ES 151 Assignment #8

Professor: Donhee Ham
Date: April 9th, 2015
Due: **12:55pm + 10 min grace period**, April 16th, 2015; slide your work under through the door at Maxwell-Dworkin 131.

Problem 1 (30pt)

Show that any arbitrary elliptically polarized wave may be broken up into two oppositely rotating circularly polarized components.

Problem 2 (30pt)

Circularly polarized light can be produced by introducing a phase shift of $\pi/2$ between two orthogonal components of linearly polarized light. Such effect can be created using a *doubly refracting* transparent crystal (*e.g.*, calcite or mica). In doubly refracting crystals, the index of refraction differs for different directions of polarization. It is possible to cut a doubly refracting crystal into slabs in such a way that an axis of maximum index n_1 (slow axis) and an axis of minimum index n_2 (fast axis) both lie at right angles to one another in the plane of the slab. The orientation of the slab is defined by the angle θ between the direction of polarization of an incident linearly polarized light and the fast axis of the slab. By choosing $\theta = 45°$, the light entering the slab can be resolved into two orthogonal linearly polarized components of equal amplitude and equal phase. Determine the thickness of the slab which introduces $\pi/2$ phase difference between the two orthogonal components in the emerging light, leading to the circular polarization. The wavelength of the light in the air is given by λ_0.

Problem 3 (150pt)

Let an isotropic dielectric be described as n valence electrons per unit volume, bound to ions harmonically with angular frequency ω_0. Assume that there is no damping in the bound-electron harmonic oscillators; this assumption is only to make analysis simpler, and will not affect the essence of what we want to study here. The dielectric is in a uniform magnetic field B_0 directed along the z-axis. Monochromatic light of frequency ω is sent through the dielectric along the z-direction.

(a) Find the index of refraction for right-circularly polarized light and left-circularly polarized light. The two polarizations should yield two different values of refractive index.

(*Hint*) You can use the same frequency-domain method as we discussed in class, but in relating the polarization density $\vec{P}$ to the electric field $\vec{E}$, you should modify the starting time-domain equation of motion into the following form:

$$\frac{d^2\vec{r}}{dt^2} = -\omega_0^2\vec{r} - \frac{e}{m}\vec{E} - \frac{e}{m}\cdot\frac{d\vec{r}}{dt}\times\vec{B}_0, \tag{1}$$

where the last term accounts for the effect of the uniform magnetic field $\vec{B}_0 = B_0\hat{z}$ (note that we justifiably ignore the weak effect of the magnetic field of the light itself, as we did in class). This equation of motion will relate the polarization density $\vec{P}(\omega) = -ne\vec{r}(\omega)$ and $\vec{E}(\omega) = (E_x(\omega), E_y(\omega), E_z(\omega))$ through a matrix, *i.e.*,

$$\begin{pmatrix} P_x(\omega) \\ P_y(\omega) \\ P_z(\omega) \end{pmatrix} = \begin{pmatrix} A_{11}(\omega) & A_{12}(\omega) & A_{13}(\omega) \\ A_{21}(\omega) & A_{22}(\omega) & A_{23}(\omega) \\ A_{31}(\omega) & A_{32}(\omega) & A_{33}(\omega) \end{pmatrix} \cdot \begin{pmatrix} E_x(\omega) \\ E_y(\omega) \\ E_z(\omega) \end{pmatrix} \tag{2}$$

where $A_{ij}(\omega)$'s are complex functions of ω. Once you establish this matrix relation, the rest of the problem is to couple this response of $\vec{P}$ to $\vec{E}$ to Maxwell's equations

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{3}$$

$$\vec{\nabla} \times \vec{H} = \frac{\partial}{\partial t}(\epsilon_0 \vec{E} + \vec{P}) \tag{4}$$

in order to find the refractive indices for the two different polarizations.

(b) Using the result of (a), show that if the incoming light is linearly polarized, the emerging light will be also linearly polarized, but the plane of polarization will be rotated by a certain angle θ after the light traveled the dielectric whose length in the z-direction is l. Compute θ with the given parameters.