# ES 151 Assignment #3

Professor: Donhee Ham

Date: February 19th, 2015

Due: **12:55pm + 10 min grace period**, February 26th, 2015; slide your work under through the door at Maxwell-Dworkin 131.

**Problem 1 (40pt)**

A coaxial cable whose cross section is shown in Fig. 1 consists of two coaxial cylindrical conductors. There are three dielectric layers between the two conductors. The radii, $a_0$, $a_1$, $a_2$, and $a_3$, are related through $a_k/a_{k-1} = e$ where $k = 1, 2, 3$ and $e$ is the base of the natural logarithm. Also, the permitivities of the dielectrics are given by $\epsilon_1 = 9\epsilon_0$, $\epsilon_2 = 18\epsilon_0$, and $\epsilon_3 = 6\epsilon_0$, where $\epsilon_0$ is the permitivity of free space. Calculate the capacitance per unit length of the coaxial cable.

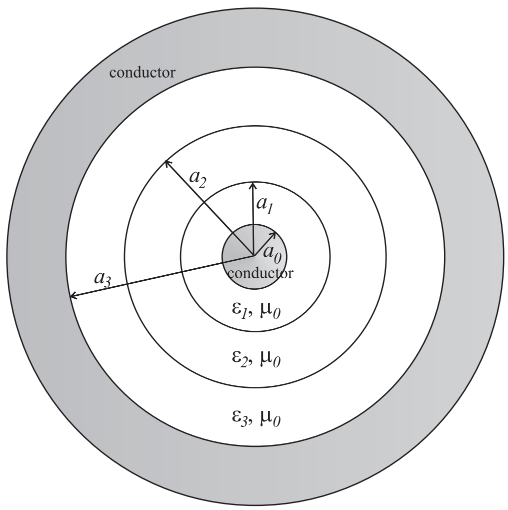


Figure 1: Cross section of a coaxial transmission line

**Problem 2 (60pt)**

(a) Consider a *polarized* dielectric sphere of radius $a$, whose density of polarization $\vec{P}$ is uniform along the $z$-direction. Calculate the electric field $\vec{E}_1$ inside and outside the dielectric sphere, which is generated by the polarization. Sketch the electric field profile. (**Hint**) For the electric field outside the dielectric, can you think of the polarized dielectric as two almost overlapping spheres, with one sphere positively and uniformly charged, and the other sphere negatively and uniformly charged? For the electric field inside the dielectric, you can use the uniqueness theorem: if you find *a* solution to Laplace's equation, it is *the* solution.

(b) Consider placing a sphere of dielectric material with permittivity $\epsilon = K\epsilon_0$ in a uniform electric field $\vec{E}_0$ between two parallel plates of an air-filled capacitor. The dielectric sphere is far from either plate, hence, the *applied* field remains to be $\vec{E}_0$ (charge distributions in the plates are not altered by the dielectric sphere). The *overall* field $\vec{E}$ is not $\vec{E}_0$ anymore; it is the superposition of $\vec{E}_0$ and $\vec{E}_1$, where $\vec{E}_1$ is the electric field produced by the polarization density $\vec{P}$, which is induced by placing the dielectric sphere in the field $\vec{E}_0$:

$$\vec{E} = \vec{E}_0 + \vec{E}_1.$$

At a great distance from the dielectric sphere, $\vec{E}$ will approach $\vec{E}_0$. Assuming that the dielectric sphere becomes uniformly polarized (you *must* check later that your result justifies this assumption, that is, you should check the *self-consistency* of your calculation) and hence using the result from part (a), and also using

$\vec{P} = \chi\epsilon_0\vec{E}$ (note that it should not be $\vec{P} = \chi\epsilon_0\vec{E}_0$), express the electric field inside and outside the dielectric sphere, in terms of $\epsilon$ and $\vec{E}_0$. Also, express the polarization $\vec{P}$ of the dielectric sphere, in terms of $\epsilon$ and $\vec{E}_0$.

**Problem 3 (60pt)**

A uniform electric field $\vec{E}_0$ is set up (in the sense of Lecture Note #10 or Problem #2 above) in a dielectric medium with a dielectric constant of $K$. This dielectric medium has a spherical cavity inside. Calculate the total electric field $\vec{E}$ inside and outside the cavity. Sketch the fields.

**Problem 4 (60pt)**

An uncharged conductor of spherical shape is placed in an uniform electric field $\vec{E}_0$ (again in the sense of Lecture Note #10 or Problem #2 above). Calculate the total electric field $\vec{E}$ outside the spherical conductor. What is the surface charge density distribution on the surface of the conductor?