ES 151 Assignment #7

Professor: Donhee Ham

Date: April 2nd, 2015

Due: **12:55pm + 10 min grace period**, April 9th, 2015; slide your work under through the door at Maxwell-Dworkin 131.

Problem 1 (30 pt)

The plates of a parallel-plate capacitor have areas of 10^{-2} m^2 each and are separated by 10^{-2} m. The capacitor is filled with a dielectric material with $\epsilon = 4\epsilon_0$, and the voltage across it is given by $V(t) = 20 \cdot \cos(2\pi \times 10^6 t)$ (volt). Find the amplitude of displacement current.

Problem 2 (60 pt)

Consider an interface between two linear isotropic non-conducting dielectric materials, with electric permittivity given by ϵ_1 and ϵ_2 and magnetic permeability given by μ_1 and μ_2. In class, we derived the interfacial boundary conditions (the relation between the electric fields on one and the other side of the interface; also the relation between the magnetic fields on one and the other side of the interface) in the static case. Now assuming a fully dynamic case, derive such interfacial boundary conditions for the electric and magnetic fields at any given time. As in the static case, you can think of the normal and tangential components separately.

Problem 3 (50 pt)

We discussed in class that the Maxwell's equation containing the Ampere's law and Maxwell's displacement current is given by:

$$\vec{\nabla} \times \vec{H} = \vec{J}_{free} + \frac{\partial \vec{D}}{\partial t} \tag{1}$$

We also noted that this is not the most general form. The most general form of the equation is written as:

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}_{total} + \epsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t} \tag{2}$$

First, justify that the second, displacement current term, on the right hand side of Eq. (2) is correct, by checking the equation's consistency with the charge conservation law. Second, prove that in the linear isotropic material Eq. (2) reduces to Eq. (1), by using the fact that the bound current $\vec{J}_{bound}$ is given by

$$\vec{J}_{bound} = \vec{\nabla} \times \vec{M} + \frac{\partial \vec{P}}{\partial t}, \tag{3}$$

where $\vec{M}$ is the magnetization and $\vec{P}$ is the polarization density (In Assignment #6, Problem #4, we did not include the second term on the right hand side of Eq. (3), because we were looking at the magnetic effect only. But if you include the dielectric effect, the second term should appear as in Eq. (3). Can you explain why the second term is needed?).