Frequency response curves:

AKA Bode Diagrams or Log Modulus Plots

Commonly have stages in cascade and frequencies which have large dynamic ranges so makes sense to make use of log plots.

Typical form of multistage networks is

$$T = T_0 \cdot T_1 \cdot T_2 \cdot T_3 \cdots \qquad T = |T|\epsilon^{j\phi} \text{ or } |T|\angle\phi$$

T_0 ↑ constant

so

$$T = |T|\angle\phi = (|T_1|\angle\phi_1)\cdot(|T_2|\angle\phi_2)\cdots$$

take logs:

$$\log|T| = \log|T_1| + \log|T_2| + \cdots$$
$$\phi = \phi_1 + \phi_2 + \cdots$$

more common to express magnitudes in db (decibels) where a number N in decibels is $20\log_{10} N$

so $|T|_{db} = |T_1|_{db} + |T_2|_{db} + \cdots$

Forms which are common are: where $n = \pm$ integer and

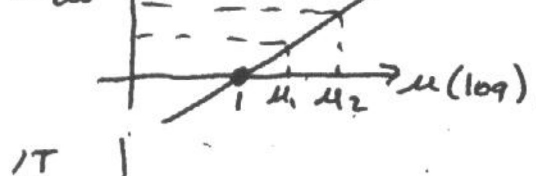
υ = normalized frequency $= \omega/\omega_n$ (not same n)

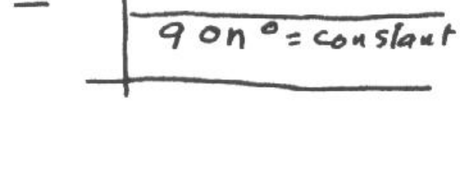
$$(j\upsilon)^n \qquad (1+j\upsilon)^n \text{ and } (1+2\zeta j\upsilon+(j\upsilon)^2)^n$$

(second order – for advanced course)

Bode diagram technique is to approximate these factors by means of two asymptotes – low freq end and hi freq end.

1. factor $T = (j\upsilon)^n$





$|T|_{db} = 20n\log_{10}\upsilon$ = straight line on $\log\upsilon$ scale

note $|T|_{db} = 0$ for $\upsilon = 1$

slope $|T|_{\upsilon_2} - |T|_{\upsilon_1} = 20n\log\upsilon_2 - 20n\log\upsilon_1$
$= 20n\log(\upsilon_2/\upsilon_1)$

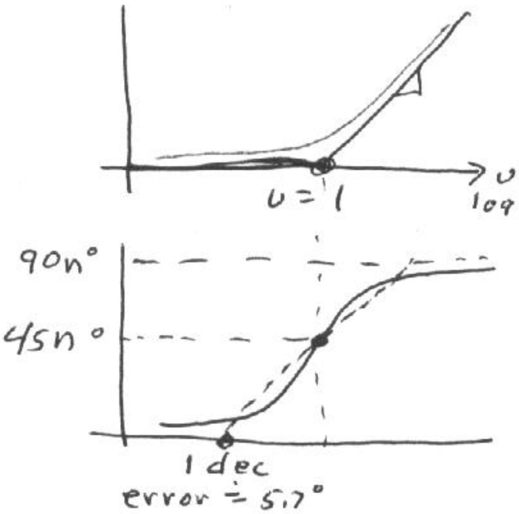
for $\upsilon_2 = 2\upsilon_1$ $(\upsilon_1 = \tfrac{1}{2}\upsilon_2)$ freqs are 1 octave apart

Choosing freqs 1 octave apart leads to slope

$|T|_{u_2} - |T|_{u_1} = 20n \log(u_2/u_1) = 20 n \log 2$

or slope is 6 n db/oct or 20 n db/decade

2. factor $T = (1 + ju)^n$



$|T|_{db} = 20 n \log \sqrt{1 + u^2}$

log freq approx $|T|_{db \atop lo\,freq} = 20 n \log 1 = 0$ db

hi freq " $|T|_{db \atop hi\,freq} = 20 n \log u$

= same form as factor 1 so

slope is 6 n db/oct

intersection at $20 n \log 1 = 20 n \log u$

or $u = 1$ = called

the "corner" freq or the "break" freq

Corrections:

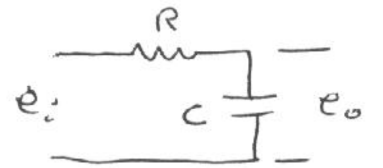
Actual curve differs from asymptotes slighty. Can be gotten if desired by applying following corrections:

$u = 1$ at corner freq: correction is 3 n db (add)

$u = 2, 1/2$ 1 octave above or below " " 1 n db

etc.

Ex:

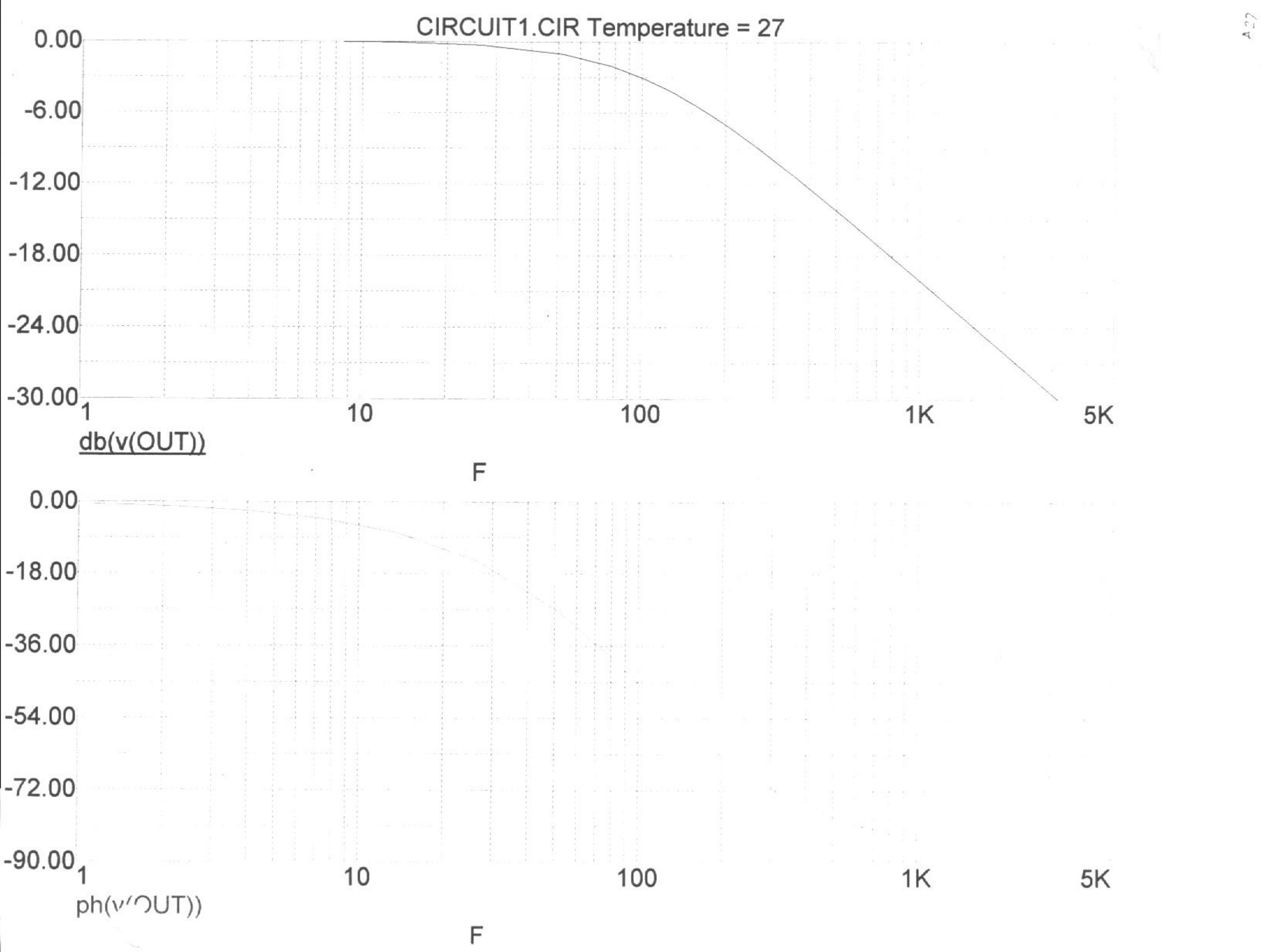


$$\frac{e_o}{e_i}(j\omega) = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC}$$

$$= \frac{1}{1 + j\left(\frac{\omega}{1/RC}\right)}$$

home exercise:

show for $R = 16K$, $C = .1\mu$ corner freq ≐ 100 hz


CIRCUIT1.CIR Temperature = 27
0.00
-6.00
-12.00
-18.00
-24.00
-30.00
1
10
100
1K
5K
db(v(OUT))
F
0.00
-18.00
-36.00
-54.00
-72.00
-90.00
1
10
100
1K
5K
ph(v(OUT))
F

Some useful network theorems:

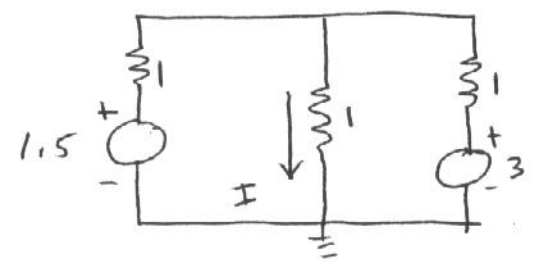
<u>Linear</u> systems - response increases by factor K if excitation increased by factor K.

<u>Super position</u>: more than one excitation

1. Take source #1, reduce all other independent sources to zero and calculate response to source #1.
2. Repeat for each source
3. Sum responses

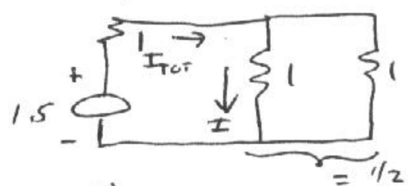
Note reducing an independent source to zero means replacing it with its source resistance. (Ideal V source becomes short ckt, ideal curent source becomes an open ckt.)

Simple example:



Want I.

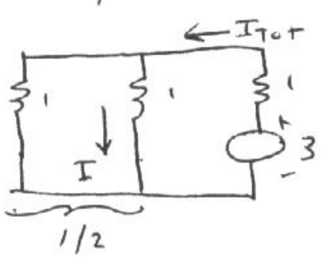
1. Get part due to 1.5 v source



$$I_{TOT}^{(1)} = \frac{1.5}{1+.5} = 1 \text{ amp}$$

so $I^{(1)} = 1/2$ amp

2. Get part due to 3.0 source



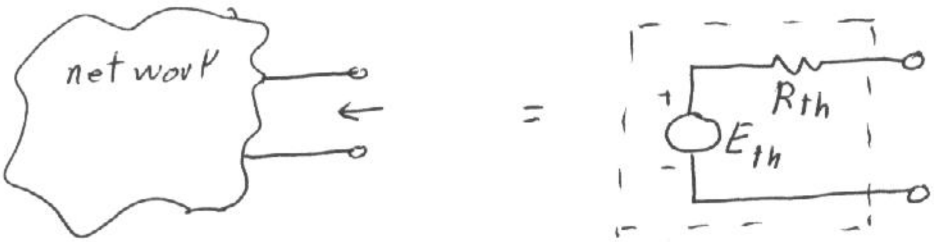
$$I_{TOT}^{(2)} = \frac{3}{1+.5} = 2 \text{ amp}$$

$$I^{(2)} = 1 \text{ amp}$$

so $I = I^{(1)} + I^{(2)} = \frac{1}{2} + 1 = 1.5a$

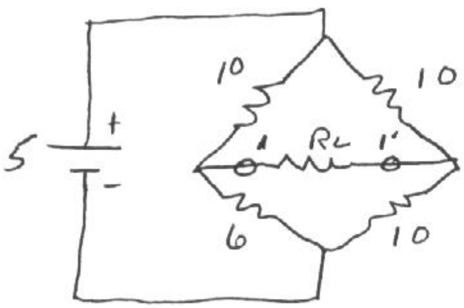
Thevinen's theorem

Any connected network composed of linear circuit elements and sources can be replaced, when viewed at a particular pair of terminals, by a voltage source in series with a source resistance (impedance in general). The impedance is the net impedance seen looking into the terminals when all independent sources have been reduced to zero. The voltage source is the voltage across the terminals when they are left open circuited.

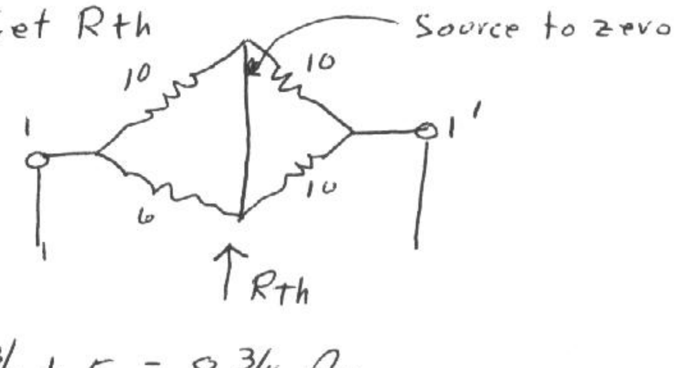


Thevinen equ. for Wheatstone Bridge:

What does load R_L see?



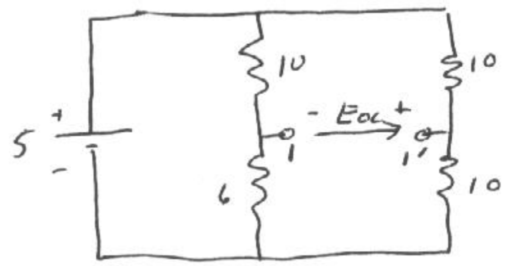
1. Get R_{th}



$$R_{Th} = 6 \| 10 + 10 \| 10 = 3\tfrac{3}{4} + 5 = 8\tfrac{3}{4}\ \Omega$$

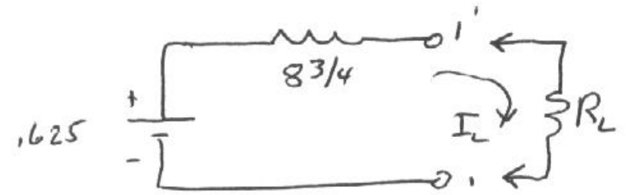
Now get E_{th}

To get $E_{th} = E_{oc}$



$$E_{oc} = \frac{10}{20} \cdot 5 - \frac{6}{16} \cdot 5 = .625 \text{ v}$$

so R_L sees:



hence if e.g. $R_L = 1\frac{1}{4}\,\Omega$

then

$$I_L = \frac{.625}{8\frac{3}{4} + 1\frac{1}{4}} = .0625 \text{ a}$$

Alternate form of Thevinen ckt

Can make a source transformation:

Note $I_{th} = \dfrac{V_{oc}}{R_{th}} = I_{sc}$ = current that would

flow thru a short ckt on the th. equi.
Sometimes easier to get V_{oc} and I_{sc} to
get equ ckt.

This is often called Norton's Theorem (and
sometimes Mayer's Theorem)

The Δ-Y (delta-Wye) transformation:

In previous example of R_{th} for bridge, if the source had a finite R_s then would have problem getting R_{th}. Would have had – not a simple series or // situation:

← Have a "delta" connection

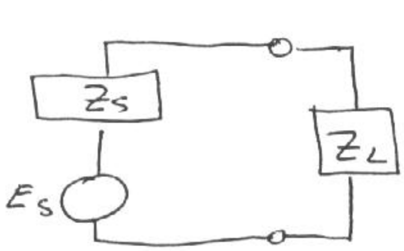
Special relation can be derived (see text)

typical part of relations: $R_a = \dfrac{R_{ab} R_{ca}}{R_{ab} + R_{bc} + R_{ca}}$ and $R_{ab} = \dfrac{R_a R_b + R_b R_c + R_a R_c}{R_c}$

etc for others

Needn't remember relations – just that the transformation exists and look up if needed.

Max power transfer Theorem



For max power transfer to load

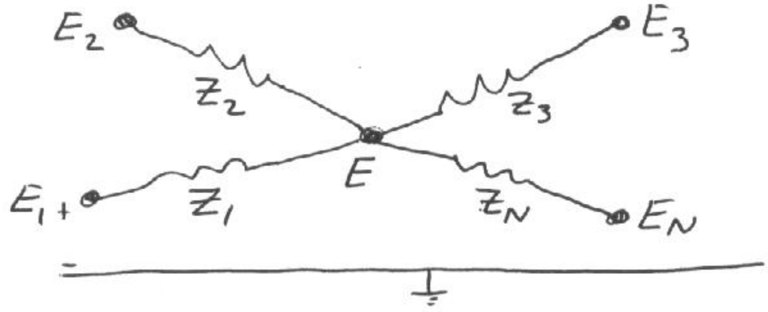
Resistive case: $R_L = R_s$

General case: $Z_L = Z_s^*$

$*$ = complex conjugate

Millman's Theorem

A common situation in many ckts is:



A number of branches are connected at a node whose voltage is "E" (all voltages w/r to gnd). The other end of these branches are at voltages shown. Want an expression for E.

Write KCL at the node:

$$\frac{E_1 - E}{Z_1} + \frac{E_2 - E}{Z_2} + \cdots + \frac{E_N - E}{Z_N} = 0$$

or
$$\frac{E_1}{Z_1} + \frac{E_2}{Z_2} + \cdots + \frac{E_N}{Z_N} = E\left(\frac{1}{Z_1} + \frac{1}{Z_2} + \cdots + \frac{1}{Z_N}\right)$$

using Y (=admittance) $= \frac{1}{Z}$ (Z = impedance)

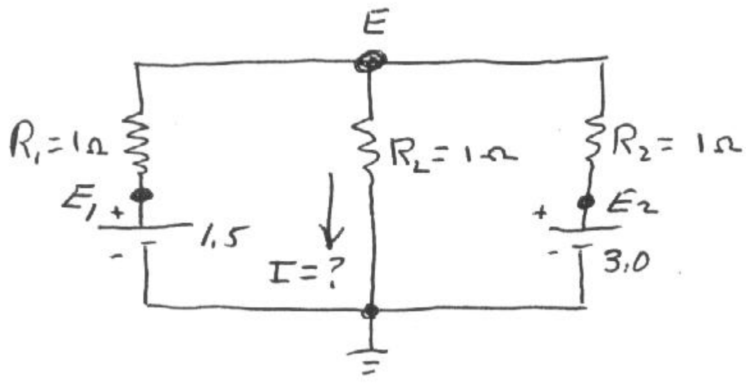
get
$$E = \frac{E_1 Y_1 + E_2 Y_2 + \cdots + E_N Y_N}{Y_1 + Y_2 + \cdots + Y_N}$$

= Millman's theorem

in practice many of the E_i are zero.

simple example: (previous superposition example:)

Millman's theorem - example



$I = \frac{E}{R_L}$ get E

$$E = \frac{E_1 Y_1 + E_2 Y_2}{Y_1 + Y_2} = \frac{1{,}5\left(\frac{1}{1}\right) + 3\left(\frac{1}{1}\right)}{\frac{1}{1} + \frac{1}{1}}$$

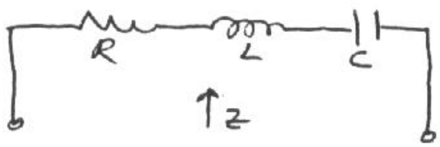
$$= \frac{1{,}5 + 3}{1 + 1} = \frac{4.5}{3} = 1{,}5\ v$$

so $I = \frac{E}{R_L} = \frac{1.5}{1} = 1{,}5\ a$

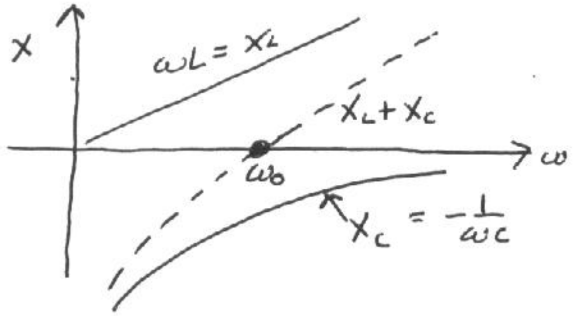
Resonant CKts

Series resonance:

Consider



$$Z(\omega) = R + j\omega L + \frac{1}{j\omega C}$$



$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$Z = R + j\left(X_L + X_c\right)$$

(X = reactance)

Lo freq ⇒ Z is capacitive
hi freq ⇒ Z is inductive

at a particular freq. $X_L + X_c = 0$ so that Z is just R, i.e. Z is purely resistive. This condition is termed "series resonance" and occurs at

$$\omega_0 L = \frac{1}{\omega_0 C} \quad \text{or} \quad \omega_0^2 = \frac{1}{LC} \quad \text{or} \quad \omega_0 = \frac{1}{\sqrt{LC}} = 2\pi f_0$$

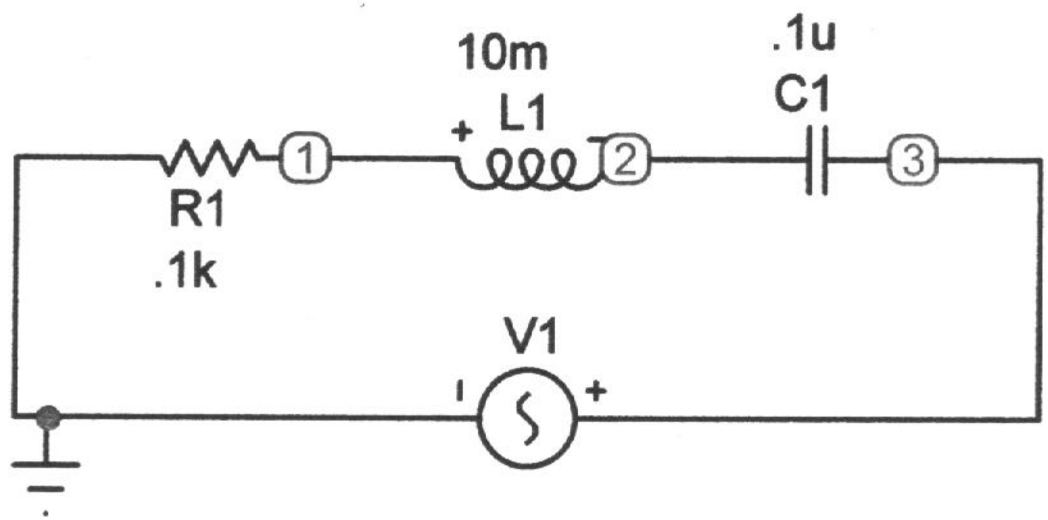
At series resonance impedance is a minimum (so current is a maximum (= $V_{applied}/R$). Note that applied voltage and resulting line current are <u>in phase</u>.

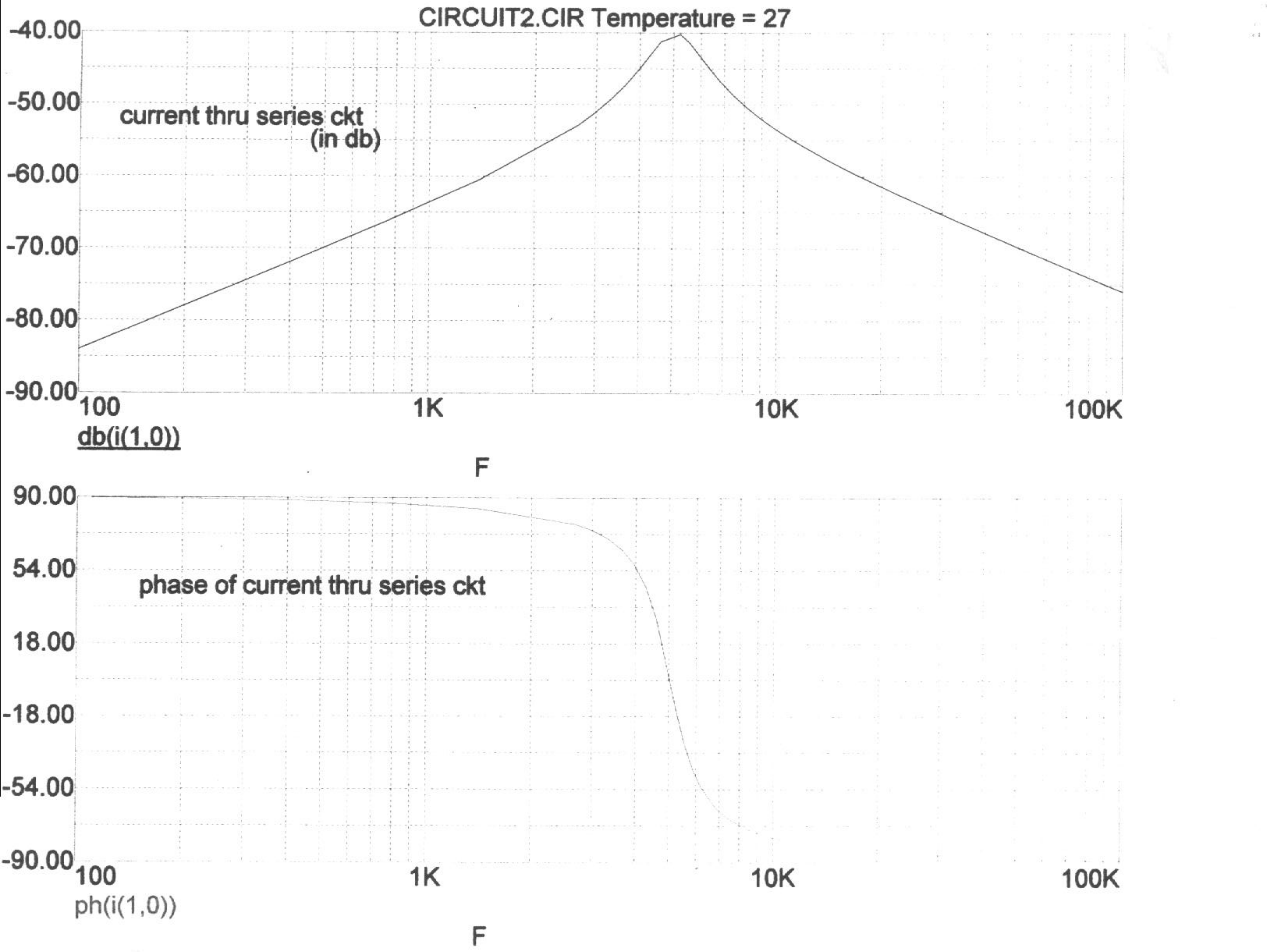
At resonance: voltage across L is $Z_L \cdot I = (j\omega_0 L)(V/R)$

$$V_L = j\left(\frac{\omega_0 L}{R}\right)V = j\,Q_s \cdot V \quad \text{where}$$

$$Q_s = \frac{\omega_0 L}{R} = \text{"quality factor"}$$

so at res. voltage across L is $90°$ ahead of V and can be large (in magnitude) series resonant ckt





CIRCUIT2.CIR Temperature = 27
-40.00
-50.00
-60.00
-70.00
-80.00
-90.00
current thru series ckt
(in db)
100
1K
10K
100K
db(i(1,0))
F
90.00
54.00
18.00
-18.00
-54.00
-90.00
phase of current thru series ckt
100
1K
10K
100K
ph(i(1,0))
F

Since at res $\omega_o L = \frac{1}{\omega_o C}$ then at resonance the voltage across C is $(1/j\omega_o C)\frac{V}{R}$ and has the same magnitude as V_L but is 90° behind V.

Summary for series resonance:

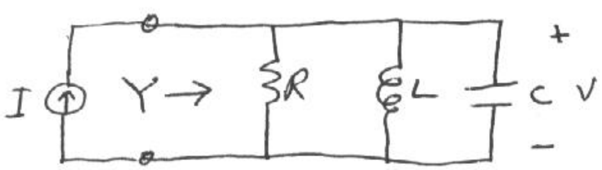
$$\omega_o = \frac{1}{\sqrt{LC}} \qquad Z = \min = R \qquad I = \max = V/R$$

$$Q_s = \frac{\omega_o L}{R} = \frac{1}{\sqrt{LC}} \cdot \frac{L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$V_L = j\,Q_s\,V \qquad V_c = -j\,Q_s\,V$$

Parallel resonance (tank ckt)

Dual of series case.



admittance Y (= 1/z) is

$$Y = \frac{1}{R} + \frac{1}{j\omega L} + j\omega C$$

$$= \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

Usual definition for resonance is for line voltage and current to be in phase i.e. Y purely resistive so freq for ∥ resonance is

$$\omega_o C = \frac{1}{\omega_o L} \quad \text{or} \quad \omega_o^2 = \frac{1}{LC} \quad \text{(same as for series case)}$$

At res voltage across tank is $V = IR$ and current thru L will be $I_L = \frac{V}{j\omega_o L} = \frac{IR}{j\omega_o L} = \frac{I}{j}\frac{R}{\omega_o L} \equiv -j\,Q_p\,I$

Q_p = quality factor $= \frac{R}{\omega_o L} = \omega_o C R$

Summary for tank ckt @ resonance

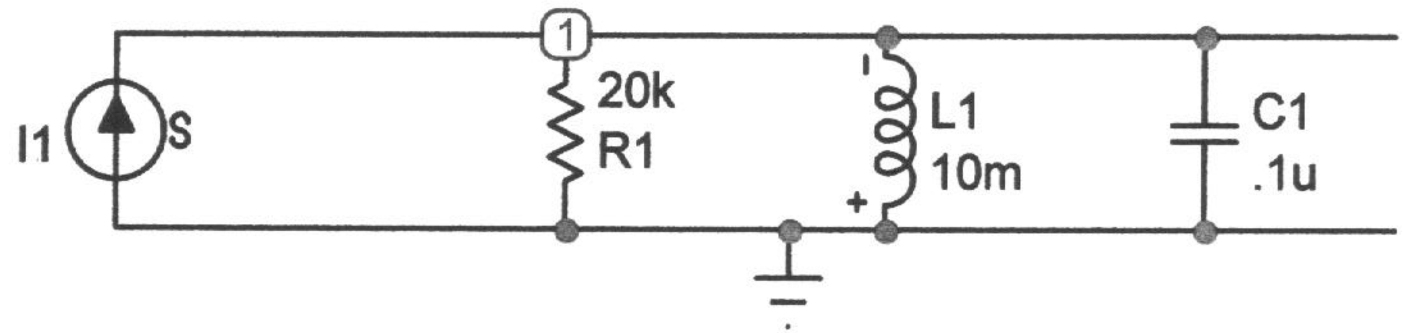
$\omega_0^2 = 1/LC \Rightarrow$ I and V in phase

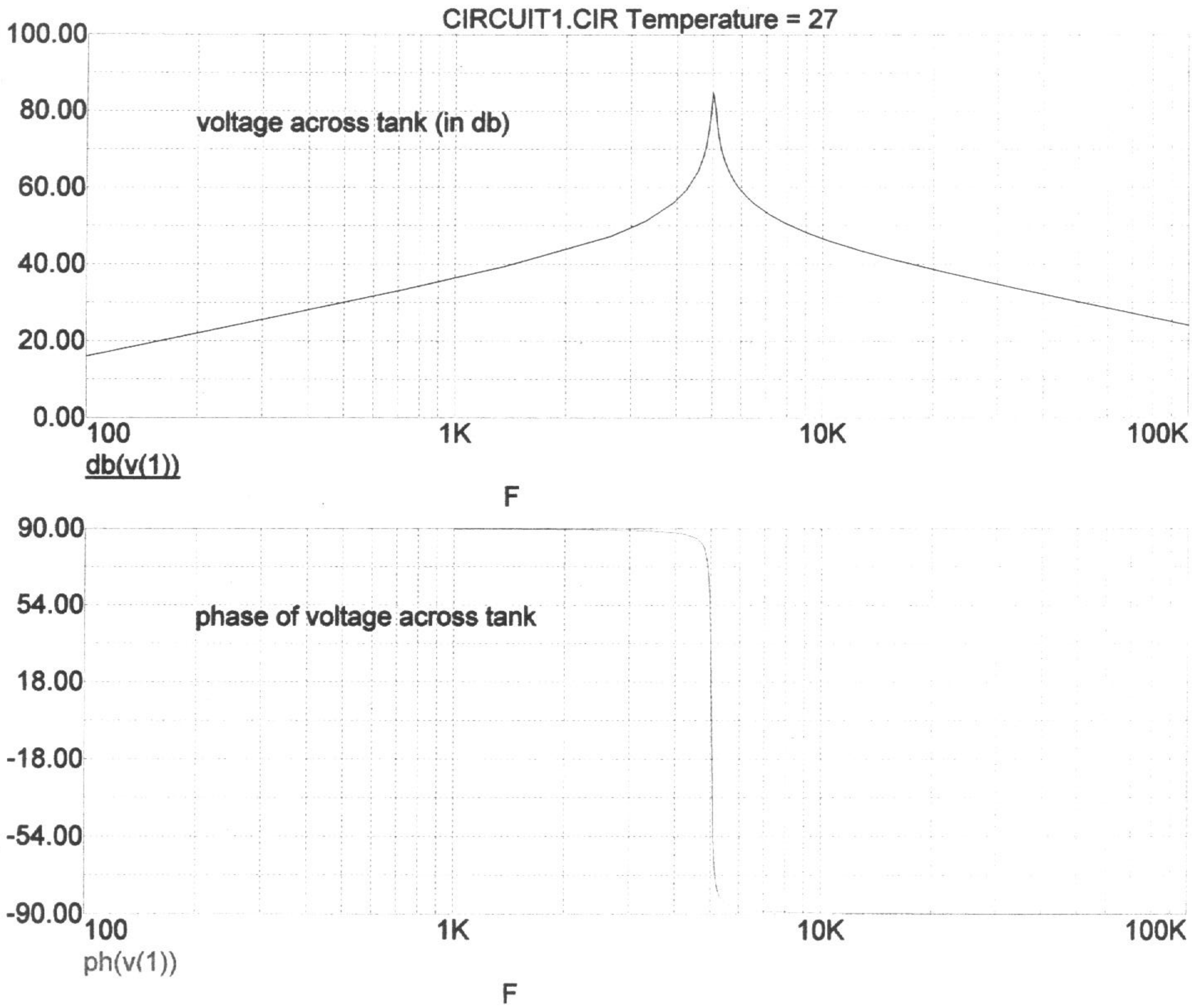
$Z = R = \text{max}$, $V = \text{max} = I \cdot R$

$$Q_P = \frac{R}{\omega_0 L} = \omega_0 C R = CR \frac{1}{\sqrt{LC}} = R\sqrt{\frac{C}{L}}$$

$I_L = -j\, Q_P\, I$ and $I_C = j\, Q_P\, I$

parallel resonant circuit (tank circuit)




CIRCUIT1.CIR Temperature = 27
100.00
80.00
60.00
40.00
20.00
0.00
voltage across tank (in db)
100
1K
10K
100K
db(v(1))
F
90.00
54.00
18.00
-18.00
-54.00
-90.00
phase of voltage across tank
100
1K
10K
100K
ph(v(1))
F