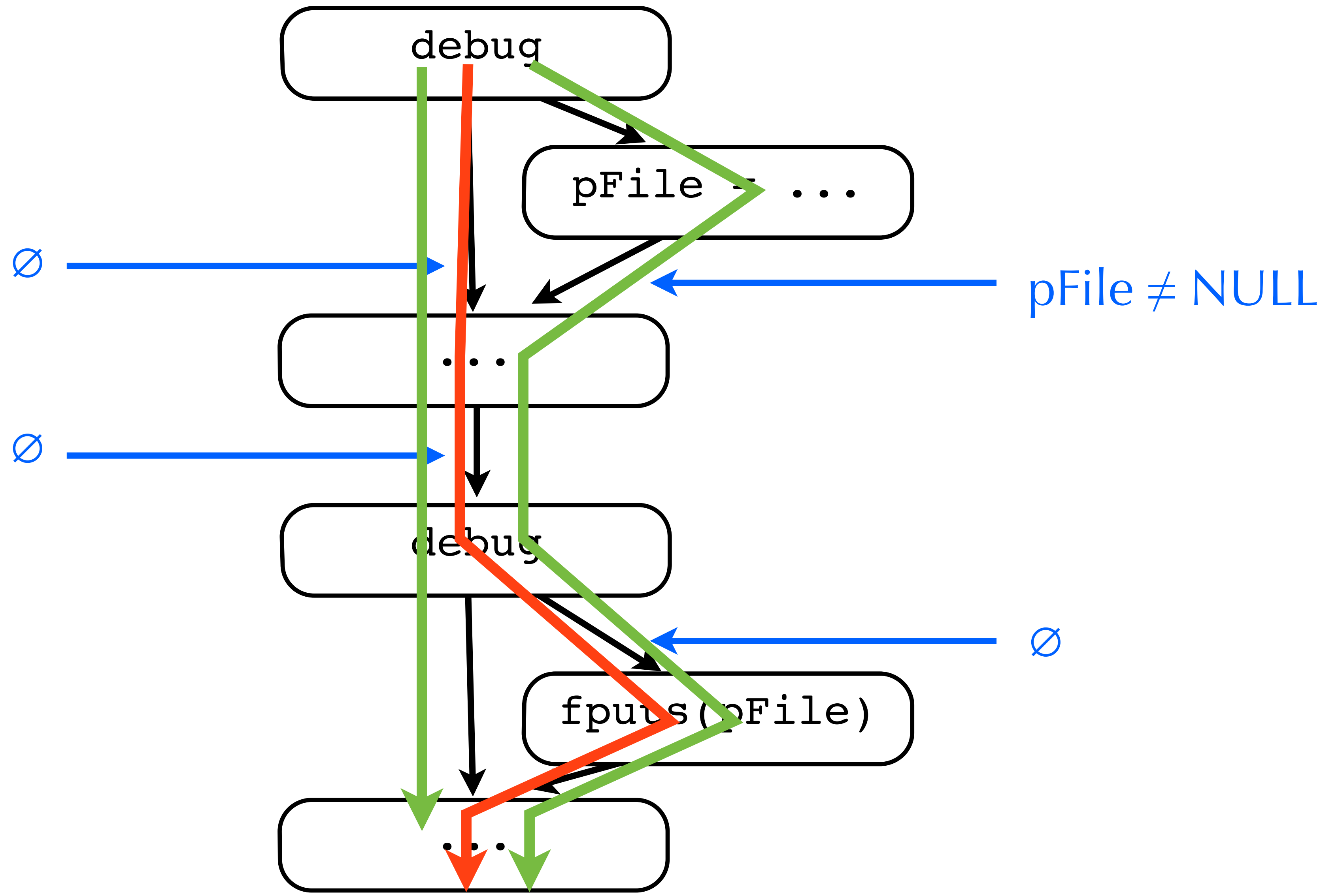
# A problem...

- Consider following program

```
FILE *pFile = NULL;
if (debug) {
    pFile = fopen("debuglog.txt", "a")
}
…
if (debug) {
    fputs("foo", pFile);
}
```

- Can `pFile` be `NULL` when used for `fputs`?
- What dataflow analysis could we use to determine if it is?

# Path sensitivity



# Path sensitivity

- A **path-sensitive** analysis tracks data flow facts depending on the path taken
  - Path often represented by which branches of conditionals taken
- Can reason more accurately about **correlated conditionals** (or dependent conditionals) such as in previous example
- How can we make a path sensitive analysis?
  - Could do a dataflow analysis where we track facts for each possible path
  - But exponentially many paths make it difficult to scale
- Some research on scalable path sensitive analyses.

# Terminology review

- Must vs. May
  - (Not always followed in literature)
- Forwards vs. Backwards
- Flow-sensitive vs. Flow-insensitive
- Path-sensitive vs Path-insensitive
- Distributive vs. Non-distributive

# Dataflow analysis and the heap

- Dataflow is good at analyzing local variables
  - But what about values stored in the heap?
  - Not modeled in traditional data flow
- In practice: `*x := e`
  - Assume all data flow facts killed (!)
  - Or, assume write through x may affect any variable whose address has been taken
- In general, hard to analyze pointers
- Pointer analysis approximates what addresses an expression may refer to
  - See next week's lecture...