ES 151 Assignment #2

Professor: Donhee Ham

Date: February 12th, 2015

Due: **12:55pm + 10 min grace period**, February 19th, 2015; slide your work under through the door at Maxwell-Dworkin 131.

Problem 1 (30pt)

Consider a point charge $q > 0$ and a remote position $\mathbf{P}$, as shown in Fig. 1(a). To reduce the electric field at $\mathbf{P}$, one can place a large grounded conducting plane distance d ($d \ll R$) below the point charge q as shown in Fig. 1(b). One way of looking at it is is because the negative charges induced in the conducting plane by q will attract much of the field generated by q. Another way of looking at it is to think of the situation with q and the conducting plane effectively as an electric dipole (do you understand this statement, from the point of view of the image charge?). With $d = 0.01R$, how much field reduction will be obtained at $\mathbf{P}$?

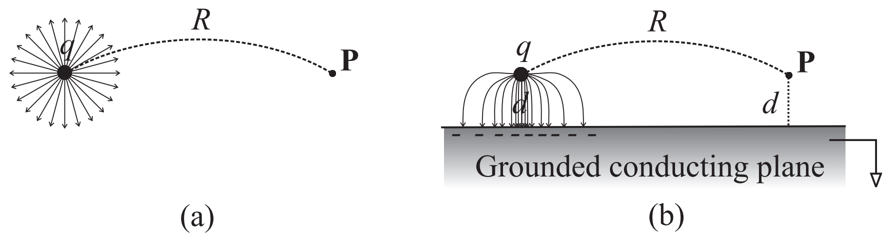

Figure 1: Shielding a point charge effect using a grounded conducting plate

Problem 2 (40pt)

A point charge q is placed at a distance b from the center of a grounded conducting sphere of radius a: see Fig. 2. Using the image method, calculate the surface density of charges induced on the surface of the sphere. You will have to first find the magnitude and position of the negative image charge (the magnitude is not q). By integrating the surface charge density, calculate the total induced charge on the sphere. The total induced charge should be equal to the image charge - can you argue why using Gauss's law? Calculate the attractive force between the point charge and the conducting sphere.

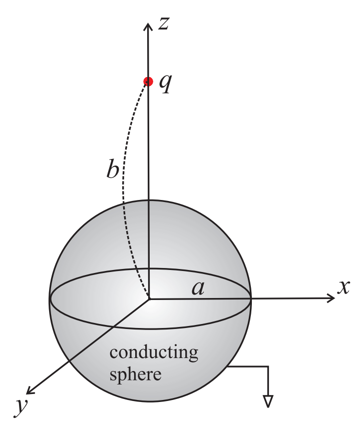

Figure 2: A point charge nearby a grounded spherical conductor

Problem 3 (40pt)

(a) Show that at a dielectric-conductor interface the surface density of dielectric's *bound* charge σ_{bound} is:

$$\sigma_{bound} = -\frac{\epsilon - \epsilon_0}{\epsilon} \cdot \sigma_{free} \tag{1}$$

where ϵ is the permittivity of the dielectric material and σ_{free} is the surface density of the conductor's charge.

(b) A conductor of arbitrary shape, carrying a total charge of q, is surrounded by an arbitrarily-shaped dielectric material of permittivity ϵ: see Fig. 3. Calculate the total *bound* charges at the inner and outer *surfaces* of the dielectric. What is the *volume* bound charge density *inside* the dielectric?

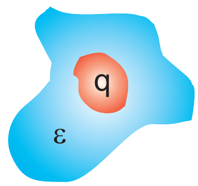

Figure 3: A conductor surrounded by a dielectric medium

Problem 4 (40pt): Charge screening in electrolyte

Consider an electrolyte at temperature T, containing positively-charged and negatively-charged ions, with each ion carrying a charge of $\pm q_{ion}$ (the sign of course depends on whether the ion is positively or negatively charged; $q_{ion} > 0$). The number concentration of the positively-charged ions and that of the negatively-charged ions with no perturbation are the same and given by n_0. Now we put a small (with a negligible radius), positively-charged particle into this electrolyte. Negative ions will rush into this particle and will *screen* the positively-charged particle, making its impact significantly smaller. Show that this screening effect manifests by making the electric potential ϕ decay *exponentially* as the distance is increased away from the positively-charged particle. Calculate the characteristic decay length (which is known as Debye length). See the notes below before you start with this problem.

Note 1: Although this problem can be solved in three dimensions, for simplicity, consider this problem in one dimension.

Note 2: The potential energy of a negative ion at position x is given by $U(x) = -q_{ion}\phi(x)$, and thus, the number density of the negative ions at x in the presence of the charged particle is given by $n_{neg}(x) = n_0 \exp[-U(x)/(k_B T)] = n_0 \exp[q_{ion}\phi(x)/(k_B T)]$, according to the Boltzmann distribution, where k_B is Boltzmann's constant. On the same token, the number density of the positive ions at x in the presence of the charged particle is given by $n_{pos}(x) = n_0 \exp[-q_{ion}\phi(x)/(k_B T)]$. The total charge density at x will be then given by $\rho(x) = q_{ion} n_{pos} - q_{ion} n_{neg}$. You can use this in conjunction with Poisson's equation to solve for $\phi(x)$.

Note 3: Although the Poisson's equation you set up can be generally solved, for simplicity, assume $q_{ion}\phi(x) \ll k_B T$, that is, the potential energy magnitude $|U|$ of a single ion is much smaller than the thermal energy $k_B T$.