

**ES128: Homework 6**  
**Due in class on Wednesday, 5 May 2010**

**Problem 1**

Calculate the mass matrix for the two-dimensional square element shown in Fig. 1. The element has uniform thickness  $t$  and uniform density  $\rho$ .

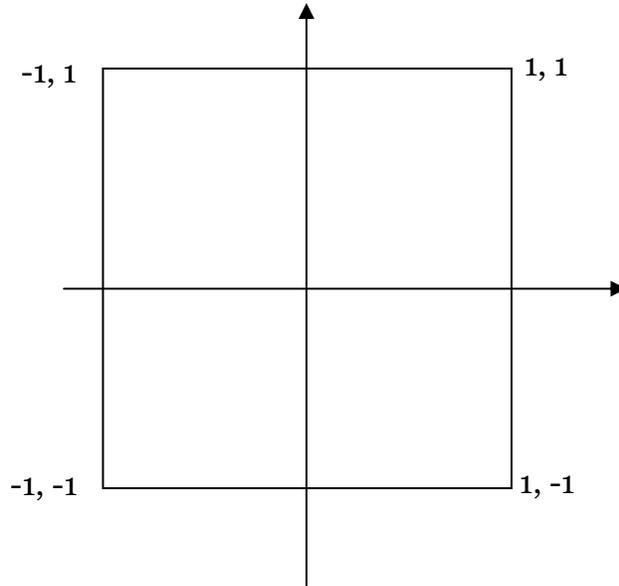


Fig.1

**Problem 2**

The four node parallelogram element shown below has uniform density and thickness. By integration, determine the consistent mass matrix (Hint: use the results of Problem 1 and the isoparametric formulation)

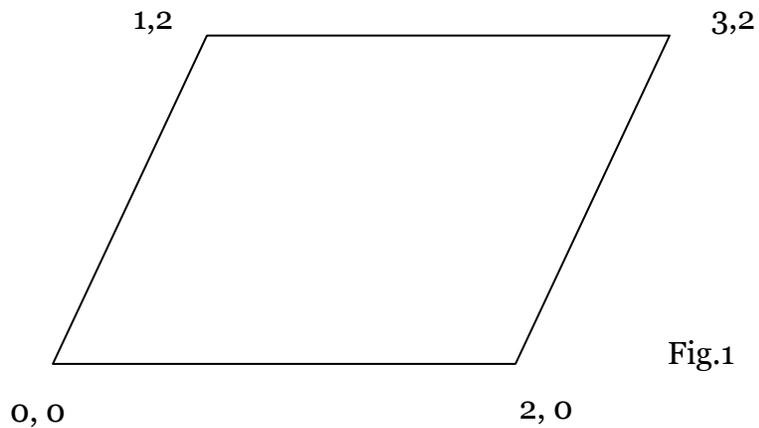


Fig.1

### Problem 3

Cross-sectional area of the bar (as shown in Fig. 2) varies linearly from  $A_0$  at the left end to  $\gamma A_0$  at the right end, where  $\gamma$  is a constant. Determine the consistent mass matrix that operates on axial degree of freedom  $u_1$  and  $u_2$ .

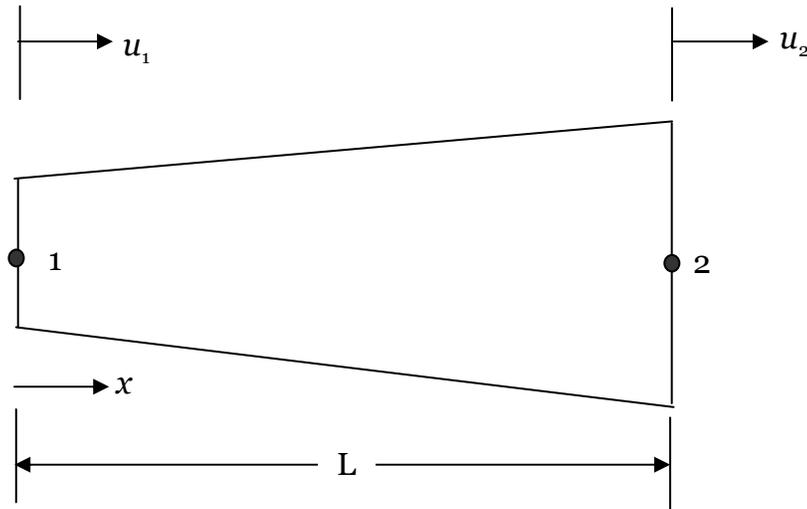


Fig. 2

### Problem 4

Only axial motion is permitted in the system shown in Fig. 3. Let  $k=1$  and  $m=2$ . Determine the fundamental vibration frequency  $\omega_1$  of the given system. Then calculate  $\omega_1$  after condensing the system to a single degree of freedom using Guyan reduction.

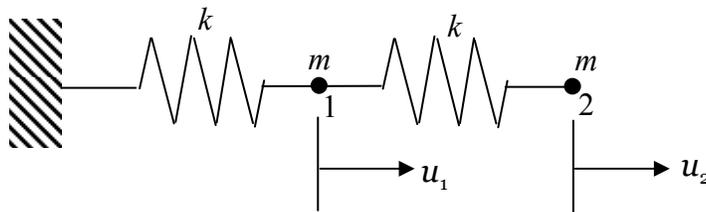


Fig. 3

### Problem 5

A particle of unit mass is supported by a spring of unit stiffness, so  $\omega_1=1$ . There is no damping. At time  $t=0$ , when the particle has zero displacement and zero velocity, a unit force is applied and maintained. Use the central difference method to calculate displacement versus time over successive time steps as follows

- 1) Use  $\Delta t=0.5$  and go to  $t=7$

- 2) Use  $\Delta t=1$  and go to  $t=7$
- 3) Use  $\Delta t=2$  and go to  $t=10$
- 4) Use  $\Delta t=3$  and go to  $t=15$

Compare the results obtained above in terms of displacements versus time.  
(Hint: implement the algorithm in Matlab)