ES 151 Assignment #5

Professor: Donhee Ham

Date: March 12th, 2015

Due: **12:55pm + 10 min grace period**, March 26th, 2015; slide your work under through the door at Maxwell-Dworkin 131.

Problem 1 (30pt)

The *Helmholtz coil pair* consists of two parallel, coaxial loops, and can provide a region of relatively uniform fields; the uniform-field region is on the axis midway between the loops. Show that the axial magnetic field, expressed as a Taylor series expansion along the axis about the point midway between the coils, will have zero first, second, and third derivatives if the loop radii a are equal to the spacing d of the loops.

Problem 2 (50pt)

(a) A magnetic dipole moment $\vec{m}$ is placed in a region where there is stationary magnetic field $\vec{B}$. You may think of the the magnetic dipole $\vec{m}$ as a small circular (or rectangular, if you like) current loop, whose area multiplied by the current gives the magnitude of $\vec{m}$ (the dipole's direction is perpendicular to the surface of the loop, while satisfying the right-hand rule as discussed in class). As the magnetic dipole is produced from moving charges in the loop, the dipole will experience a force in the magnetic field, in general. Show that this force is given by

$$\vec{F} = \vec{\nabla}(\vec{m} \cdot \vec{B}) = (\vec{m} \cdot \vec{\nabla})\vec{B}$$

(b) Two magnetic dipoles $\vec{m}_1$ and $\vec{m}_2$ are in the same plane; $\vec{m}_1$ is fixed but $\vec{m}_2$ is free to rotate about its center. Show that, in equilibrium, $\tan\theta_1 = -2\tan\theta_2$, where θ_1 and θ_2 are the angles between $\vec{r}$ and $\vec{m}_1$, $\vec{m}_2$, respectively ($\vec{r}$ is the vector displacement between $\vec{m}_2$ and $\vec{m}_1$). Assume that $\vec{m}_1$ and $\vec{m}_2$ are far away from each other enough that you may use the far-field approximation.

Problem 3 (40pt)

A two-wire transmission line consists of a pair of parallel wires of radii a and b separated by $d > a + b$. A current flows down one wire and back the other. It is uniformly distributed over the cross section of each wire (this is a reasonable model at low enough frequencies; at high frequencies, currents are crowded at the "skin" of the wire - we will discuss this later in class). Show that the total self-inductance per unity length for this transmission line is given by

$$L = \frac{\mu_0}{4\pi}\left(1 + 2\ln\frac{d^2}{ab}\right)$$

In deriving this formula, you should take into account the inductance contribution inside the wire (internal inductance).

Problem 4 (40pt)

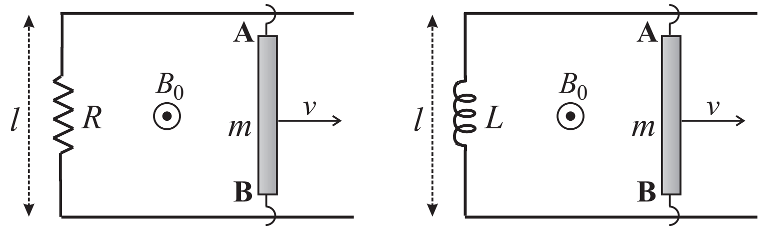
(a) Two small circular loops of wire (of radii a and b) lie in the same plane at center-to-center distance r apart. What is the mutual inductance between the loops if the distance r is sufficiently large that the dipole approximation may be used?

(b) Two small circular current loops are located at a distance r from each other that is sufficiently large so that the dipole approximation may be used. Show how one of the loops should be oriented relative to the other so that their mutual inductance is zero.

Problem 5 (40pt)

(a) A conducting rod **AB** of mass m slides without friction over two long conducting rails separated by a distance l as shown in Fig. 1(a). At the left end the rails are interconnected by a resistance R. The system is located in a uniform magnetic field, B_0, perpendicular to the plane of the loop. At the moment $t = 0$ the rod **AB** starts moving to the right with an initial velocity v_0 due to a sudden instantaneous push. Neglecting the resistances of the rails and the rod **AB**, find the distance covered by the rod until it comes to a standstill. What is the amount of heat generated in the resistance R during this process?

(b) Now in the previous problem, we replace the resistance R with an inductance L as shown in Fig. 1(b). The rod starts moving to the right with an initial velocity of v_0 due to an instantaneous energy injection into the system. Neglecting the resistance of the rails and the rod, show that the rod exhibits an oscillatory motion. Calculate the oscillation frequency.



(a) (b)

Figure 1: