Design Basics for Combinational Logic

first step: Usally a word statement specifying what outputs for what inputs.

2nd step: Construct TT from statement. If there are any ambiguities in statement resolve them at this point.

3rd step: Develop "recipe" for the design by constructing either of two "standard forms" which are called the "Canonical Forms" (AKA transmission function).

Canonical form #1:

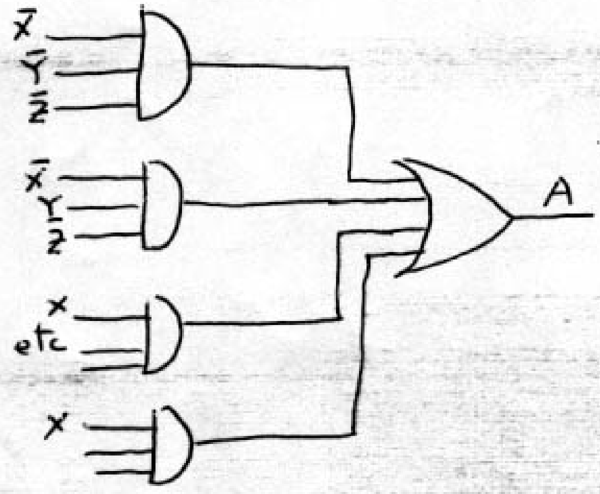
Rationale is to identify those input states which are to yield a 1 output, develop each of them in an AND gate and then sum.

input			out	
X	Y	Z	A	Product Term
0	0	0	1	$\bar{X}\,\bar{Y}\,\bar{Z}$
0	0	1	0	
0	1	0	1	$\bar{X}\,Y\,\bar{Z}$
0	1	1	0	
1	0	0	1	$X\,\bar{Y}\,\bar{Z}$
1	0	1	0	
1	1	0	1	$X\,Y\,\bar{Z}$
1	1	1.	0	

List product term for each entry where A=1. Product term has variable if it occurs as a 1, has the $\overline{\text{variable}}$ if it occurs as a 0. Then form sum:

so: $A = \bar{X}\bar{Y}\bar{Z} + \bar{X}Y\bar{Z} + X\bar{Y}\bar{Z} + XY\bar{Z}$

This the recipe for the circuit:



Called the "AND to OR" implementation

A here is called the "Sum of Products" or "SOP" Canonical Form.

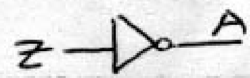
Circuit above is correct and would work when built. However:

Note: $A = \bar{X}\bar{Z}(\underbrace{\bar{Y}+Y}_{1}) + X\bar{Z}(\underbrace{\bar{Y}+Y}_{1})$

$$= \bar{X}\bar{Z} + X\bar{Z}$$

$$= \bar{Z}(\bar{X}+X)$$

so $A = \bar{Z}$ ckt is



moral: it often pays to see if expression has any obvious simplification.

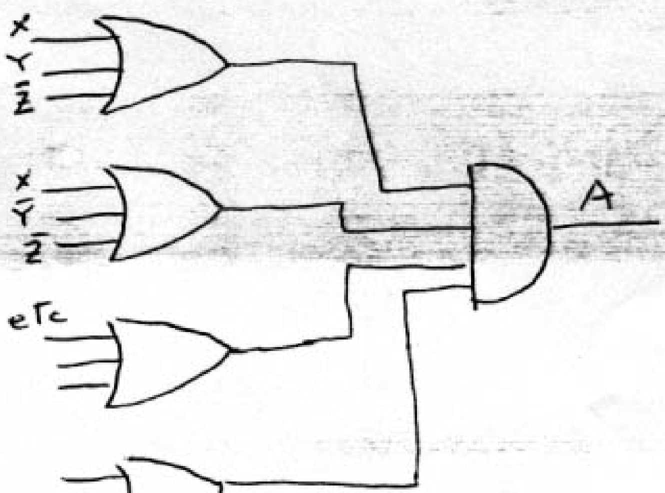
Canonical form #2

Identify input states which are to yield a 0 output. For each, form sum of input variables - uncomplemted if entry is a 0, complemented if entry is a 1. Then form product of each such sum term.

X	Y	Z	A	Sum Term
0	0	0	1	
0	0	1	0	$X + Y + \bar{Z}$
0	1	0	1	
0	1	1	0	$X + \bar{Y} + \bar{Z}$
1	0	0	1	
1	0	1	0	$\bar{X} + Y + \bar{Z}$
1	1	0	1	
1	1	1	0	$\bar{X} + \bar{Y} + \bar{Z}$

so: $A = (X + Y + \bar{Z})(X + \bar{Y} + \bar{Z})(\bar{X} + Y + \bar{Z})(\bar{X} + \bar{Y} + \bar{Z})$

This is recipe for circuit:



Called the "OR to AND" implementation.

A here is the "Canonical Product of Sums" or POS Form.

Home exercise: Show A here again reduces to $A = \bar{Z}$

Additional nomenclature:

A "literal" is the appearance of a variable in either true or complemented form.

1. For SOP form, each term is called a "min-term" and corresponds to a row in the TT.

   Given a SOP write the literals in each term in same order. Assign binary digit to each literal, 1 if appears uncomplemented, 0 if appears complemented

   here:

$$A = \underbrace{\bar{X}\bar{Y}\bar{Z}}_{\substack{0\,0\,0 \\ m_0}} + \underbrace{\bar{X}Y\bar{Z}}_{\substack{0\,1\,0 \\ m_2}} + \underbrace{X\bar{Y}\bar{Z}}_{\substack{1\,0\,0 \\ m_4}} + \underbrace{XY\bar{Z}}_{\substack{1\,1\,0 \\ m_6}} \leftarrow \text{the min-terms}$$

0, 2, 4, 6 = dec. equiv.

hence express $A = \sum m(0, 2, 4, 6)$ in min-term form

2. For POS form: assign binary digit to each literal in each sum term — 0 if appears uncomplemented, 1 if appears complemented

   here:

$$A = \underbrace{(X + Y + \bar{Z})}_{\substack{0\,0\,1 \\ M_1}}\underbrace{(X + \bar{Y} + \bar{Z})}_{\substack{0\,1\,1 \\ M_3}}\underbrace{(\bar{X} + Y + \bar{Z})}_{\substack{1\,0\,1 \\ M_5}}\underbrace{(\bar{X} + \bar{Y} + \bar{Z})}_{\substack{1\,1\,1 \\ M_7}}$$

these are "Max-terms" $\rightarrow$

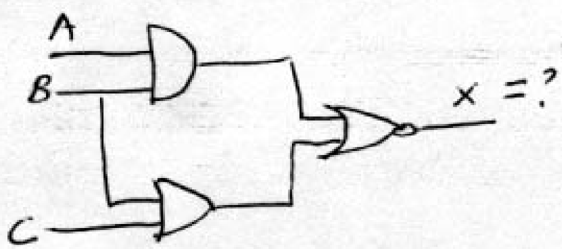
hence express $A = \prod M(1, 3, 5, 7)$ in Max-term form $\qquad \prod \Rightarrow$ continuous product

Summarize $\rightarrow$

row #	X	Y	Z	A	SOP product terms	POS sum terms	min terms	Max terms
0	0	0	0	1	$\bar{X}\bar{Y}\bar{Z}$		m_0	
1	0	0	1	0		$X+Y+\bar{Z}$		M_1
2	0	1	0	1	$\bar{X}Y\bar{Z}$		m_2	
3	0	1	1	0		$X+\bar{Y}+\bar{Z}$		M_3
4	1	0	0	1	$X\bar{Y}\bar{Z}$		m_4	
5	1	0	1	0		$\bar{X}+Y+\bar{Z}$		M_5
6	1	1	0	1	$XY\bar{Z}$		m_6	
7	1	1	1	0		$\bar{X}+\bar{Y}+Z$		M_7

Now have basic tools for both analysis and synthesis of combinational logic circuits:

Analysis straight forward, e.g.



$x = \overline{A\cdot B + (B+C)}$ directly, but good idea to try to simplify

$$x = (\overline{A\cdot B})(\overline{B+C}) = (\bar{A}+\bar{B})(\bar{B}\bar{C})$$
$$= \bar{A}\bar{B}\bar{C} + \bar{B}\bar{B}\bar{C}$$
$$= \bar{A}\bar{B}\bar{C} + \bar{B}\bar{C}$$
$$= \bar{B}\bar{C}(\bar{A}+1)$$
$$= \bar{B}\bar{C}$$