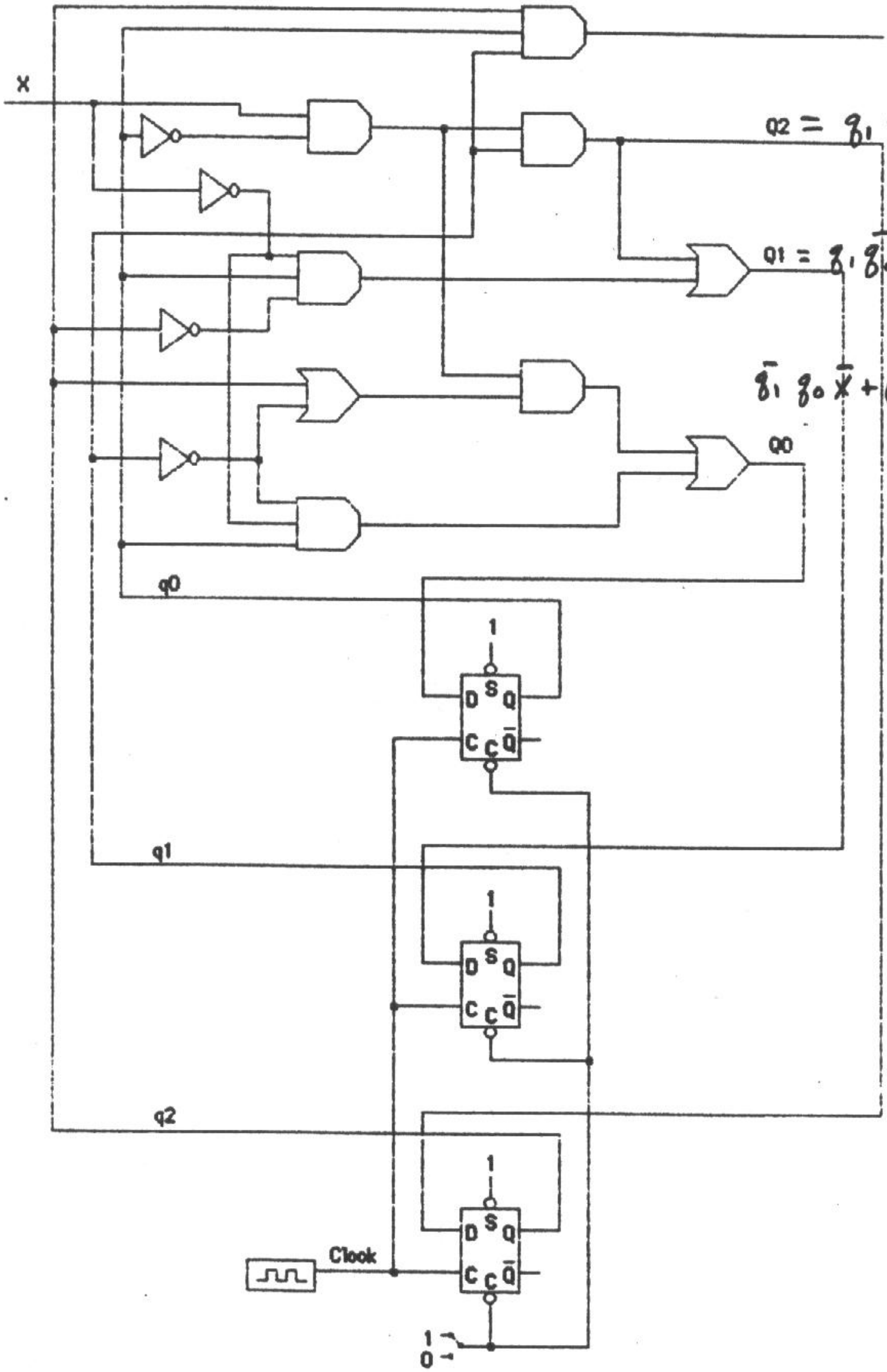


$z = q_1 q_2 q_0$

$Q2 = q_1 \bar{q}_0 X$

$Q1 = q_1 \bar{q}_0 x + \bar{q}_2 q_0 \bar{x}$

$\bar{q}_1 q_0 \bar{x} + (q_2 + \bar{q}_1) x \bar{q}_0 =$

$\bar{q}_1 q_0 \bar{x} +$

$q_2 \bar{q}_0 x +$

$\bar{q}_1 \bar{q}_0 x$

FSM's

Generic architecture previously covered.

Analysis of a FSM. Do example of this - synthesis is ~ reverse procedure.

Refer to ckt: note drawn to correspond to layout of a generic SM - would be drawn more L to R by a draftsman.

From diagram can write expressions:

$Z = q_1 q_2 q_0$ = sole primary output, independent of x so this is a Moore machine.

For excitation eqns (next state determining) get:

$$Q_2 = q_1 \bar{q}_0 x$$
$$Q_1 = q_1 \bar{q}_0 x + \bar{q}_2 q_0 \bar{x}$$
$$Q_0 = \bar{q}_1 q_0 \bar{x} + (q_2 + \bar{q}_1) x \bar{q}_0$$
$$= \bar{q}_1 q_0 \bar{x} + q_2 \bar{q}_0 x + \bar{q}_1 \bar{q}_0 x$$

Note that this ckt uses D flip-flops. Hence the present Q excitations will become the flip-flop outputs (q's) on the clock tick. This simplifies analysis. If other type (i.e. J-K) of FF were used things would be a bit harder.

Have enough to construct a "state table" which is of form:

PS	NS: here is same as present ex. (using D's)					
	for $x = 0$			for $x = 1$		
$q_2\ q_1\ q_0$	Q_2	Q_1	Q_0	Q_2	Q_1	Q_0
0 0 0	0	0	0	0	0	1

gotten from $Q_2 = q_1 \bar{q}_0 x = 0\,1\,0 = 0$ and $Q_0 = \bar{q}_1 q_0 \bar{x} + q_2 \bar{q}_0 x + \bar{q}_1 \bar{q}_0 x$

$= 1\,0\,0 \quad 0\,1\,1 + 1\,1\,1$

$= 1$

etc for all entries

Get for table: (State table)

Present State PS				Next State (= pres. ex., using D's) $x=0$			$x=1$		
	q_2	q_1	q_0	Q_2	Q_1	Q_0	Q_2	Q_1	Q_0
(A)	0	0	0	0	0	0	0	0	1
(B)	0	0	1	0	1	1	0	0	0
(C)	0	1	1	0	1	0	0	0	0
(D)	0	1	0	0	0	0	1	1	0
(E)	1	1	0	0	0	0	1	1	1
(F)	1	1	1	0	0	0	0	0	0 ← note back to beginning for either $x=0,1$
(G)	1	0	1	0	0	1	0	0	0
(H)	1	0	0	0	0	0	0	0	1

$\left.\right\}$ remaining states

Note: not showing output since so simple: $Z = q_1 \bar{q}_0 X$
Each state uniquely identified by specifying the $q_2 q_1 q_0$ values.

Now create a "state transition" table which has same information but labels each state with a letter A, B ....
So here we have assigned

state $q_2 q_1 q_0$	symbol (i.e. letter)
0 0 0	A
0 0 1	B

etc

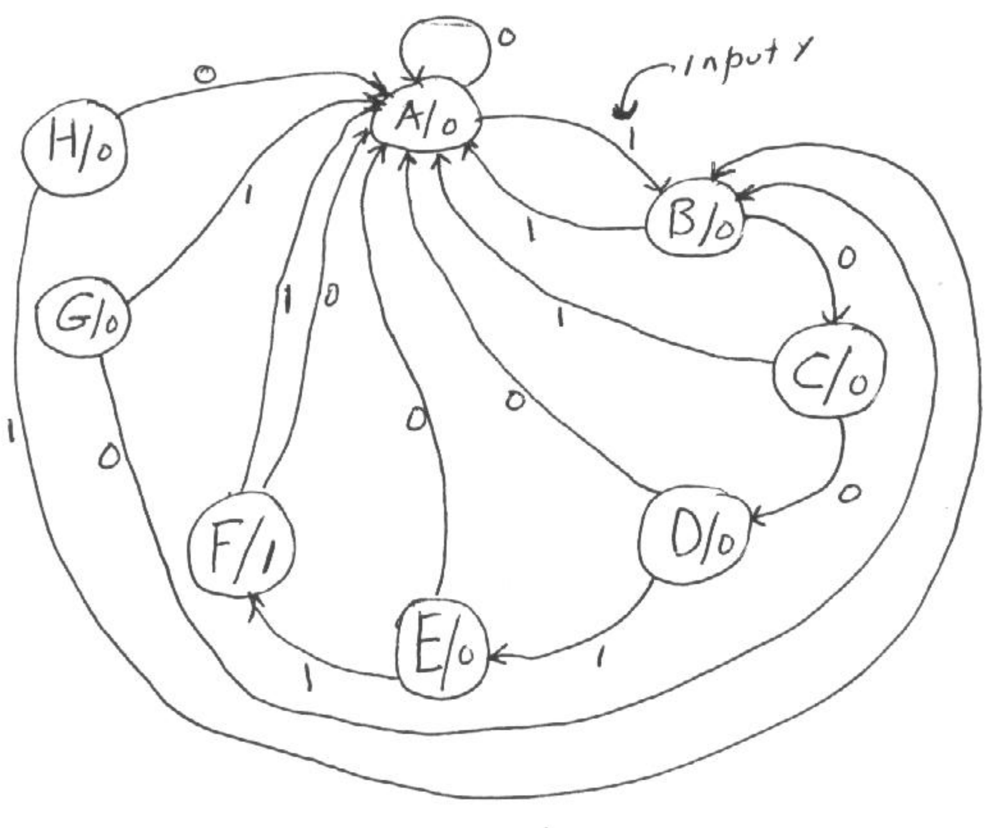
The assignment of what letter to what state not always obvious.

state transition table

PS	x = 0 NS/output	x = 1 NS/output
A	A / 0	B / 0
B	C / 0	A / 0
C	D / 0	A / 0
D	A / 0	E / 0
E	A / 0	F / 0
F	A / 1	A / 1
G	B / 0	A / 0
H	A / 0	B / 0

Note could list outputs (for x=0, 1) in two other columns. Also could omit "0" output & only list "1". etc: point is can have different forms for the table

It's easier to visualize the operation by transferring the state transition table to a state flow graph



Common practice is to include output in the state ○ since this is a Moore machine and output is f(state) only. Also commo to omit "0" for 0 output

Note that to get a $Z=1$ output a particular sequence of input values is required; starting from initial state A

$$F \xleftarrow{1} E \xleftarrow{1} D \xleftarrow{0} C \xleftarrow{0} B \xleftarrow{x=1} A$$

output $Z=1$

So x sequence detected is $x = 10011$

∴ This SM is a sequence detector.

Note that to detect next occurrence of the $x=10011$ sequence need to ensure that we start in state A again. This can be done if we require the input to come in batchs of 5 bits followed by a minimum numbers of zeroes. State which requires most 0's to get A is G which requires $G \xrightarrow[0]{} B \xrightarrow[0]{} C \xrightarrow[0]{} D \xrightarrow[0]{} A$ i.e. 4 0's preceding the first bit of sequence to be checked.

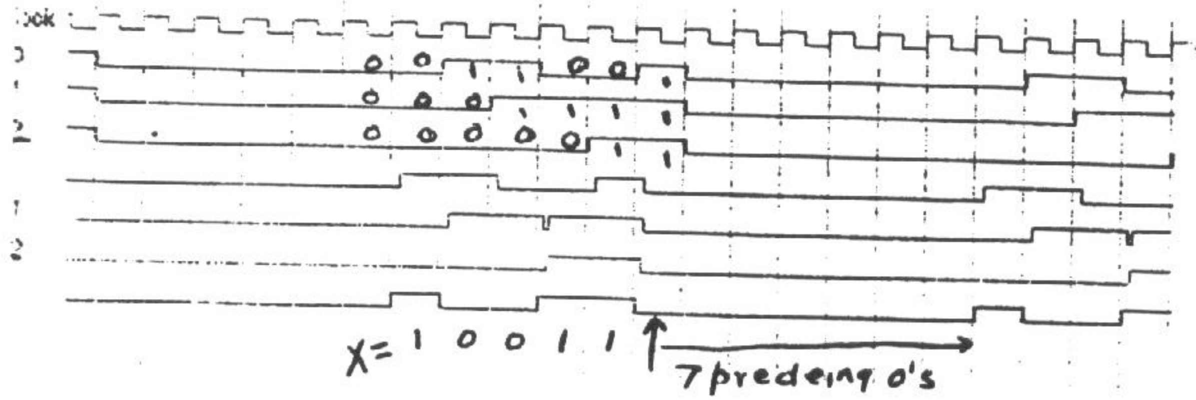
Note that if initial state was not A would get e.g.

initial $B \xrightarrow{1} A \xrightarrow{0} A \xrightarrow{0} A \xrightarrow{1} B \xrightarrow{1} \underset{\text{no output}}{A}$

(As home exercise check as above starting in other states including G and H).

So analysis procedure starts with circuit, then:

1. Get excitations
2. Get state table
3. Get state transition table
4. Get state flow graph
5. Deduce function implemented



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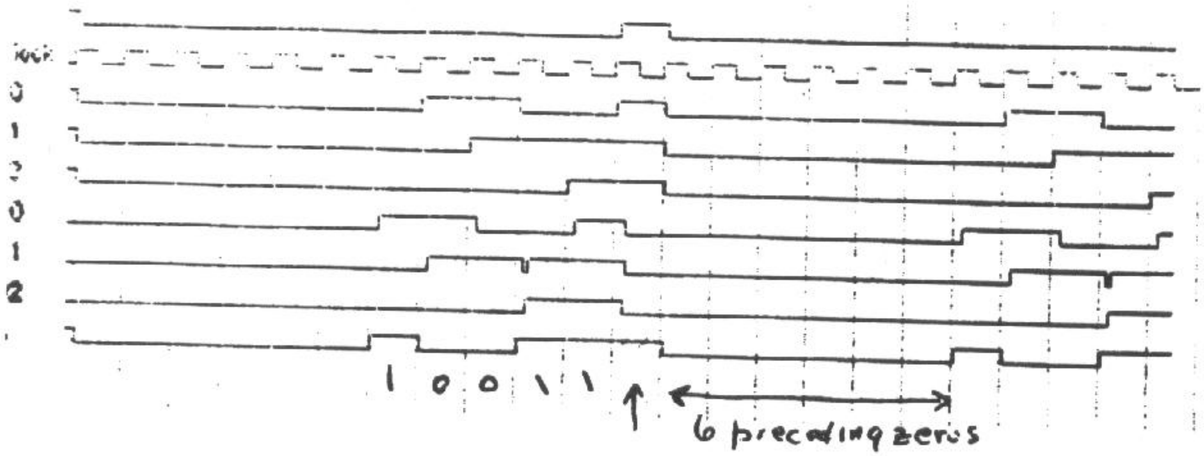

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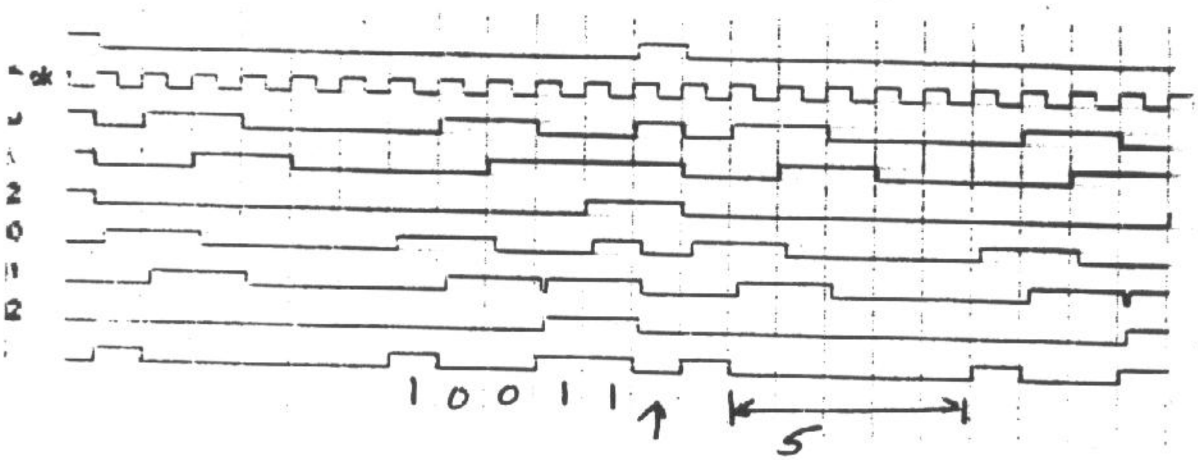

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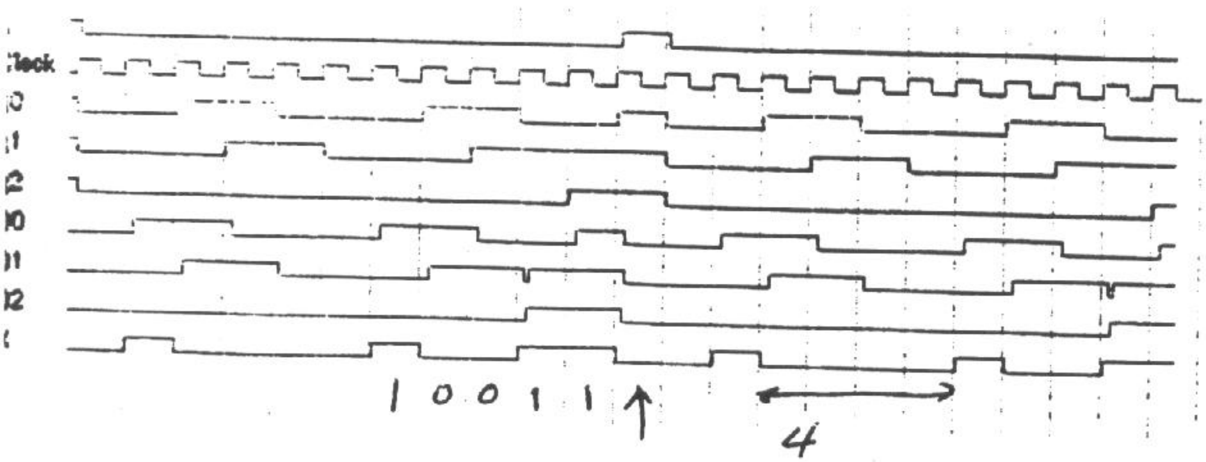

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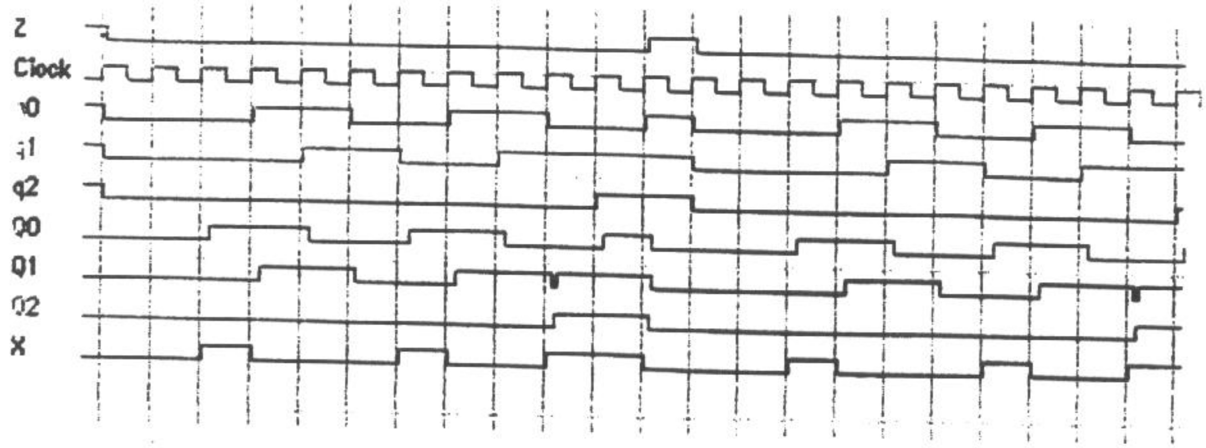
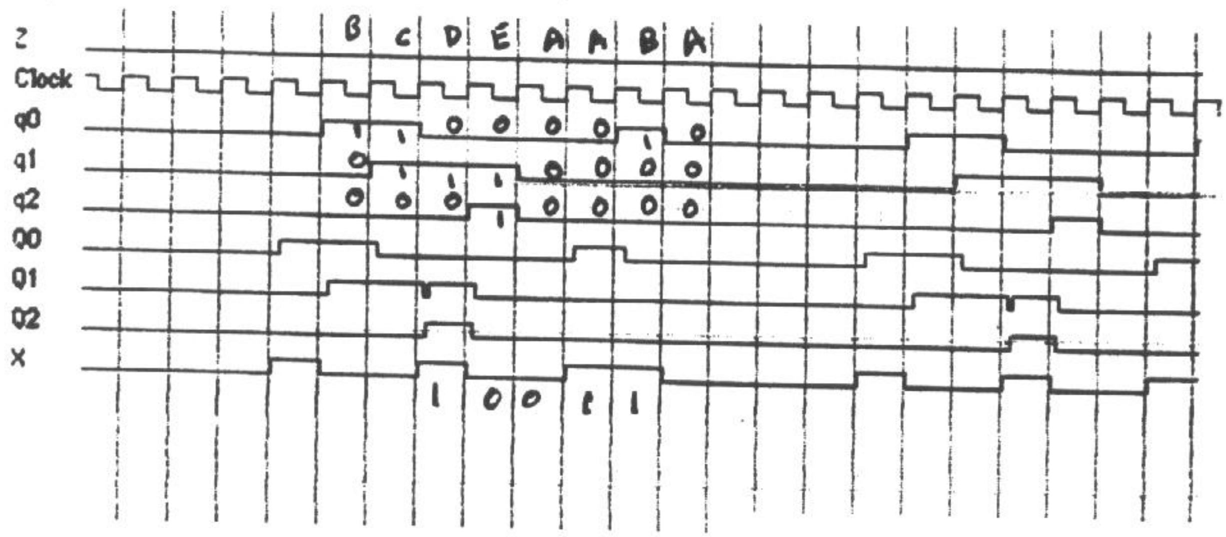
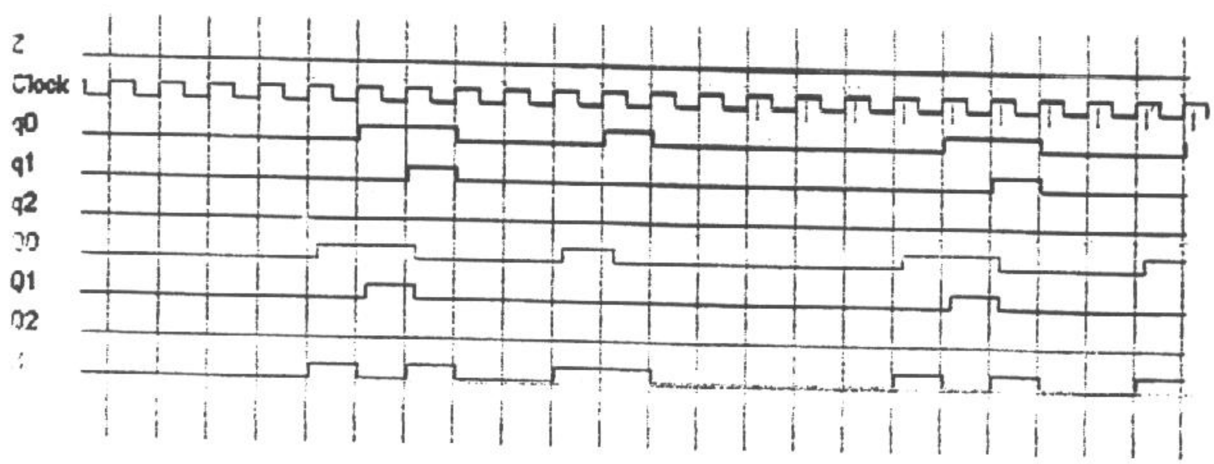

Z
Clock
q0
q1
q2
Q0
Q1
Q2
X

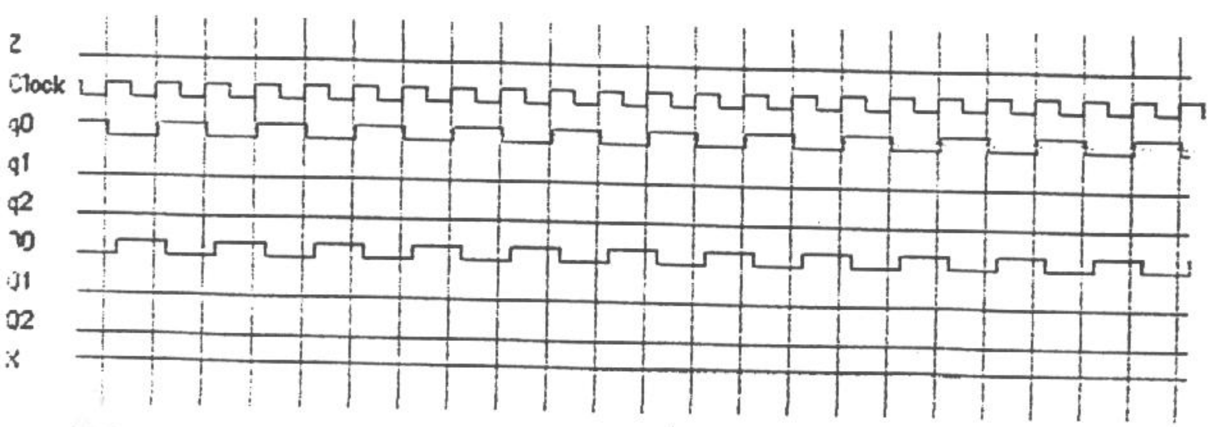
3


Z
Clock
q0
q1
q2
Q0
Q1
Q2
X
B C D E A A B A
1 0 0 1 1

2


Z
Clock
q0
q1
q2
Q0
Q1
Q2

1


Z
Clock
q0
q1
q2
Q0
Q1
Q2
X

Synthesis procedure:

1. Word statement of problem

2. Develop state flow graph – helps to visualize things.

3. Develop state transition table – letters or numbers denote states.

4. Develop state table – requires making state assignment (states uniquely identified in terms of FF outputs).

5. Develop excitation equations for the FF's. (Easiest if use D type FF, make use of excitation tables if use other types.)

(By this point have decided on # and type of FF's to be used.)

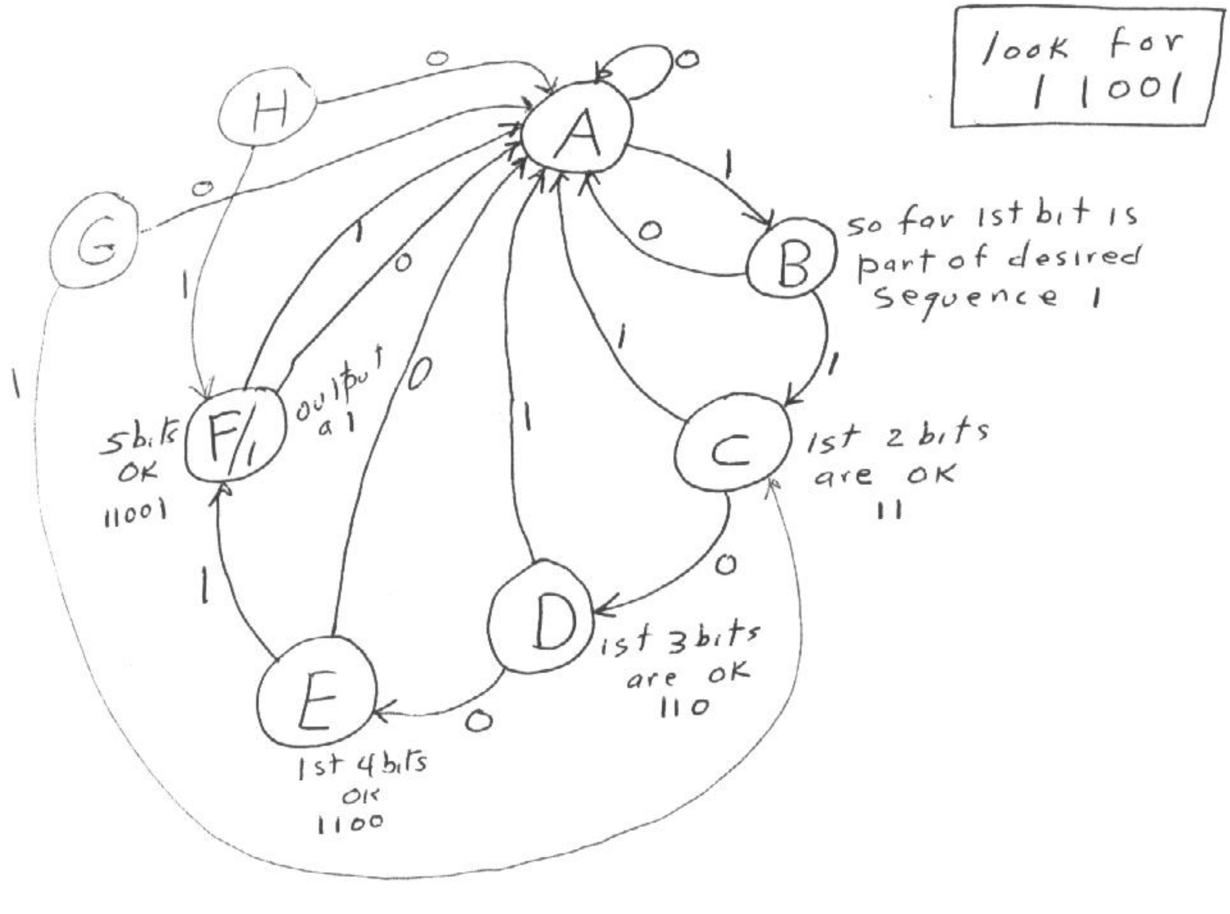
6. Generate circuit from excitation eqns.

7. Helps to run a simulation to check operation.

FSM - Synthesis procedure

Mostly just opposite of analysis with an additional step. As example - design SM to detect sequence X = 11001 (opposite of previous example).

1. Start in reset and detect (non-overlapping) sequence ⇒ 6 states ⇒ eventually need 3FF with 2 extra states. Can decide type of FF later.
2. Translate requirement into a state flowgraph: at this point states represented by letters.



At this point there are two unused (i.e. don't care) states. What happens if somehow end up in one of them determined later.

Next step

3. Translate state flow diagram into a state transition table

PS	NS/pres. out $x=0$	NS/pres. out $x=1$
A	A/0	B/0
B	A/0	C/0
C	D/0	A/0
D	E/0	A/0
E	A/0	F/0
F	A/1	A/1
G	d	d
H	d	d

don't care for now

4. State transition table now converted to a state table. Requires making state assignment : decide on using D FF's so that NS will equal present excitation

	PS $q_2\ q_1\ q_0$	NS $x=0$ $Q_2\ Q_1\ Q_0$	NS $x=1$ $Q_2\ Q_1\ Q_0$	output (z) $x=0$	output (z) $x=1$
(A)	0 0 0	0 0 0	0 0 1		
(B)	0 0 1	0 0 0	0 1 1		
(C)	0 1 1	0 1 0	0 0 0		
(D)	0 1 0	1 1 0	0 0 0		
(E)	1 1 0	0 0 0	1 1 1		
(F)	1 1 1	0 0 0	0 0 0	1	1
(G)	1 0 1	d	d		
(H)	1 0 0	d	d		

State assignment tends to get lots of attention in texts. Basic idea here is to simplify designs by, if feasible, making adjacent states (i.e. 1 bit different).

Since using D FF's, the excitations required are just = NS. So have 3 logic designs. Use K maps

$q_2q_1 \backslash q_0x$	00	01	11	10
00				
01	1			
11		1		
10	d	d	d	d

Q_2

$q_2q_1 \backslash q_0x$	00	01	11	10
00			1	
01	1			1
11		1		
10	d	d	d	d

Q_1

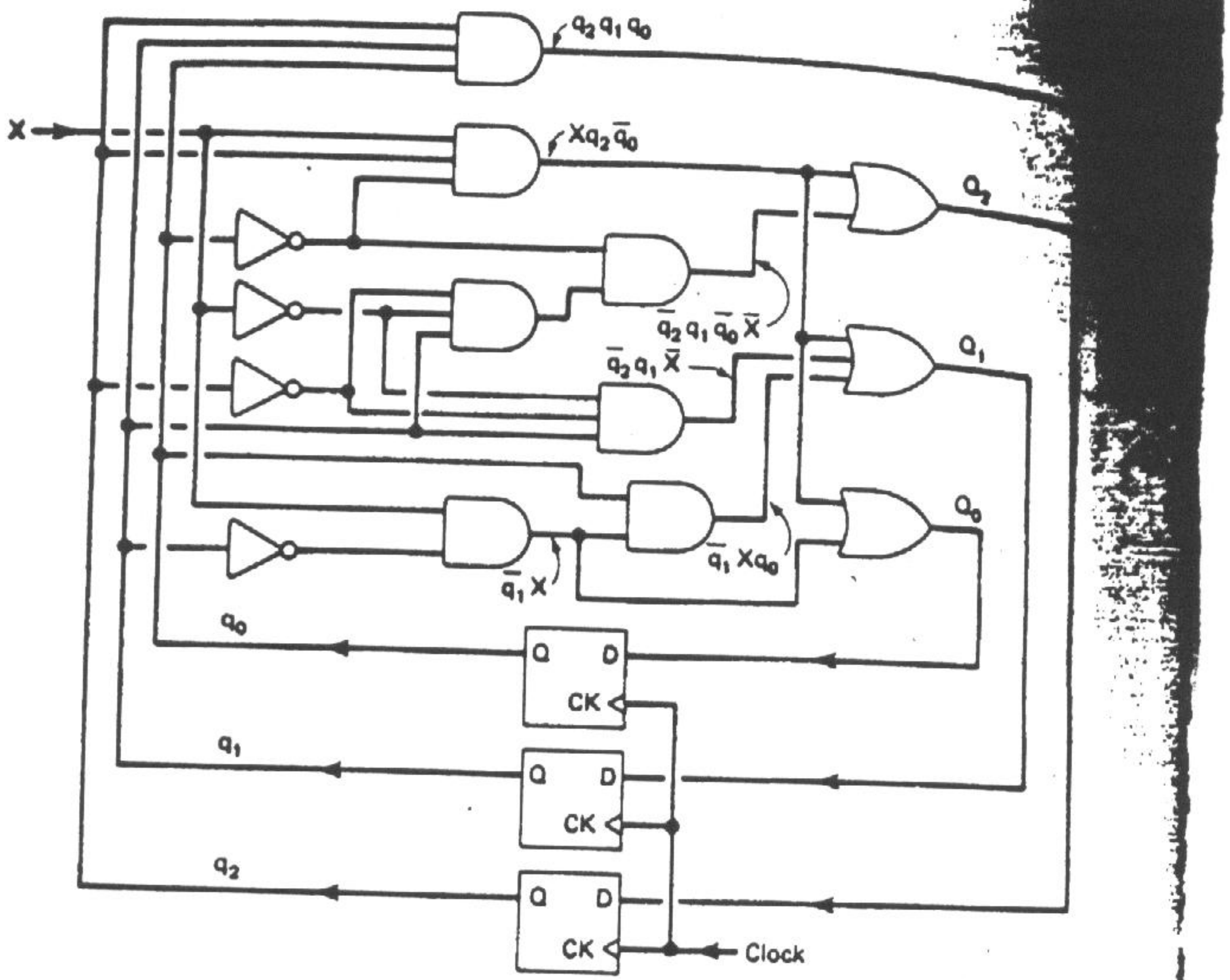
$Q_2 = \bar{q}_2 q_1 \bar{q}_0 \bar{x}$

$Q_1 = \bar{q}_2 q_1 \bar{x} + q_2 \bar{q}_0 x + \bar{q}_1 q_0 x$

As an exercise get $Q_0 = \bar{q}_1 x + q_2 \bar{q}_0 x$

Now have the recipe for the circuit.

More or less done but should see what happens if somehow land in unused states G, H. The Q eqns tell us what happens since they tell what the next state will be:



$q_2q_1q_0$
X
$Xq_2\bar{q}_0$
Q_2
$\bar{q}_2q_1\bar{q}_0\bar{X}$
$\bar{q}_2q_1\bar{X}$
Q_1
Q_0
$\bar{q}_1Xq_0$
$\bar{q}_1X$
q_0
Q
D
CK
q_1
q_2
Clock

11001

Assume a glitch puts us in state H with $q_2 q_1 q_0 = 100$. The Q eqns tell us what happens next:

<u>For</u> $x = {}^{0}_{1}$ $\quad Q_2 = \bar{q}_2 q_1 \bar{q}_0 x + q_2 \bar{q}_0 x = 001^{0}_{1} + 11^{0}_{1} = {}^{0}_{1}$

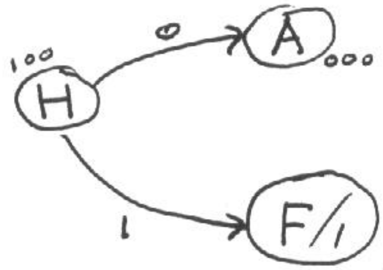
$$Q_1 = \bar{q}_2 q_1 \bar{x} + q_2 \bar{q}_0 x + \bar{q}_1 q_0 x = 0\,0^{\bar{x} \to 1}_{0} + 11^{0}_{1} + 10^{0}_{1} = {}^{0}_{1}$$

$$Q_0 = \bar{q}_1 x + q_2 \bar{q}_0 x = 1^{0}_{1} + 11^{0}_{1} = {}^{0}_{1}$$

so for $x = 0$ $\quad H \to NS \to 000 = A$

$\quad\quad\;\; = 1 \quad\quad\quad$ " $\quad 111 = F$

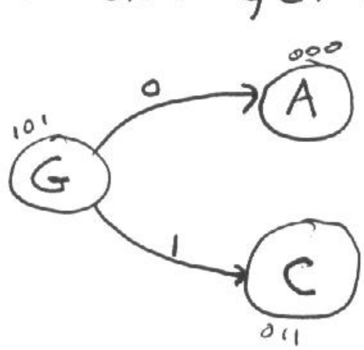
or



← note that going to F would output a 1 at first input of 11001

Note would get false output

As home exercise get when start in state G



Does this create any problem?