Analog Circuits

Review: some definitions + symbols used in EE -

time	t	sec	s
energy	E (or W)	joule	J
force	F	newton	N
power	P	watt	W
charge	Q (or q)	coulomb	C
current	I (or i)	ampere	A (or a)
electric potential (i.e. voltage)	V (or v)	volts	V
resistance	R	ohms	Ω
capacitance	C	farad	F
inductance	L	henry	H

Some unit prefixes:

atto	a	10^{-18}
femto	f	10^{-15}
pico	p	10^{-12}
nano	n	10^{-9}
micro	μ	10^{-6}
milli	m	10^{-3}
Kilo	K	10^{3}
mega	M	10^{6}
giga	G	10^{9}
terra	T	10^{12}

More basic definitions/quantities

Charge Q: in coulombs C

1 electron charge $= -1.6 \cdot 10^{-19}$ C

1 coulomb $= 6.25 \cdot 10^{18}$ electrons

-1 electron = charge on 1 proton

electron charge = smallest quantity normally observed in nature

Current I: in amperes

$I = dQ/dt$

1 amp = 1 C/sec past a given point

$= -6.25 \cdot 10^{18}$ electrons/sec

reference direction of current

$\xrightarrow{I}$ direction of conventional current flow

$\leftarrow \ominus \leftarrow$ direction of electron flow

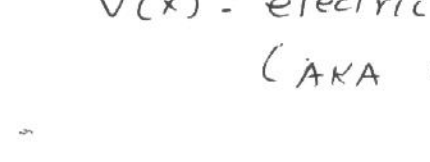
Why negative — Ben Franklin

Voltage: $E(x)$ = potential energy of a particle with charge Q located at point x. = joules = coulomb • volt

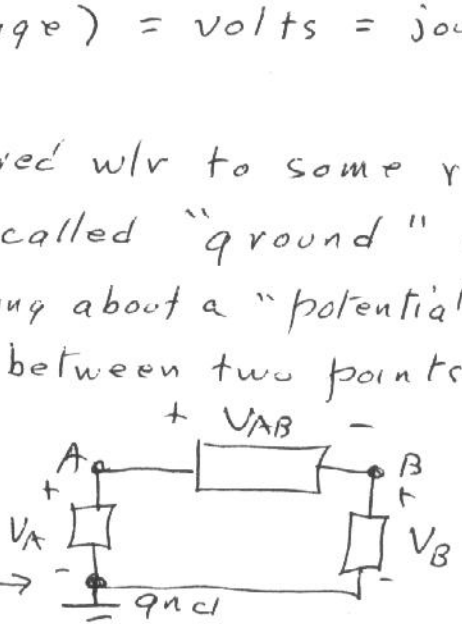
$V(x)$: electric potential at x $= E(x)/Q$

(AKA voltage) = volts = joule/coulomb

voltage always measured w/r to some reference level commonly called "ground". i.e. always really talking about a "potential difference" between two points



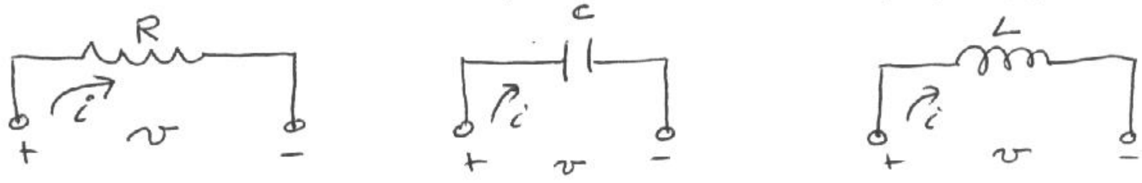
reference pt ($V=0$) →



Basic circuit elements - terminal relations are the relation(s) between the current thru and the voltage across the element. AKA "the elements' V-I characteristic".

Basic elements are:

Resistor | Capacitor | Inductor



$v = R\,i(t)$ (Ohm's Law)

$q = C \cdot v$

$i = \frac{dq}{dt} = C\frac{dv(t)}{dt}$

= open circuit for s-s dc circuits

$v = L\frac{di}{dt}$

= short ckt for s-s dc ckts

Resistor: dissipates energy (in form of heat). Note applies in general to time varying current. Unit is resist<u>ance</u> in ohms. Also note in inverted form:

$i(t) = G\,v(t)$ where constant is called conductance G, units are mho's (or siemens)

Note that R (or G) are constants (for linear ckts) which are independent of frequency.

Capacit<u>or</u>: stores energy in form of an electrostatic field. Constant is capacit<u>ance</u> in farads (1 farad is an enormous value). (coulombs/volt).

Inverted form is $q = C\,v \rightarrow i = C\frac{dv}{dt}$ and

$$v(t) = \frac{1}{C}\int_{-\infty}^{\infty} i(t)\,dt$$

$\frac{1}{C}$ called elastance with units daraf's

Inductor : stores energy in form of magnetic field
Unit of inductance = henry's (volt-sec/amp)
Inverted form is:

$$v(t) = L\frac{di(t)}{dt} \quad \text{or} \quad i(t) = \frac{1}{L}\int_{-\infty}^{\infty} v(t)\,dt$$

$\frac{1}{L}$ (called simply henry^{-1} or Γ (gamma) : not used much).

Note: two or more inductors which have magnetic coupling sometimes considered a basic circuit element

Power/energy

Power in general is product of voltage × current while energy is integral of power :

Resistor

$$P = v \cdot i = i \cdot R \cdot i = i^2 R \text{ or } \frac{v^2}{R} \quad \text{joules/sec or watts}$$

energy converted or dissipated as heat is

$$W = \int p\,dt = R\int_{-\infty}^{t} i^2 dt \qquad (\text{joules})$$

Inductor :

$$p = v \cdot i = \left(L\frac{di}{dt}\right)i = L \cdot i\frac{di}{dt}$$

Note that power supplied is zero unless current is changing. Hence energy remains constant unless current is changing

$$W_L = \int_{-\infty}^{t} v \cdot i\,dt = \int_{-\infty}^{t} L \cdot i \cdot \frac{di}{dt} \cdot dt = L\int_{i=0}^{i} i\,di = \frac{1}{2}L i^2$$

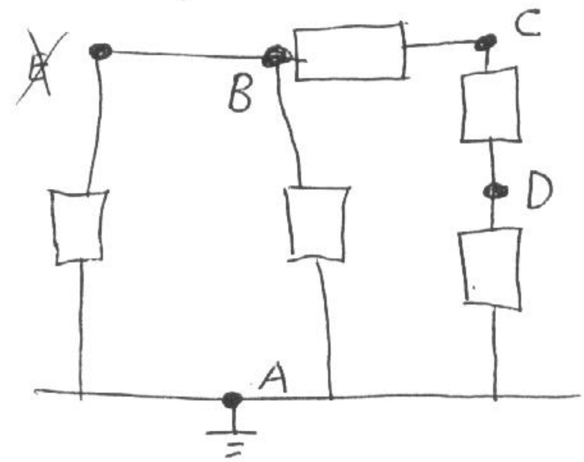
Hence energy stored at given time depends only only the current value at that time.

Circuits, and ckt analysis, consist of connections of circuit elements (the basic passive element - but more generally active devices such as transistors) plus excitation sources plus analysis "tools" i.e. math, theorems, procedures

Note that type of excitation involved dictates what Kind of analysis is appropriate + what tools are needed. Progression is from "dc" ckts, to "ac" ckts, to more general ckts.

Branches = a circuit element (together with its end pts).
Nodes = place where two or more circuit elements connect
Wires are used for connections and are perfect conductors so that all points along the wire are at the same voltage

e.g.



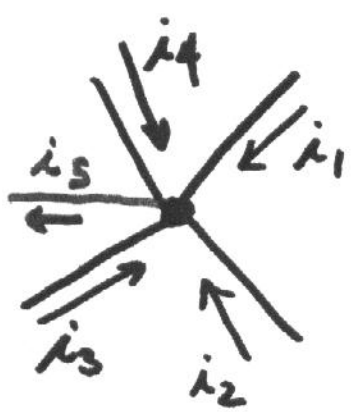
4 nodes A, B, C, D.
(pt E is same as B)

5 branches

Kirchoff's Current Law (KCL)

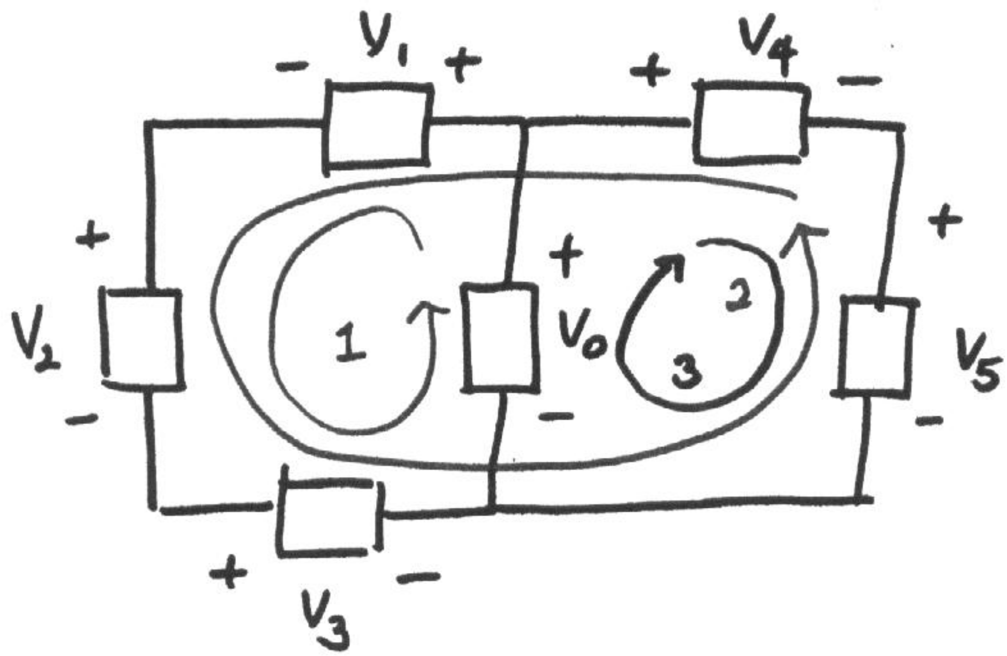
- Sum of currents into a node is always 0
- Conservation of electric charge

$$\sum i = 0 = i_1 + i_2 + i_3 + i_4 - i_5$$



Kirchoff's Voltage Law (KVL)

- sum of voltages around a loop is always 0
- electric potential (voltage) is conserved (independent of path)



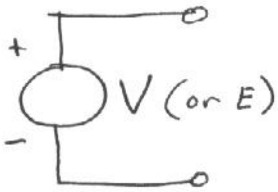
$-V_1 + -V_2 + -V_3 + V_0 = 0$ loop 1

$V_4 + -V_1 + -V_2 + -V_3 + V_5 = 0$ loop 2

$-V_4 - V_5 + V_0 = 0$

Sources: (ideal + practical)

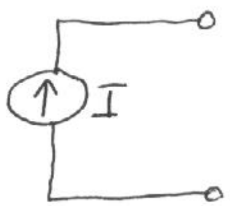
Ideal voltage source model: maintains prescribed voltage between its terminals regardless of load.



If a constant (i.e. a battery) have dc

If time varying, $E(t)$, have ac (typically sinusoidal).

Ideal current source model: maintain's prescribed current thru its terminals regardless of load.

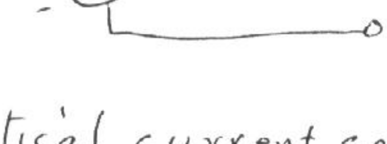


Little thought shows ideal sources must have ∞ power capability by their definition, e.g.

P in resistor is $\frac{V^2}{R}$ and as $R \to 0$, $P \to \infty$.
(short circuit)

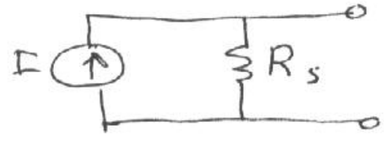
Practical sources include a resistor to limit the short ckt current or open ckt voltage.

Practical voltage source



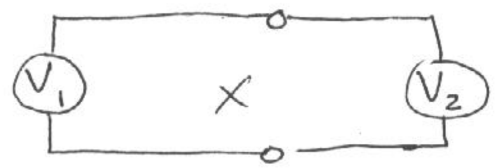
R_s = source resistance

Practical current source



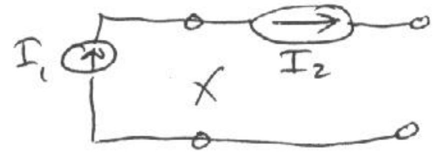
Contradictory connections of sources not allowed

e.g.



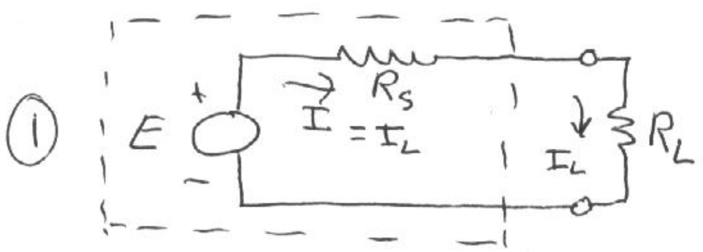
$V_1 \neq V_2$

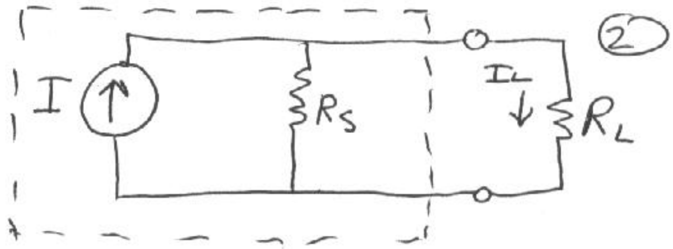
or



$I_1 \neq I_2$

<u>Source transformation</u>: either practical source may be transformed to the other – load does not know or care





$$I_L = I = \frac{E}{R_S + R_L}$$

$$I_L = I \cdot \frac{R_S}{R_S + R_L}$$ (prove later)

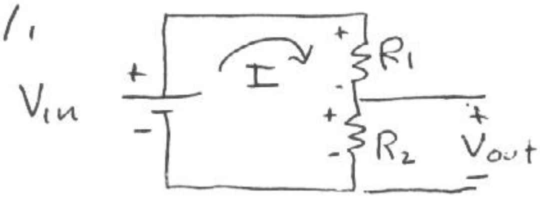
for same I_L set $E = I R_S$

Hence if have ① can transform to ② using $I = E/R_S$

" " " ② " " " ① " $E = I \cdot R_S$

Divider Rules

1.



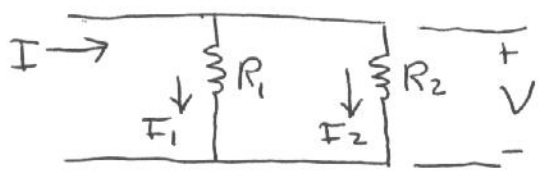
$$V_{in} = R_1 I + R_2 I$$

$$\text{or } I = \frac{V_{in}}{R_1 + R_2}$$

$$\text{so } \underline{\underline{V_{out}}} = R_2 \cdot I = V_{in} \underbrace{\frac{R_2}{R_1 + R_2}}_{}$$

voltage divider rule

2.



$$I_1 = \frac{V}{R_1} \qquad I_2 = \frac{V}{R_2}$$

$$I = I_1 + I_2$$

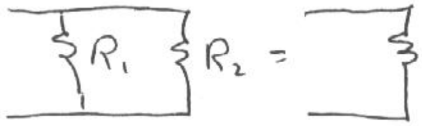
$$I = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\text{and } \underline{\underline{I_2}} = \frac{V}{R_2} = \frac{1}{R_2} \cdot I \cdot \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = I \cdot \frac{1}{1 + \frac{R_2}{R_1}} = I \cdot \underbrace{\frac{R_1}{R_1 + R_2}}_{}$$

current divider rule

Note that $\frac{1}{R_1} + \frac{1}{R_2} \equiv \frac{1}{R_{eq}}$ so $I = \frac{V}{R_{eq}}$

special, common, case where just have 2 resistors in parallel gives

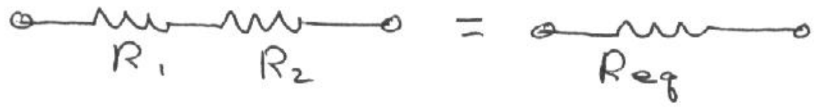


$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

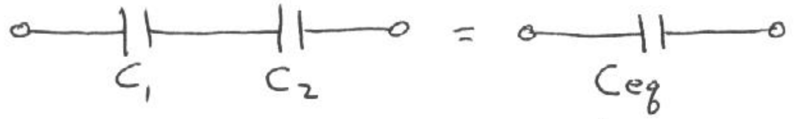
Along the way have seen that a number of the same elements in series or in parallel can be combined into one equivalent element.

Combinations are:

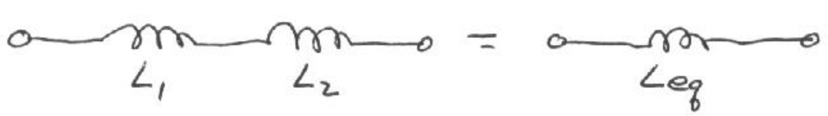
1. Series connections:



$$R_{eq} = R_1 + R_2 + \cdots$$



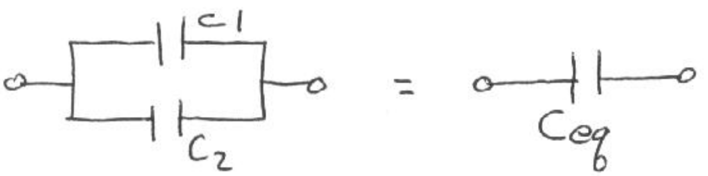
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots$$



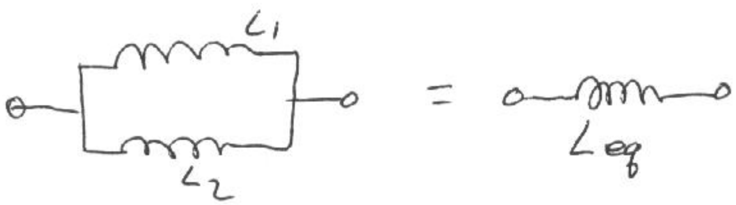
$$L_{eq} = L_1 + L_2 + \cdots$$

2. Parallel connections:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots$$

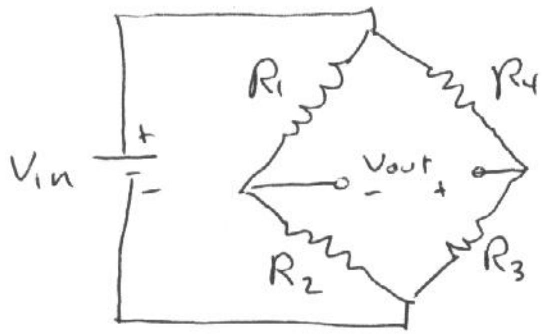


$$C_{eq} = C_1 + C_2 + \cdots$$



$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}$$

Simple ckts often can be treated with just the divider rules: e.g. get balance (out = 0) relation for a Wheatstone Bridge:



$$V_{out} = \frac{R_3}{R_3+R_4} V_{in} - \frac{R_2}{R_1+R_2} V_{in}$$
$$= \left(\frac{R_3}{R_3+R_4} - \frac{R_2}{R_1+R_2} \right) V_{in}$$

for balance: $\frac{R_3}{R_3+R_4} = \frac{R_2}{R_1+R_2}$ or $\frac{1}{1+\frac{R_4}{R_3}} = \frac{1}{1+\frac{R_1}{R_2}}$

or $\frac{R_4}{R_3} = \frac{R_1}{R_2}$ or $R_1 R_3 = R_2 R_4$ for balance

While use of series and parallel branch reductions plus divider rules can solve many circuits there is of course need for more formal analysis procedures. Note in above what happens if just put resistor across output and asked what in general is the current thru it.

The two general procedures are called mesh analysis and nodal analysis. In both cases they are set up to reduce the # of equations which would result if you simply used Kirchoff's laws and the terminal relations for the elements,