ES 151 Assignment #1

Professor: Donhee Ham

Date: February 5th, 2015

Due: **12:45pm**, February 12th, 2015; slide your submission under the door at Maxwell-Dworkin 131.

Problem 1 (40pt)

Consider two concentric spherical conductors with air ($\epsilon = \epsilon_0$, almost) between them. The inner conductor of radius a is at potential V_0 and has a positive overall charge Q, and the outer conductor of radius b is grounded (at potential zero) and has a negative overall charge $-Q$. Calculate the capacitance of this system in the following three different ways:

1) by integrating the potential contributed by each infinitesimal charge on the inner and outer conductor surfaces;

2) by using Gauss' Law;

3) by solving the Laplace's equation with the appropriate boundary condition. In the spherical coordinate, the Laplace's equation is given by (you should be able to derive this, although you don't have to do the derivation in this work):

$$\nabla^2\Phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial \Phi}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial \Phi}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\Phi}{\partial \phi^2} = 0. \tag{1}$$

Here to avoid confusion with the angle ϕ, we use symbol Φ for the electric potential. The Laplace equation is substantially simplified, if you use spherical symmetry.

Problem 2 (40pt)

Consider a coaxial cable, whose cross section is shown in Fig. 1 (treat dielectric constant ϵ in the figure as ϵ_0). Calculate the capacitance of this coaxial cable *per unit length*, by 1) using Gauss's Law, and 2) solving Laplace's equation. In the cylindrical coordinate, the Laplace operator is given by (again, you should be able to derive this, although you don't have to do it for this work):

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial}{\partial r}) + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}, \tag{2}$$

which in this problem can be greatly simplified due to symmetry.

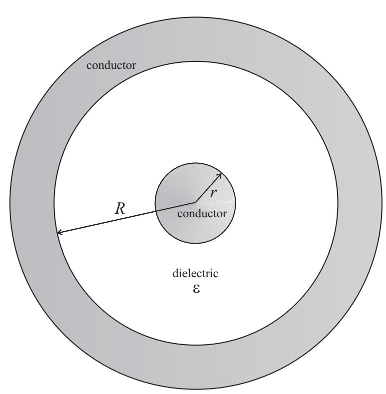

Figure 1: Cross section of a coaxial cable