ES 151 Assignment #9

Professor: Donhee Ham
Date: April 30th, 2015
Due: **12:55pm + 10 min grace period**, April 16th, 2015; slide your work under through the door at Maxwell-Dworkin 131.

Problem 1 (30pt)

Estimate the skin depth of copper, aluminum, silver, and gold at 1 gigahertz (GHz).

Problem 2 (70pt)

(a) As illustrated in Fig. 1, a linearly-polarized electromagnetic wave (angular frequency: ω) is incident at an angle of θ to the normal to a perfect conductor, and is entirely reflected from the conductor. The conductor covers the $z > 0$ space (the z-axis points downward), and the region $z < 0$ is air. The plane $z = 0$ is the air-conductor interface. The E-field of the incident wave can be expressed as $\vec{E} = E_0(\cos\theta \cdot \hat{x} - \sin\theta \cdot \hat{z})e^{j(\omega t - \vec{k}\cdot\vec{r})}$ where E_0 is the field magnitude, and $\vec{k} = k(\sin\theta \cdot \hat{x} + \cos\theta \cdot \hat{z})$ is the wavenumber ($k = |\vec{k}|$) of the incident wave. Prove that the reflection angle is the same as the incident angle. Find expressions for the surface charge density $\sigma(x, y, t)$ at $z = 0$ and the surface current density $\vec{J_s}(x, y, t)$ at $z = 0$. Show that these two quantities obey the 2-dimensional charge conservation law:

$$\frac{\partial J_{s,x}}{\partial x} + \frac{\partial J_{s,y}}{\partial y} + \frac{\partial \sigma}{\partial t} = 0 \tag{1}$$

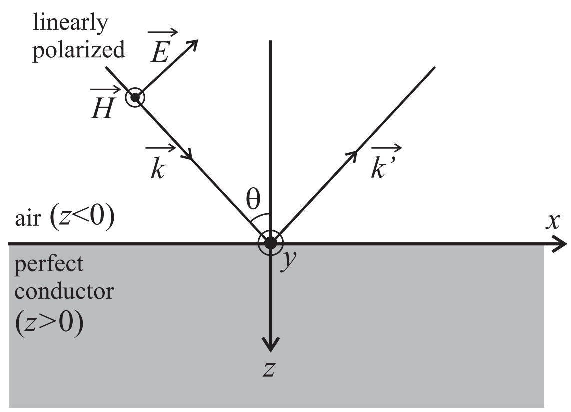

Figure 1:

(b) As shown in Fig. 2, a circularly-polarized wave is incident at angle θ to the normal to the perfect conductor. Show that the reflected wave is still circularly-polarized, but has an opposite handedness, that is, a right-circularly polarized incident wave results in a left-circularly polarized reflected wave, and vice versa.

Problem 3 (70pt)

Consider a quarter-wavelength dielectric layer of intrinsic impedances η_2 between dielectrics of intrinsic impedances η_1 and η_3 [Fig. 3]. Consider normal incidence of electromagnetic waves.

(a) Show that the 'overall' reflection coefficient, $\rho_{overall}$, of a wave incident on the boundary 1 is

$$\rho_{overall} = \frac{\eta_2^2 - \eta_1\eta_3}{\eta_2^2 + \eta_1\eta_3} \tag{2}$$

by evaluating the effective load impedance, Z_{in}, looking to the right on boundary 1 [Fig. 3], and using the reflection coefficient formula, $\rho_{overall} = (Z_{in} - \eta_1)/(Z_{in} + \eta_1)$. Equation (2) shows that for $\eta_2 = \sqrt{\eta_1\eta_3}$, there

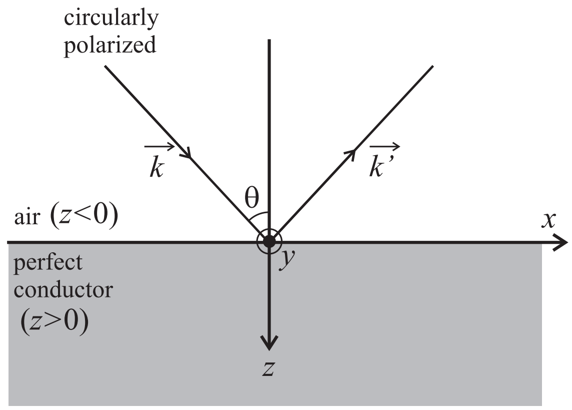

Figure 2:

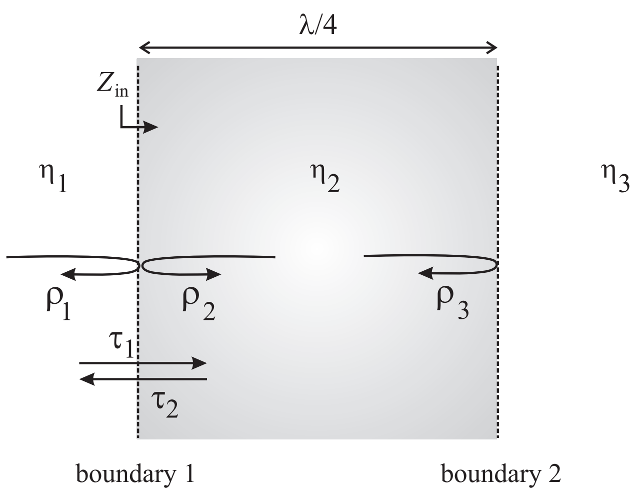

Figure 3:

is no reflection and impedance matching is achieved.

(b) In Fig. 3, imagine an electromagnetic wave traveling down the 1st dielectric towards the 2nd dielectric. When the wave first hits boundary 1, it sees only impedance η_2 since it has not yet traveled to the 3rd dielectric. Part of the wave is reflected with coefficient ρ_1, and the rest is transmitted onto the 2nd dielectric with coefficient τ_1. The transmitted wave then travels $\lambda/4$ in the 2nd dielectric towards the 3rd dielectric, is reflected with coefficient ρ_3, and travels another $\lambda/4$ back to boundary 1. Part of this wave is transmitted through to the 1st dielectric, with coefficient τ_2, and the rest is reflected back towards the 3rd dielectric with coefficient ρ_2. We summarize these 'local' reflection and transmission coefficients as follows:

- ρ_1: 'local' reflection coefficient of a wave incident on the 2nd dielectric from the 1st dielectric.
- ρ_2: 'local' reflection coefficient of a wave incident on the 1st dielectric from the 2nd dielectric.
- ρ_3: 'local' reflection coefficient of a wave incident on the 3rd dielectric from the 2nd dielectric.
- τ_1: 'local' transmission coefficient of a wave incident from the 1st dielectric into the 2nd dielectric.
- τ_2: 'local' transmission coefficient of a wave incident from the 2nd dielectric into the 1st dielectric.

Express ρ_1, ρ_2, ρ_3, τ_1, and τ_2 in terms of the intrinsic impedances, η_1, η_2, and η_3. The multiple reflection and transmission process continues with an infinite number of bouncing waves, and the 'overall' reflection coefficient, $\rho_{overall}$, at boundary 1, which we calculated in (a), can be thought of as the sum of all of the partial reflections back into the 1st dielectric. Noting that each round trip path up and down the 2nd medium results in a 180 degree phase shift (explain why), show that the overall reflection coefficient can be expressed as

$$\rho_{overall} = \frac{\rho_1 + \rho_1\rho_2\rho_3 - \tau_1\tau_2\rho_3}{1 + \rho_2\rho_3} \tag{3}$$

Show that Equation (3) is equivalent to Equation (2).

Problem 4 (50pt)

Consider two quarter-wavelength dielectric layers of intrinsic impedances η_2 and η_3 between dielectrics of intrinsic impedances η_1 and η_4. Consider normal incidence. Show that perfect matching occurs if

$$\frac{\eta_2}{\eta_3} = \sqrt{\frac{\eta_1}{\eta_4}}$$

For $\eta_4 : \eta_3 : \eta_2 : \eta_1 = 4 : 3 : 1.5 : 1$, calculate reflection coefficient at a frequency 10% below that for perfect matching.