Quiz 1 Solution Set

(10) 1. Half credit is awarded for recognition of the correct "ballpark" (underlined terms).

(2) a) essential prime implicant - a grouping in a Karnaugh map that covers at least one minterm not covered by any other grouping in the map. e.g., $F = AB + BC + \bar{A}C$: AB and $\bar{A}C$ are essential prime implicants.

(2) b) checkerboard pattern - a pattern in a Karnaugh map that suggests an XOR (or XNOR) function.

(2) c) C_{16} - a hexadecimal (base 16) digit equal to 12 in decimal.

(2) d) excitation table - a truth table for flip-flops indicating the inputs necessary to get from a given current state to a desired next state.

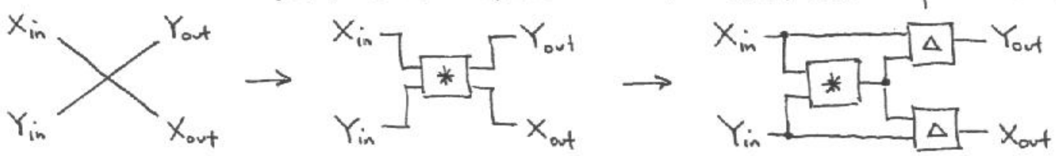
(2) e) set-up time - the minimum time that the input of an edge-triggered flip-flop must remain stable before the edge of the clock.

(15) 2. a) With no additional constraints, this is trivial because $X_{out} = X_{in}$ and $Y_{out} = Y_{in}$ — circuit is just wires (1)

b) Full credit for correct diagram; up to half credit for attempt with work shown; no credit if only incorrect diagram (14)

One approach: start from simplest possibility and work constraints in:

we want: — but no crossovers, so: — but X_{out}, Y_{out} independent, so:



truth table is symmetric in X, Y so expect circuit to be symmetric

So we have $X_{out} = (X_{in} * Y_{in}) \Delta Y_{in}$, $Y_{out} = (X_{in} * Y_{in}) \Delta X_{in}$ where $*, \Delta$ are $\cdot, +,$ or $\oplus$

if $*$ and Δ are the same, we find $X_{out} = (X_{in} \oplus Y_{in}) \oplus Y_{in} = X_{in} \oplus (Y_{in} \oplus Y_{in}) = X_{in} \oplus 0 = X_{in}$ works (similarly for Y_{out})

Another approach: work mechanically and see if that suggests anything:

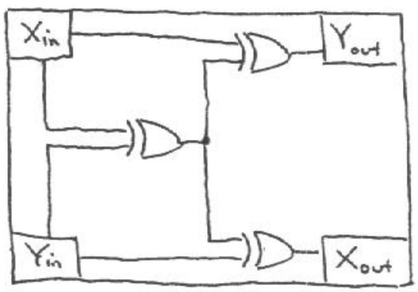
$X_{out} = X_{in}\bar{Y}_{in} + X_{in}Y_{in}$, $Y_{out} = \bar{X}_{in}Y_{in} + X_{in}Y_{in}$ (from truth table)

but inversion not possible, except using XOR: $X \oplus Y = X\bar{Y} + \bar{X}Y$, $X \oplus 0 = X$, $X \oplus 1 = \bar{X}$, $X \oplus X = 0$

this suggests a possibility: $X \oplus Y \oplus Y = X$

so we get $X_{out} = X_{in} \oplus Y_{in} \oplus Y_{in} = X_{in} \oplus 0 = X_{in}$ and similarly for Y_{out}, as above.

Solution:



(15) 3. Up to 2/3 credit awarded for a good try. Up to 3 points for a nonalgebraic proof.

Solution 1:

$$\begin{aligned}
LHS &= \bar{A}\bar{B}\bar{C}D + \bar{A}BD + ABC + A\bar{B}C\bar{D} \\
&= \bar{A}\bar{B}\bar{C}D + (\bar{A}BCD + \bar{A}B\bar{C}D) + (ABCD + ABC\bar{D}) + A\bar{B}C\bar{D} \\
&= \bar{A}\bar{C}D(B+\bar{B}) + BCD(A+\bar{A}) + AC\bar{D}(B+\bar{B}) \\
&= \bar{A}\bar{C}D + BCD + AC\bar{D} \quad = RHS.
\end{aligned}$$

Solution 2:

use these results:

$$\begin{aligned}
X + \bar{X}Y &= (XY + X\bar{Y}) + \bar{X}Y \\
&= (XY + XY) + X\bar{Y} + \bar{X}Y \\
&= (XY + X\bar{Y}) + Y(X + \bar{X}) \\
&= X + Y
\end{aligned}$$

$$\begin{aligned}
XY + \bar{X}Z &= (XYZ + XY\bar{Z}) + (\bar{X}YZ + \bar{X}\bar{Y}Z) \\
&= YZ(X+\bar{X}) + XY\bar{Z} + \bar{X}\bar{Y}Z \\
&= YZ + XY\bar{Z} + YZ + \bar{X}\bar{Y}Z \\
&= Y(Z + X\bar{Z}) + Z(Y + \bar{X}\bar{Y}) \\
&= Y(Z + X) + Z(Y + \bar{X}) \\
&= XY + YZ + \bar{X}Z \qquad \text{(consensus theorem)}
\end{aligned}$$

$$\begin{aligned}
LHS &= \bar{A}\bar{B}\bar{C}D + \bar{A}BD + ABC + A\bar{B}C\bar{D} \\
&= \bar{A}D(B + \bar{B}\bar{C}) + AC(B + \bar{B}\bar{D}) \\
&= \bar{A}D(B + \bar{C}) + AC(B + \bar{D}) \\
&= \bar{A}\bar{C}D + \bar{A}BD + ABC + AC\bar{D} \\
&= \bar{A}\bar{C}D + \bar{A}BD + BDBC + ABC + AC\bar{D} \qquad \text{(consensus theorem)} \\
&= \bar{A}\bar{C}D + \bar{A}BDD + BCD + BCD + ABCC + AC\bar{D} \qquad \text{(consensus theorem)} \\
&= \bar{A}CD + BCD + AC\bar{D} \quad = RHS.
\end{aligned}$$

(15) 4. Function must be minimal SOP expression.

The easiest way to fill the Karnaugh map is to fill in the zeroes since the function is given in product of sums form. When any of the terms is zero, the whole function is zero, so fill in the boxes corresponding to when each of the terms is zero:

$\bar{X}+\bar{Y}=0$ when $X=Y=1$

$\bar{W}+\bar{X}+Y=0$ when $W=X=1$ and $Y=0$

$\bar{W}+X+Z=0$ when $X=Z=0$ and $W=1$

All other cells must be 1 since there are no don't care states.

XY \ WZ	00	01	11	10
00	1	1	1	0
01	1	1	1	0
11	0	0	0	0
10	1	1	0	0

(10 for correct map, up to 4 if a good try except if there is a correct truth table)

$F=\bar{W}\bar{Y}+\bar{W}\bar{X}+\bar{X}Z$

(5 for correct expression, 4 if it follows from an incorrect map but must be minimal SOP, 2 if logically equivalent but not minimal SOP)

e.g.,
$F=\Sigma(0,1,3,4,5,7,8,9)$ gets 0
$F(X,Y,W,Z)=\Sigma(0,1,3,4,5,7,8,9)$ gets 2

truth table (not required):

X	Y	W	Z	F
0	0	0	0	1
0	0	0	1	1
0	0	1	0	0
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

(up to 6 for correct truth table only if Karnaugh map is incorrect or missing)

(15) 5.

(10)

A	B	D	C	F	
0	0	0	0	0	F=0
0	0	0	1	0	
0	0	1	0	1	F=1
0	0	1	1	1	
0	1	0	0	1	F=$\bar{C}$
0	1	0	1	0	
0	1	1	0	0	F=C
0	1	1	1	1	
1	0	0	0	0	F=C
1	0	0	1	1	
1	0	1	0	1	F=$\bar{C}$
1	0	1	1	0	
1	1	0	0	1	F=1
1	1	0	1	1	
1	1	1	0	0	F=0
1	1	1	1	0	

or

$\bar{C}$	0	(1)	(2)	3	4	(5)	(6)	7
C	8	(9)	10	(11)	(12)	13	(14)	15
	0	1	$\bar{C}$	C	C	$\bar{C}$	1	0

or

A	B	C	D	F
0	0	0	0	0
0	0	0	1	1
0	0	1	0	0
0	0	1	1	1
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	1
1	1	0	1	0
1	1	1	0	1
1	1	1	1	0

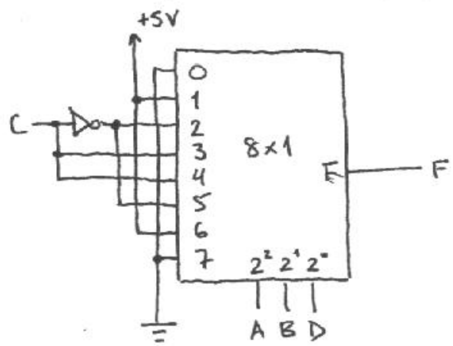
$F = \bar{A}\bar{B}\bar{D}I_0 + \bar{A}\bar{B}DI_1 + \bar{A}B\bar{D}I_2 + \bar{A}BDI_3 + A\bar{B}\bar{D}I_4 + A\bar{B}DI_5 + AB\bar{D}I_6 + ABDI_7$

$I_0 = 0$
$I_1 = 1$
$I_2 = \bar{C}$
$I_3 = C$
$I_4 = C$
$I_5 = \bar{C}$
$I_6 = 1$
$I_7 = 0$

optional only if diagram is correct

(up to 3 only, if variables mixed up or wrong variables used for select lines)

(5)

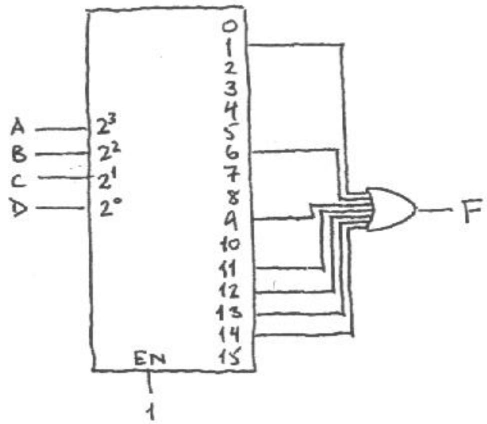


15 if first part omitted but only if diagram is completely logically correct
5 if follows correctly from incorrect first part
−2 if junctions not shown properly as +
−2 if any non-NOT gate used

(15) 6. $F = A\bar{B}CD + AB\bar{C}D + AB\bar{D} + \bar{A}BC\bar{D} + \bar{B}\bar{C}D$

$= \sum(1, 6, 9, 11, 12, 13, 14)$ (up to 5, only if diagram missing)

(may use a Karnaugh map)

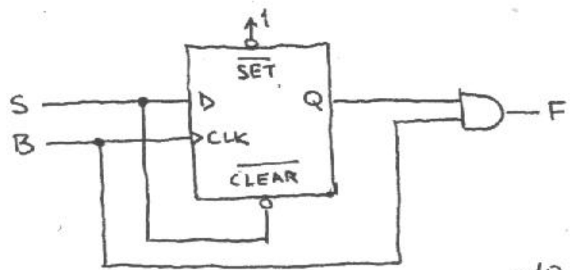


−2 for more gates used
−3 if DMUX used

(15) 7. a) The circuit has this behaviour:

		if not implemented:
neither seated nor buckled	: F = 0	−10
seated, then buckled	: F = 1	−5
seated, then buckled, then unbuckled	: F = 0	−3
then buckled again	: F = 1	−1
buckled, then seated	: F = 0	−5

(13)



−10 if not a D flip-flop
−5 if not positive edge triggered
−2 if either preset or preclear is active high
−4 if works but more than one gate

(2) b) One could cheat by buckling the seatbelt behind oneself.
Full credit (up to 2) if circuit wrong but analysis of shortcomings correct.