ES 151 Assignment #4

Professor: Donhee Ham
Date: February 26th, 2015
Due: **12:55pm + 10 min grace period**, March 5th, 2015; slide your work under through the door at Maxwell-Dworkin 131.

Problem 1 (45 pt)

(a) (5 pt) Consider two charged conductors, each of radius R, separated by a distance d ($d > 2R$). One sphere has a charge of Q, and the other a charge of $-Q$. Is the force between the spheres greater than, equal to, or smaller than the force between two *point* charges Q and $-Q$, separated by a distance d?

(b) (10 pt) An electric dipole, $\vec{p}$, is located in a uniform electric field, $\vec{E}_0$, making an angle of θ_0 with the electric field. How much work is required to rotate the dipole by 180° about an axis perpendicular to $\vec{p}$?

(c) (10 pt) What is the potential energy between two parallel electric dipoles ($\vec{p}$ and $\vec{p}$), separated by a distance d? Assume that the vector joining them is perpendicular to the direction of the dipole moments. What about two anti-parallel electric dipoles ($\vec{p}$ and $-\vec{p}$)?

(d) (20 pt) Two conducting spheres, each of radius R, are placed at a distance d from each other ($R \ll d$). A uniform electric field, $\vec{E}_0$, is applied, along the direction perpendicular to the line joining them. Find the force between the two spheres. Is it repulsive or attractive?

Problem 2 (50 pt)

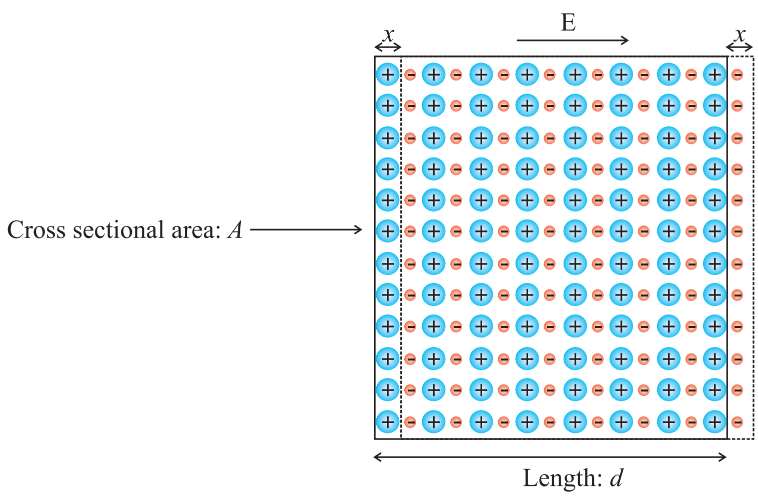

Figure 1:

(a) Consider a bulk rectangular metal, whose cross-sectional area is A, and length is d: Fig. 1. If all the conduction electrons are shifted towards right (along the direction of the length d) by displacement x as seen in the figure, surface charges will develop on both sides of the metal—the left side with positive surface charges (positive background ions fixed in the crystal lattice), and the right side with negative surface charges (conduction electrons). These surface charges will produce a long-range uniform electric field (we ignore fringing fields) across the metal to pull electrons back and thus to restore their original distribution. But electrons won't stop at their original positions due to inertia, rushing to the left. This process repeats itself, leading to collective oscillation of electrons, called *plasma oscillation*. By calculating the restoring electric force on electrons as a function of x and writing down Newton's equation of motion, calculate the plasma oscillation frequency in terms of ϵ_0, conduction electron number density n, electron mass m, and electron charge

e. Ignore electron scattering (which will give rise to the damping of the plasma oscillation) in this calculation.

(b) Repeat part (a), but now taking into account the electron scattering. To this end, you can use the Drude model, with the electron momentum relaxation time of τ. Does τ affect the plasma oscillation frequency? What is the characteristic damping time for the plasma oscillation? If substantial damping occurs before the plasma oscillation completes its one cycle, the plasma oscillation becomes difficult to observe; that is, if the damping rate is really too fast (say 10 (or more) times higher than the oscillation frequency), the plasma oscillation is completely masked by the damping. Do you expect to observe the plasma oscillation in gold?

(c) Estimate the plasma oscillation frequencies in silver and copper.

Problem 3 (45 pt)

A cylindrical hole of radius r is bored parallel to the axis of a cylinder of radius R ($R > r$) as depicted in Fig. 2, where the two axes are at a distance d apart. A current of I flows in the remaining part of the cylinder of radius R (shaded area in the figure) where the current distribution is uniform. Show that the magnitude of the magnetic field at the center of the hole is given by

$$|\vec{B}| = \mu_0 \frac{Id}{2\pi} \frac{1}{R^2 - r^2}$$

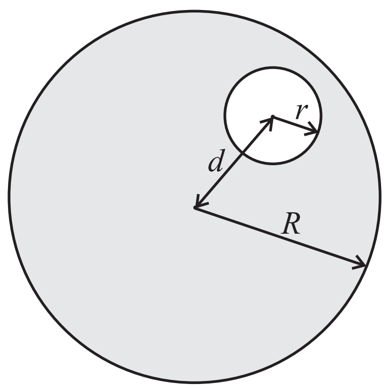

Figure 2: